REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS

67[A].—J. B. Reid & G. Montpetit, *Table of Factorials 0! to 9999!*, Publication 1039, National Academy of Sciences-National Research Council, Washington, D. C., 1962, 47p. (unnumbered), 28 cm. Price $1.50.

This table gives $n!$ in floating-point form to 10S for $n = 0(1)9999$. The exact values of $n!$ may be inferred through $n = 15$; beyond that point all the entries are rounded, approximate numbers.

We are informed in the introduction that this table was prepared in response to a request for appropriate data for the calculation of the hypergeometric distribution involving numbers exceeding 1000. For numbers not exceeding this limit, tables of factorials to 16S by Salzer [1] and 20S by Reitwiesner [2] are available. The authors, however, make no reference to these tables or any others in this field.

The calculations were carried out to 20S on an IBM 650 system, working with 30-digit products prior to truncation. The printing was done on an IBM 407 accounting machine, and no checks were applied to the printed output. On the other hand, the IBM 650 calculations were performed twice; the first time, without rounding to 20S; and the second time, adding a unit in that place prior to dropping the subsequent figures. The two values of 9999! thus obtained are displayed to 20S, and agreement to 15S is noted. This reviewer has independently computed this factorial to more than 25S; his result differs by less than 9 units in the twentieth place from the average of the two values discussed by the authors. Accordingly, the tabulated results should be reliable, except for the possibility of misprints, as the authors state.

The reviewer has compared the first thousand entries in this table with corresponding values in the table of Salzer, and has detected no discrepancies.

With respect to approximate values of the factorial function corresponding to integer values of the argument exceeding 1000, the present table constitutes a unique and valuable contribution to the literature of mathematical tables.

J. W. W.

1. H. E. Salzer, *Tables of n! and $\Gamma(n + \frac{1}{2})$ for the First Thousand Values of n*, National Bureau of Standards, AMS 16, Washington, 1951. (*MTAC*, v. 6, 1952, p. 33, RMT **957**).

2. G. W. Reitwiesner, *A Table of the Factorial Numbers and their Reciprocals from 1! through 1000! to 20 Significant Digits*, Ballistic Research Laboratories, Technical Note No. 381, Aberdeen Proving Ground, Md., 1951. (*MTAC*, v. 6, 1952, p. 32, RMT **955**.)

68[A, B].—H. T. Davis & Vera J. Fisher, *Tables of the Mathematical Functions: Arithmetical Tables*, Volume III, Principia Press, San Antonio, Texas, 1962, ix + 554 p., 25.5 cm. Price $8.75.

The "Volume III" in the title has reference to two well-known volumes [1] of Professor Davis published long ago. The present volume is of different character, however, as is indicated in the deletion of the word "higher" from the title. Its main bulk is in the following twelve tables, mostly of powers and roots:

Table 1 gives constants associated with π, e, γ, and certain roots and logarithms to a precision varying from 10D to 45D.

Table 2 lists n^2 and n^3 for $n = 1(1)10{,}000$.

Table 3 gives values of n^{-1} to 15D for $n = 1(1)100$ and to 17D for $n = 100(1)1000$. The second-order central difference is included.

Table 4 lists n^{-1} to 10S for $n = 1000(1)10{,}000$, together with first differences.

Table 5 contains values of m/n for $n = 2(1)101$ and $m < n$ to 10D.

Table 6 lists $x^{1/2}$ and $(10x)^{1/2}$ to 12D for $x = 1(1)1000$ with first differences.

Table 7 gives the same quantities to 6D for $x = 1000(1)10{,}000$.

Table 8 consists of $x^{1/3}$, $(10x)^{1/3}$, and $(100x)^{1/3}$ to 12D for $x = 1(1)1000$ with first differences.

Table 9 lists $x^{1/3}$ to 6D for $x = 1000(1)10{,}000$ with first differences.

Table 10 contains $x^{\pm 3/2}$ to 10D with first differences.

Table 11 shows the binomial coefficients ${}_nC_r = n!/r!(n - r)!$ for $n = 4(1)100$. Also given are ${}_{-n}C_r$ for $n = 1(1)10$ and ${}_nC_r$ for 22 fractions $n = \pm k/l$ with $l = 2(1)6$, k less than and prime to l.

Table 12 lists x^n for $n = 2(1)12$ and $x = 1(1)100$.

The authors do not indicate which of these tables have been newly computed and which have been compiled from previous publications. Tables 2, 7, and 9, which comprise more than one-half of these pages, are contained in the well-known *Barlow's Tables* [2], but the format here is more generous. Table 10, and perhaps Tables 8 and 3, appears to be original. The remaining tables can be found in various books, some of which, however, are long out of print.

The tables are preceded by a 96-page text in four chapters: "The Arithmetic Functions," "Mathematical Constants," "The Solution of Equations," and "Transcendental Equations".

The second chapter contains histories of π, e, γ, $\log_e \pi$, $\log_{10} e$, and some other numbers. These histories extend to about 1950 only; the NORC value of π (1955), Wheeler's value of e (1953), Wrench's value of γ (1952) and later values are not mentioned. The history of π ends with "the ENIAC computed π to the fantastic approximation of 2035 decimal places." In view of a more recent computation [3], the undersigned is reminded of an old quotation. In his first publication concerning π (1853), William Shanks remarks [4]:

"... Previous to 1831, the value of π, as the late Professor Thomson of the University of Glasgow writes, in his work on the Differential and Integral Calculus, had been calculated 'to the extraordinary extent of 140 figures!' We may here be permitted to indulge a smile at the learned writer's words, now that the ratio has been found to 607 places of decimals!"

In the third chapter two tables of $z^3 - z$ are included to facilitate the solution of cubic equations. These are for $z = 1.0000(.0001)1.2009$ to 10D and $z = 0.00(.01)10.50$ to 6D.

Several places in the text the designation "Encyclopedia" is given to the present volume together with the two previous ones [1]. But the planned extent of the "Encyclopedia" is not revealed.

For the subject matter of this third volume the reader is also referred to the following review.

D. S.

1. Harold T. Davis, *Tables of the Higher Mathematical Functions*, Vol. I and Vol. II, Principia Press, Bloomington, Indiana, 1933 and 1935.

2. L. J. Comrie, editor, *Barlow's Tables*, 4th edition, Chemical Publishing Co., New York, 1941.
3. Daniel Shanks & John W. Wrench, Jr. "Calculation of π to 100,000 Decimals," *Math. Comp.*, v. 16, 1962, p. 76–99.
4. William Shanks, *Contributions to Mathematics, comprising chiefly the Rectification of the Circle to 607 places of decimals*, G. Bell, London, 1853.

69[A, B].—C. B. Bailey & G. E. Reis, *Tables of Roots of the First Ten Thousand Integers*, Sandia Corporation Monograph, SCR-501, January 1963, 237 p., 28 cm. Price $3.00. Available from the Office of Technical Services, Department of Commerce, Washington 25, D. C.

Table I lists $N^{1/2}$, $(10N)^{1/2}$, $N^{1/3}$, $(10N)^{1/3}$, $(100N)^{1/3}$, $N^{1/4}$, $(10N)^{1/4}$, $(100N)^{1/4}$, and $(1000N)^{1/4}$ for $N = 1(1)\ 10{,}000$ to 9D.

Table II lists $N^{1/k}$ for $k = 2(1)\ 10$ and $N = 1(1)1000$ to 9D.

Table III lists $x^{1/k}$ for $k = 2(1)10$ to 11D for 48 values of x such as π, π^{-1}, e, γ, $\pi^{1/e}$, $e^{1/\pi}$, etc. In striving for symmetrical completeness, $x = \log_e 10$ and $x = (\log_{10} e)^{-1}$ are both given. Luckily, the values in these two lists coincide.

The tables were computed on a CDC 1604 with a double-precision Newton-Raphson iteration, starting from a single-precision Fortran approximation. All values were carefully rounded (in decimal). The values listed were very carefully checked in two different ways. The format is very good.

The authors are to be commended for their conscientious effort. We have seen so many machine-made tables in the past years with poor error control, mediocre format, etc., that a carefully produced table draws attention to itself at once, as the sort of thing possible if the necessary care is taken.

D. S.

70[A–E, J, M].—Robert D. Carmichael & Edwin R. Smith, *Mathematical Tables and Formulas*, Dover Publications, Inc., New York, 1962, viii + 269 p., 21.5 cm. Price $1.00.

As explicitly stated by the publisher, this is an unabridged, unaltered, paperback edition of mathematical tables and formulas compiled by Carmichael & Smith and originally published by Ginn and Company in 1931.

The material is arranged in three parts. Part I consists of an introduction devoted to linear interpolation, the elementary properties of logarithms, and a brief description of some of the fourteen tables therein, which are "necessary in the study of college algebra and trigonometry." These tables include: common logarithms to 5D, arranged in a single-entry table; natural and logarithmic trigonometric functions to 4 and 5D; conversion tables for use with sexagesimal and radian angular measurement; and well-known constants, generally to 7 and 8D, except for π and e and their logarithms, which are separately listed to 30D.

Part II consists of five tables "not generally accessible to students of college mathematics," together with brief introductory explanations of their contents and use. These tables include: 6S values of $n^{-1}, n^2, n^3, n^{1/2}, (10n)^{1/2}, n^{1/3}, (10n)^{1/3}, (100n)^{1/3}$ for $n = 1(0.01)10$; ln n to 5D for $n = 0.01(0.01)10(0.1)100(1)1000$; $e^{\pm x}$, sinh x, cosh x, generally to 5S, and their common logarithms to 5D, for $x = 0(0.01)3(0.05)$-$4(0.1)6(0.25)10$; the first 100 multiples of M and $1/M$ to 6D; and finally 10D common logarithms of primes less than 1000.

Part III consists of: formulas from algebra, elementary geometry, trigonometry, analytic geometry, and the calculus; graphs for reference; a compilation of 323 indefinite integrals and 37 definite integrals; and a concluding selected list of infinite series (including the well-known infinite-product expansions for sin πx and cos πx).

It is interesting to note that a number of similar collections of tables and formulas appeared shortly after the first edition of the present work. Two of these are by Burington [1] and by Dwight [2], which together include such additional material as interest and mortality tables, formulas and tables relating to elliptic functions and integrals, the gamma function, probability integral, Legendre polynomials, and Bessel functions.

Further expansion and elaboration of such information appears in the recent compilation published by the Chemical Rubber Company [3]. For example, herein we find statistical tables, dictionaries of Laplace and Fourier transforms, and a number of other tables not to be found in the references previously cited.

Thus a comparison of the tables of Carmichael and Smith with similar books published subsequently reveals the continual growth of applied mathematics. In brief, the book under review, although acceptable as an inexpensive elementary reference, cannot be considered adequate as a general reference for mathematical formulas and numerical information, more than thirty years after its initial appearance.

J. W. W.

1. R. S. Burington, *Handbook of Mathematical Tables and Formulas*, Handbook Publishers, Inc., Sandusky, Ohio, 1933; second edition, 1940; third edition, 1953.

2. H. B. Dwight, *Tables of Integrals and other Mathematical Data*, The Macmillan Company, New York, 1934; revised edition, 1947; third edition, 1957; fourth edition, 1961. [See *MTAC*, v. 1, 1943–45, p. 190–191, RMT **154**; v. 2, 1946–47, p. 346, RMT **447**; v. 16, 1962, p. 390–391, RMT **42**.]

3. S. M. Selby, R. C. Weast, R. S. Shankland, & C. D. Hodgman, Editors, *Handbook of Mathematical Tables*, Chemical Rubber Publishing Company, Cleveland, Ohio, 1962. [See *Math. Comp.*, v. 17, 1963, p. 303, RMT **34**.]

71[A, I].—Alan Bell & Adele Higgins, *Table of Stirling Numbers of the Second Kind $S(n, k)$, $k = 1(1)n$, $n = 1(1)100$*, Sylvania Electric Products, Inc., Reconnaissance Systems Laboratory Report RSL-1330-1 SN, Mountain View, California, 2 October 1961, 18 p., 28 cm.

The table consists of 6S values of the Stirling numbers of the second kind, presented in floating-point form over the range indicated in the title. The necessary calculations were performed on a Burroughs 220 computer, using a program written in BALGOL capable of producing a similar table of $S(n, k)$ to $n = 350$, if a core memory of 10,000 words is fully utilized.

The authors define these numbers and give some of their properties, referring the reader to books by Richardson [1] and Riordan [2] for further information. No reference is made, however, to the considerable existing literature of tables of these numbers. For example, tables of *exact* values up to $n = 50$ have been prepared by Gupta [3] and Miksa [4]. A number of smaller tables are referenced in the new edition of the FMR *Index* [5].

Numerous rounding errors in the table under review have been revealed by a comparison with Miksa's table. Apparently the editing routine consistently neglected later figures, so that the inaccurate published data invariably err in defect.

With this realization that the last figure is unreliable by as much as a unit, the table-user can still derive useful information from these tables, especially for values of n exceeding those in previous publications.

J. W. W.

1. C. H. Richardson, *An Introduction to the Calculus of Finite Differences*, D. Van Nostrand Company, Inc., New York, 1954.
2. John Riordan, *An Introduction to Combinatorial Analysis*, John Wiley & Sons, Inc., New York, 1958.
3. H. Gupta, *East Panjab University Research Bulletin*, No. 2, 1950, p. 13–44.
4. Francis L. Miksa, *Table of Stirling Numbers of the Second Kind*, ms. deposited in UMT File. See RMT **85**, *MTAC*, v. 9, 1955, p. 198.
5. A. Fletcher, J. C. P. Miller, L. Rosenhead & L. J. Comrie, *An Index of Mathematical Tables*, Second edition, Addison-Wesley Publishing Co., Inc., Reading, Massachusetts, 1962, p. 106–107.

72[F].—C. A. Nichol, John L. Selfridge, & Lowry McKee, under the direction of Richard V. Andree, *A Table of Indices and Power Residues for All Primes and Prime Powers Below 2000*, W. W. Norton & Co., New York, 1962, 20 + approx. 700 unnumbered pages, 22 cm. Price \$10.00.

This is the published form of the tables [1] previously available on magnetic tape. They list, for each of the 302 odd primes $p < 2000$, and $i = 0(1)p - 2$, the *power residues*,

$$n \equiv g^i \pmod{p}, \tag{1}$$

where g is the smallest positive primitive root of p. In parallel tables are listed the *indices* i that satisfy (1) for every $n = 1(1)\ p - 1$. Following these 302 pairs of tables are 22 more pairs corresponding to the odd prime powers $p^a (a > 1)$ from $9 = 3^2$ to $1849 = 43^2$, inclusive. In these latter tables the g chosen is again the smallest and also is that corresponding to p. (This would not always remain possible if these tables were to be much extended. Thus, in Miller's table [2] one finds that 5 is the least positive primitive root of $p = 40487$, while Hans Riesel has determined that $5^{p-1} \equiv 1 \pmod{p^2}$ for this prime. But such instances, where the smallest positive primitive root of p is *not* also a primitive root of p^2, are no doubt very rare; this is the only example known to this reviewer.)

Jacobi's famous *Canon Arithmeticus* [3] (usually mentioned with the adjective "monumental") gave similar tables for p and $p^a < 1000$, but did not generally use the smallest positive g.

This volume includes an historical and theoretical introduction by H. S. Vandiver and a shorter, unsigned preface. In the latter we learn that the tables were computed on an IBM 650–653. Checking was accomplished by sum checks, echo-checking of the printer, re-printing, and spot checks.

These tables are highly useful in many number-theoretic computations and are the best of their kind available. The figures are clear and legible, but exhibit the usual variations in darkness so common in photographically reproduced tables.

There is an assortment of minor inelegancies in the format and printing, including: no space between the headings and the arguments; faulty zero suppression, so that some blanks appear instead as 0 or 0000; failure of the page eject on the primes $p = 1531$ and 1543; listing of the prime powers as $p = 9, 25$, etc.; and the continued listing of the arguments in some power-residue tables after the table has ended. However, these are demerits in aesthetics, and while they should have been corrected, they do not nullify the high utility of the tables.

D. S.

1. L. McKee, C. Nichol & J. Selfridge, *Indices and Power Residues for all Primes and Powers Less than 2000*; reviewed in RMT **64**, *Math. Comp.*, v. 15, 1961, p. 300.
2. J. C. P. Miller, *Table of Least Primitive Roots*; one copy deposited in UMT File. (See *Math. Comp.*, v. 17, 1963, p. 88–89, RMT **2**.)
3. K. G. J. Jacobi, *Canon Arithmeticus, sive tabulae quibus exhibentur pro singulis numeris primis vel primorum potestatibus infra 1000 numeri ad datos indices et indices ad datos numeros pertinentes*, Berlin, 1839.

73[F].—Daniel Shanks, *Solved and Unsolved Problems in Number Theory*, Vol. 1, Spartan Books, Washington, D.C., 1962, ix + 229 p., 24 cm. Price $7.50.

This book is an excellent introduction to number theory, well motivated by an entertaining and instructive account of the origin and history of the classical problems connected with perfect numbers, primes, quadratic residues, Fermat's Last Theorem, and other topics.

Superb in every respect, as an introductory account, as a history of number theory, as an essay in mathematical and scientific philosophy, this volume can be used either as a textbook in high school or college, as a book for self-study, or as a gift to the educated layman with the perennial query, "What does a mathematician do?"

This delightful and stimulating book should be on the shelf of anyone interested in mathematics.

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74[G].—I. M. Gel'fand, *Lectures on Linear Algebra*, Interscience Publishers, Inc., New York. 1961 ix + 185 p., 23 cm. Price $6.00.

The author presents in this book a very clearly written shorter text on linear algebra which would generally be suitable for a one-semester course at the junior level in the United States. The contents consist of four chapters (Chapter 1, n-Dimensional Spaces; Chapter 2, Linear Transformations; Chapter 3, The Canonical Form of an Arbitrary Linear Transformation; and Chapter 4, Introduction to Tensors), the first two chapters comprising about three-fourths of the book. The author is to be congratulated for his lucid discussions and proofs. The notation and the printing are excellent.

For those who wish to use this as a text, it should be mentioned that the author frequently assumes knowledge of results from matrix theory that American students, as opposed to Russian students, do not possess at this level.

R. S. V.

75[I, L].—F. W. J. Olver, *Tables for Bessel Functions of Moderate or Large Orders* (National Physical Laboratory, *Mathematical Tables*, v. 6), Her Majesty's Stationery Office, London, 1962, iii + 51 p., 28 cm. Price 17s. 6d. (In U.S.A.:

British Information Services, 845 Third Avenue, New York 22, New York. Price \$3.50.)

Various tables directly tabulating Bessel functions of fairly high order exist, and are briefly described in this volume, but such direct tabulation cannot be extended indefinitely, and needs to be supplemented by a method of computing any Bessel function of high order. After describing J. C. P. Miller's algorithm, the author sets out his own asymptotic expansions, which form the basis of the present work. The slim volume packs in a small compass a great deal of information, divided almost equally between text and tables, and it would exceed the proper limits of a review to describe either in full detail. The author's aim is to give tables to facilitate the computation of $J_n(nx)$, $Y_n(nx)$, $I_n(nx)$, $K_n(nx)$, and their first derivatives to ten significant figures (except in the immediate neighborhood of zeros) when $n \geqq 10$.

In the case of J, Y, J', and Y', the asymptotic expressions used involve an auxiliary variable ζ along with Airy functions Ai, Bi and their derivatives Ai', Bi' of argument $n^{2/3}\zeta$; here

$$\frac{2}{3}\zeta^{3/2} = ln\,\frac{1 + (1 - x^2)^{1/2}}{x} - (1 - x^2)^{1/2} \qquad (0 < x \leqq 1)$$

$$\tfrac{2}{3}(-\zeta)^{3/2} = (x^2 - 1)^{1/2} - \sec^{-1} x \qquad (x \geqq 1)$$

ζ is tabulated against x, and various coefficients are tabulated against ζ. The British Association tables of Airy integrals are assumed to be available, and indeed it may be noted that the whole work grew out of the prolonged British Association and Royal Society labors on the tabulation of Bessel and related functions.

In the case of I, K, I', and K', the asymptotic expressions used involve an auxiliary variable ξ and exponential functions of $\pm\, n\xi$, where

$$\xi = (1 + x^2)^{1/2} - ln\,\frac{1 + (1 + x^2)^{1/2}}{x},$$

but it is found convenient to take the argument of all the tables to be $t = (1 + x^2)^{-1/2}$, so that

$$\xi = \frac{1}{t} - \frac{1}{2}\,ln\,\frac{1 + t}{1 - t}.$$

Either $\xi - x$ or $\xi - ln\,x$ is tabulated against t, as are various coefficients. The author forms exponentials in his worked examples by using the well-known National Bureau of Standards tables, but any other logarithmic or antilogarithmic tables (either common or natural) with sufficient figures could be pressed into service.

An interesting feature is that provision for interpolation is made by tabulating coefficients in "economized" polynomials, as described by Clenshaw & Olver [1]. The tables of $\zeta(x)$ give coefficients c_i for use with the formula

$$f_p = f_0 + c_1 p + c_2 p^2 + \cdots + c_n p^n \qquad (0 \leqq p \leqq 1)$$

where n never needs to exceed 5. The remaining tables give coefficients d_2, d_4 (no more than these two ever being needed) which are derived from even central differences and allow interpolation by the formula (akin to a modified Everett formula)

$$f_p = qf_0 + q(1 - q^2)\,d_{2,0} + q^3(1 - q^2)\,d_{4,0} + pf_1 + p(1 - p^2)\,d_{2,1} + p^3(1 - p^2)\,d_{4,1}$$

where $q = 1 - p$. The author believes that the present tables are the first to use these particular aids to interpolation, and states that he welcomes comments and criticisms by users.

A useful bibliography has some forty references. The whole work constitutes a powerful tool, not to be overlooked by anyone concerned with numerical values of Bessel functions of high order.

A. F.

1. C. W. Clenshaw & F. W. J. Olver, "The use of economized polynomials in mathematical tables," *Proc. Camb. Phil. Soc.*, v. 51, 1955, p. 614–628.

76[K].—Donald Mainland, Lee Herrera & Marion I. Sutcliffe, *Tables for Use with Binomial Samples*, Department of Medical Statistics, New York University College of Medicine, New York 16, N. Y., 1956, xix + 83 p.

These tables are a consolidation of tables previously published (Mainland [1], Mainland and Murray [2], Mainland and Sutcliffe [3]). They contain many more entries than the original versions, and sections have been recalculated to give finer precision. All the tables are for use with qualitative data, that is, of the A, *not* A type.

Tables I–IV are for the comparison of two binomial samples arranged in a 2×2 contingency table.

Sample	A	not A	
1 2	a b	c d	N_1 N_2
	a + b	c + d	$N_1 + N_2$

The labels A and *not* A are assigned arbitrarily.

Tables I and II give minimum contrast pairs a, b, $a < b$, which are significant at the two-tailed 5% and 1% levels, respectively. Such pairs a, b are tabulated for $N_1 = N_2 = N = 4(1)20(10)100(50)200(100)500$. For $N \leqq 30$, some pairs a, b were omitted because they can be obtained quickly on sight by interpolation. The portion up to $N = 20$ was based on the exact hypergeometric distribution, whereas, for $N \leqq 30$, the chi-square with Yates' correction was generally used, and the significance was tested by Table VIII of Fisher and Yates [4].

Table III contains single-tail exact probabilities to 4D of 2×2 contingency tables for equal samples up to $N = 20$. Pairs a, b and corresponding exact probabilities are tabulated for all pairs a, b such that the tail probabilities are less than or equal to one-half. These pairs are those for which $a + b \leqq N$ and $a < b$.

Table IV gives minimum contrasts and probabilities for unequal samples of size up to $N = 20$. For given sample sizes N_1 and N_2, $N_1 > N_2$, the table gives (i) the pairs a, b $\left(a \leqq \frac{N_1}{2}\right)$ which provide a minimum contrast for significance at the single-tail 2.5% and 0.5% levels, and (ii) 4D of the corresponding exact probabilities.

Tables V–IX provide upper and lower confidence limits for a single binomial ratio such as (number of A's)/N, where N is the number of individuals in a random sample all of which have been classified as A or *not* A. Tables V–VIII were prepared from Table VIII_1 of Fisher and Yates [4], whereas Table IX was calculated directly from binomial expansions.

Table V gives 95% and 99% confidence limits for samples in which the number of A's is 1 through 14, A being the label of the class having the fewer number of individuals. The range of N is approximately $N = 2A(1)2A + 20$, and thereafter by increasingly large intervals to $N = 1000$. The limits are given to at least two decimal places and at least two significant figures.

Table VI provides 95% and 99% confidence limits for samples in which the number of A's is greater than 14. The tabulation is according to values of N and the percentage of A's in the sample, $P = 100$ (number of A's)/N. The range of P (in per cent) is $P = 0.1(.1)1(1)50$. The range of N is not the same for all values of P; but in all cases N extends, by increasingly large intervals, to $N = 100{,}000$. As P increases, the range of N increases downward and the number of values of N for which limits are given increases; i.e., for $P = 0.1\%$, N has 14 values over the range 3,000 to 100,000, whereas for $P = 50\%$ there are 55 values of N over the larger range, 30 to 100,000. The limits are given to at least two decimal places and, in all but a few cases, to at least two significant figures.

Table VII has the same description as Table V except that the confidence limits given are 80% limits.

Table VII is the same type as Table VI. This table contains 80% confidence limits, but for fewer values of P than Table VI. The values of P are $P = 0.1, 0.5, 1.0, 5(5)50$. The values of N are approximately equal to those in Table VI.

Table IX gives *upper* 95, 99, and 80% confidence limits for samples in which the number of A's is zero, the lower limit being zero per cent A's. The limits are given to 2D for $N = 1(1)30(5)100(10)200$, 220, 250, 300, 400, 500, 700, and 1,000.

Table X is entitled "Percentages of Successful Experiments (%S) in Relation to Sample Size (N) and to Percentages of A's in Populations V and W". Specifically, the table answers the following: If a sample of size N is taken from each of two effectively infinite populations which have P_1 and P_2 per cent A's, and if the sample percentages of A's are tested by means of Table I for a significant difference at the 5 *per cent level*, then, what percentage of such experiments would show the sample percentages to be significantly different and in the same direction as P_1 and P_2. Values S in per cent are given to 1D for all pairs of P_1 and P_2, $P_1 < P_2$, obtained from the following set: 1, 5, 10, 15, 25, 33, 50, 67, 75, 85, 90, 95, and 99. The sample sizes are $N = 5, 10, 15, 20, 30, 50, 70$, and 100.

The introduction to the ten tables contains fourteen examples which demonstrate the use of the tables. Several of these examples show how to interpolate or extrapolate in certain of the tables. Included also in the introduction is a section on the preparation and reliability of the tables.

For use in testing 2×2 contingency tables with equal samples, Tables I and II are certainly more convenient for the user than other existing tables. The test is done easily and quickly by eye. If, instead of "assigning the label A arbitrarily", we pick a to be the smallest of all four entries, a large number of entries in Tables I and II can be eliminated, making the test seem still easier.

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1. D. MAINLAND, "Statistical methods in medical research. I. Qualitative statistics (enumeration data)", *Canad. J. Res.*, Series E, v. 26, 1948, p. 1–166.
2. D. MAINLAND & I. M. MURRAY, "Tables for use in fourfold contingency tests," *Science*, v. 116, 1952; p. 591–594.
3. D. MAINLAND & M. I. SUTCLIFFE, "Statistical methods in medical research. II. Sample sizes required in experiments involving all-or-none responses," *Canad. J. Med. Sci.*, v. 31, 1953, p. 406–416.
4. R. A. FISHER & F. YATES, *Statistical Tables for Biological, Agricultural, and Medical Research*, Hafner Publishing Company, Inc., New York, 1953.

77[K].—A. M. YAGLOM, *An Introduction to the Theory of Stationary Random Functions*, Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 1962, xiii + 235 p., 23 cm. Price $10.60.

This book is an exceedingly well written account of the subject of stationary processes and linear prediction theory. It does not require much mathematical background on the reader's part. The small quantity of Hilbert space theory needed for prediction theory is supplied by the author. Bochner's fundamental theorem (also due to Khinchin) on the representation of positive definite functions as Fourier transforms of finite nonnegative measures is not proved but is discussed at length. The theorem due to Szëgo, and Kolmogoroff's generalization, that

$$\inf_p \frac{1}{2\pi} \int_{|z|=1} |1 - zp(z)|^2 f(\theta)\, d\theta = \exp\left\{\frac{1}{2\pi} \int \log f(\theta)\, d\theta\right\},$$

where p runs through all polynomials in z, $f(\theta) \geqq 0$ in L_1, and its corollaries characterizing deterministic discrete and continuous processes, are also examined but not proved. The problem of linear extrapolation is therefore restricted to cases in which the optimal estimate can be expressed as an infinite series of past values:

$$\hat{\xi}_m = a_1\xi_{-1} + a_2\xi_{-2} + \cdots + a_k\xi_{-k} + \cdots,$$

$m \geqq 0$, in which the series converges in $L_2(f(\theta))$, f being the power density. The case in which $f(\theta)$ is a rational function of $e^{i\theta}$ is treated in detail. The extrapolation problem is followed by a chapter on linear filtering; that is, given ζ_n for $n \leqq -1$, with $\zeta_n = \xi_n + \eta_n$, find the optimal estimate of ξ_m. The problem is solved in detail for the case in which ξ and ζ have power densities rational in $e^{i\theta}$. The problems of extrapolation and filtering for random functions (instead of sequences) come next, with a chapter for each. This new edition concludes with two short appendices: one on generalized random processes, in which "white noise," for example, can be defined rigorously; and the other, written by D. B. Lowdenslager, on some recent developments, in particular, vector-valued processes.

Although the reviewer does not read Russian, it must be concluded on the basis of the beautiful style of the book that the translator, R. A. Silverman, has done an excellent job of translating or writing, or both. This book will provide both for beginners and for more experienced mathematicians a fundamental grasp of the applied aspects of linear prediction theory, and is highly recommended.

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78[K].—A. H. ZALUDOVA, "The non-central t-test (q-test) based on range in place of standard deviation," *Acta Technica*, v. 5, 1960, p. 143–185.

This paper presents tables of percentage points of the non-central q-distribution and additional tables which were used to compute these percentage points. The non-central q is a non-central t-type statistic, which is based on the mean sample range. Because of the ease with which it is used, the non-central q-test is proposed as a substitute for the non-central t-test as given by Johnson and Welch [1], especially when the application is in sampling inspection by variables.

The statistic q is given by

$$q(m, n) = \frac{T - \bar{x}}{\bar{w}(m, n)},$$

where $\bar{w}(m, n)$ is the mean of the m ranges of m sub-samples, each of size n, drawn from a normal population $N(\mu, \sigma)$; $\bar{x}$ is the over-all sample mean; and

$$T = \mu + K_p\sigma,$$

where K_p is the normal deviate exceeded with probability p. The distribution of $q(m, n)$ is derived by the author.

Tables 1, 2, 4, and 5 give percentage points q_ϵ to 3D for percentiles $\epsilon = .05, .95, .25$, and .75. For each value of ϵ the points are tabulated for $m = 1(1)5, n = 3(1)12$, and $p = .20, .10, .05, .02, .01$, and .001. Some details of the computation of the values and remarks on their accuracy are given.

In order to calculate the percentage points q_ϵ it was necessary to tabulate the frequency functions $f_m(\bar{w}_m)$ of the sample range and mean range, $\bar{w}_m$ denoting the mean range in m independent sub-samples from a normal population having unit standard deviation. This was done only for combinations $m = 1, 2, 4$ and $n = 3, 4, 6, 8, 10, 12$. These values of $f_m(\bar{w}_m)$ are given in Tables 7, 8, and 9. All values are given to at least 5D. They are tabulated for the following intervals of $\bar{w}_m$: $\bar{w}_1$ (single range) $= 0.0(0.1)8.2$, $\bar{w}_2 = 0.0(0.1)6.5$, and $\bar{w}_4 = 0.1(0.1)5.5$. These three tables were used to calculate a framework table of values of q_ϵ from which values of q_ϵ for other combinations of m and n were obtained by interpolation. Remarks on the calculation and accuracy of the values of $f_m(\bar{w}_m)$ are made.

It was found desirable for interpolation to know the limiting value of q_ϵ for $m = \infty$. The limiting distribution of q is derived and found to be concentrated at the point $E(q) = K_p/d_n$, where d_n is the expected value of the range in samples of size n from a normal population with unit standard deviation. Table 3 gives values of K_p/d_n to 5D for $n = 3(1)12$ and values of K_p corresponding to $p = .20, .10, .05, .02, .01$, and .001.

One additional table (Table 6) is presented; it shows a comparison of the non-central q-distribution with the power function of Lord's [2], [3] central t-test (u-test). This is done by tabulating $q_{.05}$ against an approximation to it which is based on the power function of Lord's u-test. The comparison is made for combinations $m = 1, 2, 4$ and $n = 3, 4, 6, 8, 10$. For each of the fifteen combinations a value of K_p is chosen such that α(error of first kind) $= \beta$(error of second kind) $= .05$, approximately. Values of $q_{.05}$ corresponding to these K_p are compared with the approximated values as a rough check on the tabulated values of q_α.

In conjunction with the tables of percentage points, twelve figures are given which show the relation (almost linear) between q_ϵ and K_p for p ranging from .001 to .20 (extended to .50 in some figures). For each value of ϵ and m, the relationship is shown for various values of n from 3 to 12. These graphs make possible the determination of q_ϵ for other than the six tabulated values of p.

Several examples are given which demonstrate the application of the non-central q-test in industrial sampling inspection. These illustrations indicate that the non-central q-test and corresponding tables of percentage points can be very useful in such work.

JOHN VAN DYKE

1. N. L. JOHNSON & B. L. WELCH, "Applications of the non-central t distribution," *Biometrika*, v. 31, 1940, p. 362–389.
2. E. LORD, "The use of range in place of standard deviation in the t-test," *Biometrika*, v. 34, 1947, p. 41–67.
3. E. LORD, "Power of the modified t-test (u-test) based on range," *Biometrika*, v. 37, 1950, p. 64–77.

79[L].—S. L. BELOUSOV, *Tables of Normalized Associated Legendre Polynomials*, Pergamon Press, Ltd., Oxford, distributed by The Macmillan Company, New York, 1962, 379 p., 26 cm. Price $20.00.

This is a republication in an attractive binding of Russian tables of normalized associated Legendre polynomials previously reviewed in this journal (*MTAC*, v. 11, 1957, p. 276, RMT **115**).

For convenience the contents are here summarized again. The polynomials $\bar{P}_n^m(\cos\theta)$ considered are related to the associated Legendre polynomials P_n^m by the normalizing factor $\left[\frac{2n+1}{2}\frac{(n-m)!}{(n+m)!}\right]^{1/2}$, so that $\int_{-1}^{1} [\bar{P}_n^m(x)]^2\,dx = 1$, and are herein tabulated to 6D for $m = 0(1)36$, $n = m(1)56$, and $\theta = 0(2^\circ.5)90^\circ$. No tabular differences are given.

The introduction has been translated into English by D. E. Brown.

These useful tables remain the most extensive of their kind published to date.

J. W. W.

80[L].—O. S. BERLYAND, R. I. GAVRILOVA, & A. P. PRUDNIKOV, *Tables of Integral Error Functions and Hermite Polynomials*, Pergamon Press, Ltd., Oxford, England, distributed by The Macmillan Co., New York, 1962, 163 p., 26 cm. Price $15.00.

This volume of the Pergamon Mathematical Tables Series is an English translation by Prasenjit Basu of original tables of integral error functions and Hermite polynomials published in Minsk in 1961 by the Byelorussian Academy of Sciences.

The authors separately tabulate $I_n \operatorname{erfc} x = A_n i^n \operatorname{erfc} x$, where $A_n = 2^n\Gamma\left(1 + \frac{n}{2}\right)$, and $H_n^*(x)$, which is defined in terms of the standard Hermite polynomials by the relations $H_{2n}^*(x) = H_{2n}(x)/B_{2n}$ and $H_{2n-1}^*(x) = H_{2n-1}(x)/B_{2n}$, where $B_{2n} = (-1)^n(2n)!/n!$, so that $H_{2n}^*(0) = 1$.

In a separate table $I_0 \operatorname{erfc} x \equiv \operatorname{erfc} x$ is given in floating-point form to 6S for $x = 0.01(.01)3.50$. On succeeding pages appears the tabulation of $I_n \operatorname{erfc} x$ for $n = 1(1)30$, at an interval of 0.01 in x. The precision ranges from 6S initially to 2S near the end of the table. The upper limit to the argument x depends upon n, and varies monotonically from 3.50, when $n = 1$ and 2, to 1.00 when $n = 26$–30. A preliminary table of A_n to 9S is given in floating-point form for $n = 0(1)30$; this has terminal-digit errors, beginning with A_1, which is simply the well-known constant $\sqrt{\pi}$. The table of $I_0 \operatorname{erfc} x$ is seriously infested with errors, which apparently arose from the retention of a fixed number of significant figures instead of a fixed number of decimal places. This loss of accuracy was also observed in the table of $I_n \operatorname{erfc} x$, $n \geqq 1$. Moreover, the table-user will be annoyed to discover that certain columns have been filled out with zeros, with an attendant loss of all significant figures in those tabulated data.

Following this is a table of the coefficients B_{2n}, which are given exactly for $n = 0(1)9$ and are truncated (without rounding) to 9S for $n = 10(1)15$. The value for B_{22} contains a more serious error; namely, the sixth most significant figure is given as 0 instead the correct digit, 5.

The second principal table gives 6S values of $H_n^*(x)$ for $n = 1(1)30$, $x = 0(.01)10$. The floating-point format is retained for the entries in this table.

An introduction describes the fundamental properties of the tabulated functions, the methods used in calculating the tables, and their arrangement and use. A list of ten references includes papers by Hartree and by Kaye that contain related tables of $i^n \operatorname{erfc} x$.

It is regrettable that the accuracy of these extensive tables does not match the very attractive appearance of the binding.

J. W. W.

81[L].—L. K. Frevel & J. W. Turley, *Tables of Iterated Bessel Functions of the First Kind and First Order*, The Dow Chemical Company, Midland, Michigan, 1962. Deposited in UMT File.

The authors have continued their study and tabulation of iterated functions, which has included the iterated sine (*Math. Comp.*, v. 14, 1960, p. 76), the iterated logarithm (*ibid.*, v. 15, 1961, p. 82), the iterated sine-integral (*ibid.*, v. 16, 1962, p. 119), and now this report on the iterated Bessel function of the first kind and first order.

Two tables of decimal values of $J_1^n(x)$ are presented, as computed on a Burroughs 220 system, supplemented by Cardatron equipment to permit on-line printing of the final format.

Table 1 consists of 15D values of $J_1^n(x)$ corresponding to $n = 1(1)10$ and $x = 0(0.2)10$. Table 2, comprising the bulk of the report, gives 12D values of $J_1^n(x)$ for $n = 0(0.05)10$, $x = 0.2(0.2)1.8$, and for $n = 1(0.05)10$, $x = 2(0.2)10$.

In the heading of this table, the increment in n is erroneously given as 0.02, although it is correctly stated in the abstract.

The prefatory text of three pages defines the iterated Bessel function under consideration and describes the method of computation employed in the construction of the tables. It is there stated that the entries in Table 2 were calculated to 17D prior to rounding to the tabular precision of 12D.

These data constitute another original contribution to the rapidly increasing number of new mathematical tables.

J. W. W.

82[P, S, X].—JOHN W. DETTMAN, *Mathematical Methods in Physics and Engineering*, McGraw-Hill Book Company, New York, 1962, xii + 323 p., 23 cm. Price $9.75.

In the author's introduction to this very well written textbook he states that, whereas the traditional course in advanced mathematics for engineers and physicists is intended for students who wish to pick up additional techniques not covered in the elementary calculus, these topics are often presented in a very heuristic fashion because the students lack a solid background in analysis. Eventually, he claims, most graduate physics and engineering students will need a thorough understanding of applied mathematics.

The purpose of this book, then, is to fill the need for an introduction to mathematical physics for which a foundation has been prepared by a solid "mathematician's" advanced calculus course.

The author has done yeoman service to his announced aims. There is at least enough material here for a two-semester course, and it is characterized by a good continuity of development. Further, it is an order of sophistication beyond the aforementioned advanced engineering mathematics course.

The style is terse; perhaps this follows from his implied discontent with heuristic presentations. If so, it is not an unusual viewpoint but regrettable, nonetheless, because heuristic discourse can be quite rigorous and still serve to stimulate investigation. It merely implies a high redundancy level which is all too often disdained in textbooks.

Chapter 1 mainly prepares the algebraic foundations for later material, moving through linear algebra into infinite-dimensional vector spaces, orthonormal functions, Fourier series, quadratic forms, and vibrations problems. The second chapter covers variational methods, from maxima and minima of functions and functionals through Lagrange's equations and Hamilton's principle to boundary-value problems and eigenvalue problems.

Chapter 3 discusses separation of variables, Sturm-Liouville systems and the method of Frobenius, while chapter 4 concerns itself with Green's functions in boundary-value problems. Chapter 5 includes a lucid, but still very terse, treatment of integral equations. The final chapter treats of Fourier transforms and their applications, with mention of Laplace and other integral transforms.

Each chapter is divided into several sections, each of which is followed by a set of moderately difficult exercises. The book abounds with excellent examples of the applications of the techniques which have been developed.

While this textbook certainly will serve its intended purpose and is, in fact, the fruit of just such a course taught at Case Institute of Technology, there may be preferable alternatives open to the physicist or engineer preparing himself for a career of research. Perhaps the following remarks would apply more to the theoretician than the experimentalist, but the principle still persists.

Limits on the size of a book and the teaching time available during a two-semester course severely curtail the amount of material which can be encompassed. The end result of survey courses too often appears to be that when the physicist or engineer returns home from his brief sojourn in the land of applied mathematics he makes two discoveries: (a) when he really needs a mathematical implement it happens to be one he didn't bring home with him, and (b) many of the ones he brought back could have been bought more cheaply at home in the first place.

The goal then should be the development of a mathematical maturity to temper and complement one's physical intuition, rather than the acquisition of a repertoire of a few mathematical *dei ex machinae*. It often becomes necessary to "roll your own" analysis, and the ability to do this can't be achieved by a crash program.

An alternative program might well include, beyond the bed-rock analysis, separate courses in complex function theory (statistical mechanics, plasma physics, field theory, fluid dynamics), real variables (for Lebesgue integration occurring in other contexts), linear spaces (field theory, quantum mechanics, elementary particle interactions), abstract algebra (transition probabilities, quantum mechanics, crystal lattices), and partial differential equations (all continuum mechanics).

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83[P, X].—R. D. STUART, *An Introduction to Fourier Analysis*, John Wiley & Sons, New York, 1962, 126 p., 19 cm. Price $3.00.

This book is written from the viewpoint of a physicist or communications engineer and, typically, it deals almost entirely with applications. The first and third chapters are supposed to provide the basic material in the theory of Fourier series and Fourier integrals, but the "proofs" are almost everywhere fallacious, even for continuous functions. The second and fourth chapters contain descriptions of the "usual" functions and their transforms, as, for example, the square wave, saw-tooth, unit impulse, and exponential decay. Chapters V and VI treat applications to circuit analysis and wave motion, including filters, the capacitance-resistance circuit, bandwidth, diffraction, amplitude modulation, and phase modulation (in which the presumably immature reader is suddenly expected to know some Bessel function theory).

The book cannot be recommended for serious students of waveform analysis, and it is hard to see where its value does lie.

JOSEPH BRAM

84[P, X].—P. P. TEODORESCU, *Probleme Plane in Teoria Elasticitatii*, Vol. I, Editura Academiei Republicii Populare Romine, Romania, 1960, 995 p., 23 cm. Price Lei 42,80.

This book of just under 1000 pages is in Romanian. It is concerned only with plane problems in linear elasticity. The book presents a number of the most important and best known methods for treating plane problems, and gives numerous examples by way of applying the various methods. The book contains a preliminary chapter on the basic formulation and ideas of linear elasticity, four chapters on plane problems in elasticity, and seven chapters on the deep-beam problems for members having rectangular boundaries. In addition, there are appendices, tables of calculations and diagrams, as well as abstracts both in Russian and in English.

This book is actually the first of two volumes which the author is devoting to plane problems in elasticity. The second volume is to consider deep beams with general boundaries, beams of variable thickness, thermal effects as well as vibrations. Some consideration is to be given to viscoelastic and plastic behavior as well as various other nonlinear phenomena.

Chapter I of Volume I deals with the notion of stress and strain, Hooke's law, and thermal effects. The author defines the basic boundary-value problems and discusses the standard variational principles, with brief mention of such questions as existence, uniqueness, St. Venant's principle, etc.

Chapter II deals with the notions of plane stress, plane strain, generalized plane stress, and deep beams of variable thickness.

In Chapters III and IV the author introduces the Airy stress function, the Marguerre displacement function, the formulation of Love, and a number of other scalar functions in terms of which the solutions to plane problems may be presented (or approximated).

Chapter V contains various methods for solving or approximating the solution of plane problems, such as complex or hypercomplex-variable methods, variational methods, methods of operational calculus, finite-difference methods, experimental methods, etc.

In Chapter VI the author gives a systematic treatment of straight beams. Much of the chapter is devoted to the technical theory of bending of beams. Some interesting reciprocity theorems are presented.

Chapter VII is devoted to the study of semi-infinite beams, in particular to the study of half-plane, quarter-plane, elastic-strip, and half-strip problems. A number of particular cases of practical importance are examined.

In Chapters VIII, IX, and X a systematic study of deep beams is made. Approximate calculation methods are used, one of which is due to the author.

Chapter XI concerns itself with the treatment of beams on an elastic foundation. Also included is the rigid punch problem. The final chapter discusses the problem of beams subjected to elastic loads.

In the appendices the author presents much helpful information, such as expressions for biharmonic polynomials up to the 13th degree, Fourier-series and Fourier-integral representations of different functions, and lists of various functions encountered frequently throughout the book. In addition to this, there follow a number of useful tables and graphs.

At the end of each chapter the author furnishes an extensive bibliography. References are made to papers not widely known in this country. If this book were written in a language which was more widely understood, it would serve as an excellent reference book for engineers working in the area of theoretical and applied mechanics. Supplemented by other pertinent material it could actually be used as a text for an advanced course in Strength of Materials or a course in Plane Problems in Elasticity.

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85[S, X].—G. BIRKHOFF & E. P. WIGNER, Editors, *Proceedings of Symposia in Applied Mathematics, vol. XI, Nuclear Reactor Theory*, American Mathematical Society, Providence, R.I., 1961, v + 339 p.

This book contains the nineteen papers presented at the Symposium on Nuclear Reactor Theory jointly sponsored by the American Mathematical Society and the Office of Naval Research which was held in New York City, April 23–25, 1959. The expressed purpose of the present volume is to increase the number of mathematicians who will devote "serious effort to the mathematical problems of nuclear reactor theory," by indicating a variety of mathematical problems encountered in this field. There is a considerable diversity in the content and approach taken in the various papers, ranging from papers oriented towards the physical aspects of the problems to those of a purely mathematical nature.

The volume begins with an excellent paper entitled "Reactor Types" by A. M. Weinberg, which furnishes a background for the symposium. As is pointed out there, despite the diversity of neutron chain reactor types, "In every case neutrons induce the basic energy-liberating fission reaction, and they are themselves produced by the fission reaction. It is this property that gives to nuclear reactors their name—'chain reactors'—and to the mathematical theory of nuclear chain reactors a beautiful unity." The general chain reactor equation is set up in terms of the neutron flux as a function of the basic variables of position, energy, velocity direction, and time. In this paper problems associated with the treatment of the general equation are discussed, and various simplifying assumptions appropriate for the several types of reactors are outlined.

E. P. Wigner surveys some of the more interesting mathematical problems of nuclear reactor theory. Papers treating particular mathematical problems associated with the reactor equations and processes include: G. Birkhoff, on "Positivity and Criticality"; G. J. Habetler and M. A. Martino, on "Existence Theorems and Spectral Theory for the Multigroup Diffusion Model"; and a paper by G. M. Wing on spectral theory problems associated with transport theory.

The problem of the deep penetration of radiation, which is important in reactor shielding problems, is discussed in a paper by U. Fano and M. J. Berger.

J. E. Wilkins, Jr. derives the diffusion approximation to the transport equation.

A set of papers on numerical methods includes: R. Ehrlich, concerning one-dimensional multigroup diffusion calculations; R. S. Varga, on solving the multi-dimensional, multigroup diffusion equations; R. D. Richtmyer, concerning the application of Monte Carlo methods; B. Carlson, concerning the solution of the neutron transport equation; and R. Bellman and R. Kalaba, on the application of invariant imbedding to the solution of some one-dimensional problems of neutron multiplication.

The treatment of two problems associated with the determination of the energy dependence of neutron flux is discussed in a paper on neutron thermalization by M. S. Nelkin and in one on resonance absorption by L. W. Nordheim.

Papers treating problems associated with time-dependent behavior of reactors include: H. Soodak, surveying some of the problems of reactor kinetics; H. L. Garabedian, on low-power core kinetics; H. Brooks and also T. A. Welkin, on the stability of reactors at higher powers.

From the range of subjects and the list of authors given above, it can be seen that in this symposium volume a broad range of problems of nuclear reactor theory has been surveyed by a well chosen set of authors.

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86[W, Z].—ROBERT S. HOLLITCH & BENJAMIN MITTMAN, *Computer Applications—1961*, The Macmillan Company, New York, vii + 198 p., 23 cm. Price $8.95.

This book contains the proceedings of the 1961 Computer Applications Symposium sponsored by the Armour Research Foundation. On the whole, the papers on business and management applications deal primarily with the adoption of standard automatic data processing procedures and computational techniques. For example, one speaker decried the fact that extensive research was being carried out in the fields of indexing and literature searching rather than in the area of management control for carrying out routine library operations such as catalog preparation. Papers on engineering and scientific applications include the following fields: real-time control in space flight, communications engineering, medical diagnosis, and teaching machines.

The papers dealing with automatic programming for numerically controlled tools, the implementation of ALGOL in Europe, and the use of decision tables in problem definitions provide an informative description of the current status of important types of problem-oriented programming languages.

This small volume contains the following presented papers plus transcripts of the discussions:

"Management of Records in a Large-Scale Collaborative Research Program (Honeywell 800)" by Bernard H. Kroll.

"A Method for Systematic Documentation—Key to Improved Data Processing Analysis" by Orren Y. Evans.

"Automation of Library Operations" by Louis A. Schultheiss.

"Man-Machine Communications in the Coming Technological Society" by Simon Ramo.

"The Coming Impact of Computers on Advertising" by Edward F. Andresen.

"Computer Techniques in Assembly Line Balancing (IBM 1620, IBM 650, UNIVAC Solid State 80)" by David I. Scheraga.

"BUWEPS PERT-Milestone System—A Tool for Management" by Yukio Nakayama.

"Description of the Mercury Real Time Computing System" by James Donegan.

"The Progress of ALGOL in Europe" by Peter Naur.

"Scientific Applications for the UNIVAC LARC" by Cecil E. Leith, Jr.

"Digital Computers in Communications Engineering" by Robert M. Fano.

"Automatic Programming for Numerically Controlled Tools—APT III" by Edgar A. Bates.

"Medical Diagnosis Aided by Digital Computers" by Robert S. Ledley.

"CLASS—The Automated Classroom (Philco 2000)" by Donald E. Englund and D. P. Estavan.

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87[X].—WOLFGANG HAHN, *Theory and Applications of Liapunov's Direct Method*, 1963, Prentice-Hall, Inc., Englewood Cliffs, New Jersey, x + 182 p., 23.5 cm. Price $9.00.

Take a differential equation

$$\dot{x} = X(x, t) \tag{1}$$

where x and X are n - vectors (vectors with n components), $X(0, t) = 0$ for $t \geqq 0$, and X satisfies standard conditions, continuity included, to guarantee existence and uniqueness of solutions in a certain spherical region $S : \|x\| < A$. How stable are the solutions relative to the "trivial" solution $x = 0$? The problem of stability thus faced is of great theoretical and practical importance. It has given rise to a widely developed theory whose creator is the great Russian mathematician Liapunov (around 1890). Owing in part to the language barrier (in spite of a French translation in 1907), this theory lay practically forgotten by the world at large. It woke up very suddenly about 40 years ago and has been vigorously pursued in the USSR since then. In recent years it has at last reached these shores, with the result that many younger scientists in the U.S. are now participating in its development.

The present volume, a translation of the 1959 German edition of the Ergebnisse series, is a most valuable and timely addition to the literature in English on the subject. It is part careful treatise, part summary of many contributions, for a large part from the Soviet Union, the leader in this field. Definitions and theorems are carefully stated and proved, making this monograph a very good guide in the subject, particularly in view of its ample bibliography. The following chapter titles will give an idea of the extensive ground covered:

1. Fundamental concepts.
2. Sufficient conditions for stability or instability of equilibrium.
3. Application of the stability theorems to concrete problems.
4. The converse of the main theorems.
5. Liapunov functions with certain properties of rate of change.
6. The sensitivity of the stability behavior to perturbations.
7. Critical points.
8. Generalizations of the concept of stability.

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88[X].—WILLIAM KARUSH, *The Crescent Dictionary of Mathematics*, The Macmillan Co., New York, 1962, x + 313 p., 24 cm. Price $6.50.

From the introduction: "The subject matter of the 1422 entries in this dictionary covers two general categories. First, a detailed treatment is provided of the following standard high school and college mathematics material: arithmetic; elementary, intermediate, and college algebra; plane and solid analytic geometry; differential and integral calculus. Second, a wide selection of items is provided from more advanced mathematics, including the following fields: logic and fundamental concepts; theory of equations, theory of numbers, and modern higher algebra; advanced calculus; geometry and topology; probability and statistics; recent areas such as computer sciences, information theory, operations research, and so on."

The coverage in the first "general category" is thorough, but most branches of advanced mathematics are hardly attempted. For instance, even advanced calculus, including such terms as "Jacobian", "Stieltjes Integral", "Uniform Convergence" and "Cauchy's Theorem", is not covered. On the other hand some modern specialties, e.g., Theory of Games, Linear Programming, Computers and Their Coding, Logic, Machine Intelligence, Markoff Process, etc. are covered briefly but well. This class of specialties is apparently favored because of the author's position on the staff of the System Development Corporation.

The coverage is not as broad as that of the larger volume [1] of James and James. Perhaps because of this narrower range, the present volume is more uniform in character. The definitions are uniformly good and are carefully written and printed. With its limited extent in mind, the dictionary can be recommended to the several audiences for which it was intended.

D. S.

1. GLENN JAMES & ROBERT C. JAMES, *Mathematics Dictionary*, 2nd ed., D. Van Nostrand Co., Princeton, New Jersey, 1959. (Reviewed in RMT 66, *MTAC*, v. 13, 1959, p. 331–332.)

89[X].—N. N. KRASOVSKII, *Stability of Motion*, Stanford University Press, Stanford, California, 1963, vi + 188 p., 24 cm. Price $6.00.

One of the most powerful and flexible techniques available for the study of the stability of solutions of functional equations is that based upon the use of Liapunov functions.

To illustrate the basic idea, first presented by Liapunov in his fundamental memoir of 1892, consider the system of equations

$$\frac{dx_i}{dt} = g_i(x_1, x_2, \cdots, x_N), \qquad i = 1, 2, \cdots, N, \tag{1}$$

or, in vector notation, $dx/dt = g(x)$. Considering $V(x)$, where V is as yet undetermined, as a function of t, we have

$$\frac{dV}{dt} = \sum_i \frac{\partial V}{\partial x_i} g_i = (\text{grad } V, g). \tag{2}$$

Suppose that V, as a function of x, has been chosen so that

$$(\text{grad } V, g) \leqq -kg, \qquad k > 0, \tag{3}$$

for all x. Then (2) and (3) yield the result that $V \leqq V(0)e^{-kt}$, whence $V(x) \to 0$ as $t \to \infty$. If $V(x)$ is a function such as $\sum_i x_i^2$, with the property that $V(x) \to 0$ if and only if $x \to 0$, we have deduced in this way the important fact that $x(t) \to 0$ as $t \to \infty$, a stability result.

The problem, of course, lies in obtaining $V(x)$, given $g(x)$. Although there is no uniform approach, there exists a vast literature of results due to mathematicians such as Cetaev, Malkin, Persidskii, Massera, Letov, and others. An excellent survey may be found in another recent book in this area, namely, J. P. LaSalle and S. Lefschetz, *Stability by Liapunov's Direct Method with Applications*, Academic Press Inc., New York, 1962.

The great merit of Krasovskii's book is to contain not only a more complete and detailed account of the research of this nature in the field of ordinary differential equations, but also to present a thorough discussion of the application of these methods to differential-difference and more general time-lag equations.

The book is wholeheartedly recommended to all those interested in the modern theory of differential equations and in modern control theory.

The format is attractive, the price is reasonable, and the translation by J. L. Brenner is excellent.

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90[X].—JAMES B. SCARBOROUGH, *Numerical Mathematical Analysis*, Fifth Edition, Johns Hopkins Press, Baltimore, Md., 1962, xxi + 594 p., 23.5 cm. Price $7.00.

This is a revised edition of the well-known text by James B. Scarborough. In addition to a number of corrections and minor changes, the Fifth Edition contains a chapter on Newton's interpolation formula for unequal intervals. It is gratifying that the author has been able to find time periodically to review and improve one of the oldest and most popular elementary texts in the field of Numerical Analysis.

H. P.

91[X, Z].—GEORGE S. SEBESTYEN, *Decision-Making Processes in Pattern Recognition*, The Macmillan Company, New York, 1962, viii + 162 p., 24 cm. Price $7.50.

Pattern recognition is a subject which is currently receiving considerable attention. It is important in a variety of situations ranging from the need of the Post Office for mechanical reading devices to speed up sorting of the mails to the need of the Military to be able to decide whether an incoming radar or sonar signal comes from a harmless object such as a meteor or a fishing boat, or whether it comes from a threatening source such as a missile warhead or a hostile submarine. In any situation, the problem to be solved is how to organize one's knowledge about the object in question and how to be able to compare this with similarly organized knowledge about the possible categories to which the object can be assigned.

In the book under review, the author attempts to exploit a geometrical point of view. Data describing a given object consist of numerical values assigned to N

parameters. These are regarded as coordinates of a point in N-dimensional Euclidean space. Data from a set of such objects, all members of some class, constitute a cluster of points in N-space. This cluster is characterized by a single parameter: the mean-square distance between all possible independent pairs of points in the cluster. The question whether a new sample data point belongs to the class in question is decided on the basis of its mean-square distance from all points of the cluster.

Pursuing this theme, the author considers first linear, then nonlinear, transformations of the N-dimensional parameter space so as to achieve such intuitively clear and plausible goals as bringing as close together as possible the points of a cluster representing a single class or separating as far as possible the clusters representing two or more classes. Further, he brings in likelihood ratios and some elementary decision theory in his discussion of the problem of setting up criteria for determining whether a given sample data point does or does not belong to a particular class (cluster) or to which of several classes it should be assigned.

Among additional topics taken up is the application of the methods developed here to "learning machines," which could be implemented either as special hardware or as computer programs. This naturally leads to a brief discussion of such closely related topics as neural nets and Perceptrons. There is also a very informative and well illustrated section on the geometric interpretation of various classification procedures.

As a primary example of the application of his methods, the author presents the results of what must have been extensive experimental work on speaker recognition by means of analysis of the audio frequencies involved. Sample problems: to decide which of two speakers uttered a given phrase; to classify a spoken sound as voiced or unvoiced; to distinguish among spoken numerals, whether spoken by male or female voices. The results of these experiments appear to be quite encouraging. The similarity of these speaker recognition problems to the militarily important recognition problems mentioned in the first paragraph of this review need hardly be belabored.

The book as a whole is a well written account of important research in a fascinating field which touches several disciplines: electrical engineering (which is the author's specialty), statistical decision theory, learning devices, and linear algebra and its geometrical interpretations. In the non-mathematical portions the writing is very clear, leaving no doubt as to what the author is trying to say.

In the mathematical portions (Chapters 2 and 3), where the fundamental ideas are developed, the situation is, unfortunately, otherwise. Here the standards of presentation and the logical orderliness of developments are much poorer than in the rest of the book. To the careful reader it becomes obvious that the author lacked sufficient familiarity and facility with linear algebra and matrix theory to handle competently the problems with which he dealt. For example, he failed to make use of a simple theorem of which he actually seems to have been aware, which would have shown up his class separation criteria as being rather poorly formulated. This theorem asserts that the mean-square distance between all pairs of points of a finite set is twice the mean-square distance of all points from the centroid, that is, twice the variance of the set. Furthermore, if, in forming the mean-square distance between all pairs of points, he had counted pairs whose members coincided as well as pairs of independent points, the equations resulting from almost all of his class membership or class separation criteria would have simplified enormously and been capable of *simple explicit solution.* As it is, they are hanging on the brink of multiple degeneracy and, presumably, numerical instability. This applies particularly to the transformation for improving the separation of two classes (pages 40–42), which is left to be determined by the numerical solution of the eigenvector problem for a matrix pair.

These same Chapters 2 and 3 are, furthermore, sprinkled with a number of false statements, whose nature indicates that the author simply had not assimilated all he had crammed from some algebra text. In at least one place (middle of page 46) he finds himself in a bind and resorts to sheer bluff.

Unfortunately, there is not room enough in a review such as this to present all the evidence to support the preceding comments. To do so properly would involve rewriting the two chapters in question. This really should have been done, with competent coaching, of course, before the book was accepted for publication. To the prospective user of this book the reviewer's advice is, "Caveat emptor!"

On the conceptual side, there is a point which should be brought out: in setting up his class membership and class separation criteria, the author has made use of only a small part of the information about a set of points which is contained in the covariance matrix associated with the set, namely, the trace of this matrix, which is the variance of the set. There are N-1 other parameters, the remaining coefficients of the characteristic equation, which also have geometrical interpretations as mean-square areas of triangles formed by three points, mean-square volumes of tetrahedra formed by four points, and so on. Alternatively, there are N eigenvalues whose sum is the variance of the set. Certain well-known symmetric functions of these are again the coefficients of the characteristic equation. Surely, the information contained in these other parameters ought to be usable in building sharper criteria for determining class membership.

An interested reader will probably find, as the reviewer did, that studying this book is a rewarding experience. Perhaps its greatest contribution lies in the stimulation of further research.

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92[Z].—Thomas C. Bartee, Irwin L. Lebow, & Irving S. Reed, *Theory and Design of Digital Machines*, McGraw-Hill Book Company, Inc., New York, 1962, ix + 324 p., 23 cm. Price $11.50.

This is a senior or first-year graduate level introductory text on the logic design of digital machines. It does not cover electronic circuit design, components, programming, or arithmetic algorithms, and it discusses numerical representations only briefly in an appendix.

For the purposes of this book, a digital machine is viewed as a system of registers that store binary scalars or vectors and associated combinational switching circuits that produce binary scalar or vector functions of the contents of registers. The basic process is the transfer of the contents or a function of a register into another register. Functionally the machine is completely described by a list of all possible transfers, with the timing and control signals on which they occur. "The design process is then divided into three phases: (1) the system design, which sketches in the general configuration of the machine and specifies the general class of hardware to be used; (2) the structural design, which describes the system in terms of transfer relations; and (3) the logic design, which realizes the transfer relations by means of Boolean equations." This design method is developed largely by examples in five chapters, which include descriptions of a simple general-purpose computer, a radar data digitizer, and a digital differential analyzer.

Five chapters comprise an introduction to switching theory. One introduces Boolean algebra informally by the method of perfect induction, while another develops it axiomatically via Boolean rings and fields. One on minimization techniques describes the Quine-McCluskey method and extends it to multiple output networks. Sequential machines of the Moore and Meahly types are described by flow tables and state diagrams. In the last chapter, flow tables are reduced by merging equivalent or compatible states and a final section is devoted to Turing machines.

At the end of each chapter is a set of problems and a usually extensive bibliography that will lead the reader on to topics not covered in the book.

This very readable book presents probably as uniform and systematic a technique as has been devised for the design of digital machines; though, of course, a host of system design considerations lie outside the scope of the method.

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93[Z].—Bernard A. Galler, *The Language of Computers*, McGraw-Hill Book Company, Inc., New York, 1962, x + 244 p., 23 cm., Price $8.95.

The reviewer must confess to a certain excitement as he first opened this book. What could be *the* language of computers? As it turns out, the choice of the title can only be laid to literary license, perhaps with the marketplace in view. How else could one account for the page headings opposite one another on pages 196–197: "The Language of Computers"—"Other Computer Languages"? So let us be honest at the outset and recognize that this is a book written to promote a particular computer language, namely, MAD.

The author has written "for the person who is interested in learning how problems are solved on electronic computers." To achieve his purpose, he has organized the body of the book into cycles consisting of the formulation of a problem, the development of an algorithm for solving the problem, discussion of the special linguistic requirements imposed by the algorithm on the programming language that is being used (essentially MAD), and finally, as the culminating phase, the display of a program in MAD that effects the desired solution. By this procedure the introduction of the various features of the language takes place in a natural and persuasive manner, and the student gains practice in the mode of thinking that one must adopt in order best to exploit the contribution of the computer.

Somewhat different critical standards must be used to measure those portions of the book that are substantially independent of any particular programming language and those that are MAD-oriented. To those in the first category, comprising, say, 60% of the first ten chapters, the author has brought the full power of a talent for vivid, thorough exposition. Here we find rapport with the reader established at once, along with the clear impression that the writer is sympathetically aware of the difficulties that face the novice in the field. As by-products we have little introductory essays on certain topics, as, for example, arithmetic congruence, random numbers, sorting, switching functions, and the solution of systems of equations—gems in their own right and skillfully set into the main structure.

However, the author's inclination toward hucksterism is manifested not only in naming the book, as already noted, but in titling his chapters as well. One feels that the material under "The Secret-code Problem" was contrived to fit the title rather than included because of its intrinsic suitability, and to call his excellent ninth chapter "A Program to Produce Programs" instead of "A Program to Produce Network Descriptions" smacks of hypocrisy, which the opening remarks of that chapter fail to mitigate.

One's estimate of the success of the portions of the book in which MAD is described or used will naturally be colored by his own bias as to the merits of MAD vis-à-vis alternative languages. Under "Other Computer Languages" we find FORTRAN and ALGOL briefly treated, and there are appendices giving a feature-by-feature translation from MAD to each of these insofar as translation is possible.

We are entering an era of books about programming in specific programming languages, and those who are building libraries of such books will want to have MAD represented by this interesting, well executed work.

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94[Z].—G. M. Hartley, *An Introduction to Electronic Analog Computers*, John Wiley & Sons, New York, 1963, vii + 155 p., 19.5 cm. Price $4.50.

The analog computer is presented in its simplest form, as a tool to solve linear differential equations with constant coefficients. As a result, the operational amplifier is the only active device on which the book concentrates. The fact that the author is an engineer is apparent because the bulk of the material analyzes the characteristics of d.c. amplifiers, while comparatively little material is spent on the application of equipment.

The level of the book is such that it could easily be understood by a junior in electrical engineering. With prerequisites of differential equations and a course in electronics, the material in the book could be covered in eight one-hour lectures.

The first two chapters are background in nature; the first deals with the history of the analog and digital computers, while the second is a general survey of applications for various types of computers. The essence of the book is contained in the next four chapters. Chapter 3 illustrates the role of the operational amplifier in performing mathematical operations. Chapter 4 describes in detail the programming, scaling, and wiring of a second-order differential equation with constant coefficients for a typical computer. This chapter also touches on time scaling, amplifier balancing, and potentiometer loading. Chapter 5 and 6, which comprise roughly half of the book, give a relatively simple but comprehensive coverage of vacuum tube and transistorized d.c. amplifiers. The operation of vacuum tubes and transistors as amplifiers is described, and gain equations are derived. Included are descriptions of typical operational amplifier circuits and the derivation of equations of errors resulting from finite open-loop gain and from some drift sources. On the other hand, the dynamics of the amplifier (stability, bandwidth, etc.) are not investigated. The final chapter is a very short survey of some nonlinear computing equipment, including multipliers and function generators. It also includes a discussion of analog-to-digital and digital-to-analog conversion. No breakdown, however, is given of the various types of errors (bias, gain error, dynamic lag, etc.), nor is the use of multipliers to perform division discussed.

The book contains no new material, and has comparatively little value as a reference book. While the book does not cover a large number of topics such as error detecting techniques, recording equipment, and proper balance of computing elements, it does describe the central role played by the higher-gain amplifier in an analog computer.

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