REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS

1[A-F, H-M, R, SX].—MILTON ABRAMOWITZ & IRENE A. STEGUN, Editors, *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables*, Applied Mathematics Series, No. 55, U. S. Government Printing Office, Washington, D. C., 1964, xiv + 1046 p., 27 cm. Price $6.50.

This elaborate, definitive handbook represents the impressive consummation of a decade of planning and preparation by many persons under the broad supervision of a committee originally elected by the participants in a Conference on Tables held at the Massachusetts Institute of Technology in September 1954.

The report of this Committee set forth the suggestion that, with the financial assistance of the National Science Foundation, the National Bureau of Standards undertake the production of "a Handbook of Tables for the Occasional Computer, with tables of usually encountered functions and a set of formulas and tables for interpolation and other techniques useful to the occasional computer." The existence of a real need was thus recognized for a modernized version of the classical tables of functions of Jahnke-Emde.

As early as May 1952 Dr. Abramowitz had mentioned preliminary plans for such an undertaking, and it was under his planning and supervision that active work on the project began at the Bureau of Standards in 1956. Following his untimely death in 1958, the project has been carried to its successful completion under the editorial direction of his former co-worker, Miss Stegun.

According to the Introduction, "this present Handbook has been designed to provide scientific investigators with a comprehensive and self-contained summary of the mathematical functions that arise in physical and engineering problems." To this end, this book extends the work of Jahnke-Emde by presenting more extensive and more accurate numerical tables and by giving more extensive compilations of mathematical properties of the tabulated functions. The number of functions considered has been increased by the inclusion of Coulomb wave functions, hypergeometric functions, parabolic cylinder functions, spheroidal wave functions, orthogonal polynomials, Bernoulli and Euler polynomials, arithmetic functions, Debye functions, Planck's radiation function, Einstein functions, Sievert's integral, the dilogarithm function, Clausen's integral, and vector-addition (Wigner or Clebsch-Gordan) coefficients.

The scope of this handbook may be inferred from the following enumeration of the titles and names of the contributors of the 29 chapters comprising the body of it.

1. Mathematical Constants—David S. Liepman,
2. Physical Constants and Conversion Factors—A. G. McNish,
3. Elementary Analytical Methods—Milton Abramowitz,
4. Elementary Transcendental Functions—Ruth Zucker,
5. Exponential Integral and Related Functions—Walter Gautschi & William F. Cahill,
6. Gamma Function and Related Functions—Philip J. Davis,
7. Error Function and Fresnel Integrals—Walter Gautschi,
8. Legendre Functions—Irene A. Stegun,

9. Bessel Functions of Integer Order—F. W. J. Olver,
10. Bessel Functions of Fractional Order—H. A. Antosiewicz,
11. Integrals of Bessel Functions—Yudell L. Luke,
12. Struve Functions and Related Functions—Milton Abramowitz,
13. Confluent Hypergeometric Functions—Lucy Joan Slater,
14. Coulomb Wave Functions—Milton Abramowitz,
15. Hypergeometric Functions—Fritz Oberhettinger,
16. Jacobian Elliptic Functions and Theta Functions—L. M. Milne-Thomson,
17. Elliptic Integrals—L. M. Milne-Thomson,
18. Weierstrasse Elliptic and Related Functions—Thomas H. Southard,
19. Parabolic Cylinder Functions—J. C. P. Miller,
20. Mathieu Functions—Gertrude Blanch,
21. Spheroidal Wave Functions—Arnold N. Lowan,
22. Orthogonal Polynomials—Urs W. Hochstrasser,
23. Bernoulli and Euler Polynomials, Riemann Zeta Function—Emilie V. Haynsworth & Karl Goldberg,
24. Combinatorial Analysis—K. Goldberg, M. Newman & E. Haynsworth,
25. Numerical Interpolation, Differentiation and Integration—Philip J. Davis & Ivan Polonsky,
26. Probability Functions—Marvin Zelen & Norman C. Severo,
27. Miscellaneous Functions—Irene A. Stegun,
28. Scales of Notation—S. Peavy & A. Schopf,
29. Laplace Transforms.

Within each chapter devoted to a function or a class of functions the material has been uniformly arranged to include mathematical properties, numerical methods, references, and tables, respectively.

We are informed in the Introduction that the classification of mathematical functions and the organization of the chapters in the Handbook has been based on *An Index of Mathematical Tables*, by A. Fletcher, J. C. P. Miller, and L. Rosenhead, which has been published in a second, two-volume edition in 1962, with L. J. Comrie added as a co-author. The mathematical notations have followed those adopted in standard texts, in particular, *Higher Transcendental Functions*, Volumes 1–3, by A. Erdélyi, W. Magnus, F. Oberhettinger, and F. G. Tricomi.

In the numerical tables no attempt has been made to fix the number of significant figures presented throughout the Handbook, because of the prohibitive labor and expense required to do so. However, the great majority of the tables provide at least five significant figures, at tabular intervals sufficiently small to permit linear interpolation accurate to four or five figures. Exceptions include certain tables designed to furnish key values, such as Table 9.4, entitled Bessel Functions—Various Orders, which gives $J_n(x)$ and $Y_n(x)$ to 10 significant figures for $n = 0(1)20(10)50, 100$ and $x = 1, 2, 5, 10, 50, 100$.

In those working tables of functions wherein linear interpolation is inadequate, Lagrange's formula or Aitken's method of iterative linear interpolation is recommended. These procedures, as well as others, are discussed in the Introduction and in Chapter 25. Tables are not provided with differences, so as to effect a saving of space that has been used for the tabulation of additional functions. However, at the foot of most of the tables there appears a symbolic statement of the maximum error arising from linear interpolation and the number of function values required in Lagrange's formula or in Aitken's method to interpolate to full tabular accuracy, as illustrated on p. x in the Introduction.

The 184 numerical tables appearing in this volume are too numerous to describe individually in a review, although their extensive range may be inferred from the listing of chapters given above. Approximately one-third of these tables have been extracted or abridged, with appropriate acknowledgment, from the numerous well-known tabular publications of the National Bureau of Standards, another third were taken from tables of the British Association for the Advancement of Science, the Harvard Computation Laboratory, H. T. Davis, L. M. Milne-Thomson, A. J. Thompson, C. E. Van Orstrand, and many others, and the remainder are the results of new computations.

The claim is made on p. ix that the maximum end-figure error is 0.6 unit in all tables of elementary functions in the Handbook, and is 1 unit (or in rare cases, 2 units) in tables of higher functions. This reviewer has carefully examined Table 1.1 (Mathematical Constants) and discovered several errors exceeding this limit. These corrections and others submitted by other users are presented in the appropriate section of this issue.

Despite such minor flaws, which are almost unavoidable in a work of this magnitude, the Handbook is a truly monumental reference work, which should be in the possession of all researchers and practitioners in the fields of numerical analysis and applied mathematics.

J. W. W.

2[D].—NORTON GOODWIN, *Seven Place Cosines, Sines, and Tangents for Every Tenth Microturn*, Society of Photographic Scientists and Engineers, Washington, D. C., 1964, 79 p. (unnumbered), 26 cm. Price $2.00.

According to the author, these tables were designed primarily to facilitate desk-calculator transformations of the coordinates of artificial earth satellites. However, as he states, they should also prove useful in space navigation and in electrical engineering, where cyclical coordinate changes are encountered.

The tables consist of sine $2\pi x$ and cos $2\pi x$ for $x = 0(0.00001)0.25000$ and tan $2\pi x$ for $x = 0(0.00001)0.12500$, all to 7D. The (linearly interpolable) values of the sine and cosine are arranged semiquadrantally, without differences, on facing pages, each containing 500 distinct entries, arranged in the conventional ten columns, supplemented by an eleventh, which gives the same tabular value in any row as the first column in the succeeding row, thereby facilitating the use of the tables in obtaining functional values for complementary arguments. Economy of space is attained by separation of the first two decimal digits and listing only the last five decimal digits in all the columns after the first. Change in the second decimal place occurring within a line is signalled by boldface numerals.

The author has communicated to this reviewer the information that these tables were computed on an IBM 7090 system at The Rand Corporation, using double-precision arithmetic to evaluate the functions by Taylor series prior to final rounding.

In the Preface to these tables we are referred to a comparable, unpublished table of the sine and cosine to 15D prepared by Bower [1], which is available in listed and punched-card form.

The present tables were photographically composed from digital-computer tape records. The typography was prepared by a commercial printer on a conventional photocomposition unit controlled by perforated paper tapes produced by a converter designed to process magnetic-tape records in a form suitable for general-purpose phototypesetting machines.

The resulting typography is uniformly excellent and the arrangement of the data is attractive. This compilation constitutes a valuable contribution to the limited existing literature [2] of trigonometric tables based on the decimal subdivision of the circle.

J. W. W.

1. E. C. Bower, *Natural Circular Functions for Decimals of a Circle*, ms., 1948. Listed and punched-card copies available at nominal cost from The Rand Corporation, Santa Monica, California. [For a review, see *MTAC*, v. 3, 1949, p. 425–426, UMT **77**.]

2. *MTAC*, v. 1, 1943, p. 40; also, A. Fletcher, J. C. P. Miller, L. Rosenhead & L. J. Comrie, *An Index of Mathematical Tables*, 2nd ed., Addison-Wesley Publishing Co., Reading, Mass., 1962, v. I, p. 177–178.

3[G, X].—Marvin Marcus & Henryk Minc, *A Survey of Matrix Theory and Matrix Inequalities*, Allyn and Bacon, Inc., 1964, xvi + 180 p., 24 cm. Price $9.75.

This book is a compendium of many important facts about matrices. Moreover, it starts out, as the authors state, "with the assumption that the reader has never seen a matrix before." It proceeds then, in a logical sequence and in condensed, systematic notation, to state definitions and theorems, the latter generally without proof. Since the purpose is evidently to condense as much material as possible in a short space, "certain proofs that can be found in any of a very considerable list of books have been left out."

It would be indicative of the extent of the coverage to say that if one needed to look up all of the missing proofs, he would have to consult all of a by no means inconsiderable list of books. This was, of course, not the expectation, but the instructor who considers adopting the book as a class text would be well advised to make sure that he can himself supply the proofs that are not readily available to him.

There are three chapters, the first, Survey of Matrix Theory, comprising slightly more than half of the book. Here one finds the expected topics: determinants, linear dependence, normal forms, etc. In addition, one finds somewhat nonstandard material such as permanent, compound and induced matrices, incidence matrices, property L, among others. The next chapter, Convexity and Matrices, develops such inequalities as those of Hölder, Minkowski, Weyl, Kantorovich, and also discusses the Perron-Frobenius theorem, and Birkhoff's theorem on doubly stochastic matrices. The final chapter, Localization of Characteristic Roots, deals almost exclusively with what the reviewer calls exclusion theorems, by contrast with inclusion (e.g., theorems of Temple, of D. H. Weinstein, and of Wielandt). Other topics briefly dealt with are the minimax theorems for Hermitian matrices, and the field of values (one of the omitted proofs is that of the convexity of the field of values).

The few algorithms presented are given solely as constructive existence proofs, and not as computational techniques. Nevertheless, the numerical analyst would find it a handy reference book, with much information condensed into a very small volume. The student will find many challenges, and the careful documentation will permit him to look up the proofs, when necessary, if his library is adequate. The proofreading seems to have been very carefully done, for which the reader can be doubly grateful in view of the compactness.

A. S. H.

4[H, X].—J. F. Traub, *Iterative Methods for the Solution of Equations*, Prentice-Hall, Inc., Englewood Cliffs, N. J., 1964, xviii + 310 p., 24 cm. Price $12.50.

As "iterative methods," the author includes Newton's and the method of false position, but he excludes Graefe's, and even Bernoulli's. By "equations," he means nonlinear equations, no consideration being given to linear systems.

There is a wealth of literature buried in journals on the subject of the numerical solution of equations, but remarkably little in books. In this book there is a fourteen-page bibliography, but only five items, or possibly six, are books devoted exclusively or primarily to the numerical solution of nonlinear equations. Of these, perhaps the best known, and the earliest one to appear in English, is by Ostrowski, published in 1960. Books on numerical methods in general usually do no more than summarize three or four of the standard methods, and sometimes not even that.

The author attempts to develop a general theory of the particular class of methods under consideration. Accordingly, the initial chapters present rather general theorems on convergence, and outline methods of constructing functions for iteration. Subsequent chapters deal with particular types (e.g., one-point), or with particular complexities (e.g., multiple roots). One short chapter deals with systems, and a final chapter gives a compilation of particular functions. Several appendices give background material (e.g., on interpolation), some extensions (e.g., "acceleration"), and discussion of some numerical examples. But except for this, very little is said about computational error.

The author has attempted to trace the methods of their sources, and references can be found in the bibliography to Halley (1694) and Lambert (1770), though not to Newton! An interesting feature of the bibliography is the listing with each item of each page in the text where reference is made to this item.

The rather elaborate systematic notation permits greater compactness, but may seem a bit forbidding to the casual reader who wishes to use the book mainly for reference. As a text, its value could have been enhanced by the addition of some problems. But as a systematic development of a large and important class of methods, the book is by far the most complete of anything now to be found in the literature.

A. S. H.

5[I].—D. S. Mitrinović & R. S. Mitrinović, *Tableaux d'une classe de nombres reliés aux nombres de Stirling*, (a) II. *Publ. Fac. Elect. Univ. Belgrade* (*Serie*: *Math. et*

Phys.) No. 107, 1963, 77p., (b) III. *Belgrade, Mat. Inst., Posebna izdanja,* Knjiga 1 (*Editions speciales*, 1), 1963, 200 p.

A number of tables by these authors have been reviewed in *Math. Comp.* from time to time. In recent years the tables have most commonly appeared, as does (a) above, in *Publ. Fac. Elect. Univ. Belgrade*, whereas in (b) we now have the first "book" in a new series of occasional special publications of the Mathematical Institute at Belgrade; the new series is destined to contain monographs, extended original articles and original numerical tables.

In (a) and on p. 13–156 of (b) we find two continuations of tables (*Publ. Fac. Elect.*, No. 77, 1962) already reviewed in *Math. Comp.*, v. 17, 1963, p. 311. The integers ${}^pP_n{}^+$ defined by

$$\prod_{r=0}^{n-1} (x - p - r) = \sum_{r=0}^{n} {}^pP_n{}^+x^+,$$

previously listed for $p = 2(1)5$, are now listed in (a) for $p = 6(1)11$ and in (b) for $p = 12(1)48$. In both (a) and (b) the values of the other arguments for given p are $n = 1(1)50 - p$, $r = 0(1)n - 1$; when $r = n$, the value of ${}^pP_n{}^r$ is obviously unity.

In the second part (p. 159–200) of (b) are tables of the integers ${}^\nu S_n{}^k$ defined by

$$t(t - 1) \cdots (t - \nu + 1)(t - \nu - 1) \cdots (t - n + 1) = \sum_{k=1}^{n-1} {}^\nu S_n{}^k t^{n-k},$$

where it is to be noted that the left side contains $(n - 1)$ factors, $(t - \nu)$ being omitted. The table is for arguments $n = 3(1)26$, $\nu = 1(1)n - 2$, $k = 1(1)n - 1$.

The tabular values were computed on desk calculating machines, and all are given exactly, even when they contain more than 60 digits. Various spot checks were made in the Instituto Nazionale per le Applicazioni del Calcolo at Rome and in the Computer Laboratory of the University of Liverpool. Details of some of the verificatory computations are given.

A. F.

6[I, X].—Peter Henrici, *Error Propagation for Difference Methods*, John Wiley & Sons, Inc., New York, 1963, vi + 73 p., 24 cm. Price \$4.95.

This little monograph is a sequel to the author's now classic *Discrete Variable Methods in Ordinary Differential Equations*, published by Wiley in 1962. The subject here is the use of multi-step methods for systems of equations, and the treatment, though in the spirit of the previous volume, is independent of it. The author remarks, however, that to pass from one to several variables was "not a mere exercise in easy generalization," so that the reader would be well advised to read the volumes in the order of their appearance. The two together provide a unified treatment of the subject that will not soon be surpassed.

A. S. H.

7[K].—I. G. Abrahamson, *A Table for Use in Calculating Orthant Probabilities of the Quadrivariate Normal Distribution*, 5 p.+ 71 computer sheets, ms. deposited in UMT File.

This manuscript table gives the probability that four jointly normally distributed random variables will be simultaneously positive (orthant probability) when the distribution has a mean of zero and a correlation matrix of the form

$$\begin{bmatrix} 1 & A & 0 & 0 \\ A & 1 & B & 0 \\ 0 & B & 1 & C \\ 0 & 0 & C & 1 \end{bmatrix}$$

where A, B, and C are non-negative.

The values of this probability are tabulated to 6D for $A = 0(0.05)0.95$, $B = 0(0.05)0.95$, and $C = 0(0.01)0.99$, consistent with the correlation matrix being positive definite. The author claims accuracy of the tabular values to at least 5D, on the basis of a number of checks. She briefly discusses the question of interpolation, and presents a method for using this table to calculate the orthant probability in the general case.

J. W. W.

8[K].—Norman T. J. Bailey, *The Elements of Stochastic Processes with Applications to the Natural Sciences*, John Wiley & Sons, Inc., New York, 1964, xi + 249 p., 23 cm. Price $7.95.

This book is highly recommended reading, and is a good introductory text in applied stochastic processes for three reasons:

(1) It is clearly written, proceeding by examples; it is very readable and contains a number of exercises.

(2) It attempts to be broad, covering a number of areas, and has chapters on recurrent events, random walks, Markov chains and processes, birth-death processes, queues, epidemics, diffusion, and some non-Markovian processes.

(3) It does not belabor any one topic; it is, therefore, not too voluminous, and hence is challenging to the interested reader.

The author's experience in the field has produced a very fine contribution.

T. L. Saaty

United States Arms Control and Disarmament Agency
Washington 25, D. C.

9[K].—Statistical Engineering Laboratory, National Bureau of Standards, *Table of Percentage Points of the χ^2-Distribution*, Washington, D. C., August 1950, 1 + 7 p. Deposited in UMT File.

This is a composite table made up from three previously published tables and by transformation or by interpolation in them.

The table uses the format of Thompson [2] and gives the percentage points of χ^2 for the following values of ν and P:

ν	P and $1 - P$
1(1)30	.005, .01, .02, .025, .05, .10, .20, .25, .30, .50
31(1)100	.005, .01, .025, .05, .10, .25, .50
102(2)200	.01, .10, .25, .50
2(2)200	.000001, .0001

All entries are given to three decimal places except 2(2)200, .000001 and .999999 which are to two.

Special features of the present table are coverage of even values of ν from 102 to 200 as well as values of P and $1 - P$ equal to .0001 and .000001.

It is noted that the Greenwood and Hartley *Guide to Tables in Mathematical Statistics* in its list of tables of percentage points of χ^2, p. 140–143, makes no mention of the fact that Campbell's Table II may be used to obtain percentage points of the χ^2, taking $2c$ in Campbell as ν and $2a$ in Campbell as χ^2. This was done in obtaining certain entries of the present table. The Greenwood and Hartley *Guide* does, however, list Campbell on p. 151 under "Percentage points of the Poisson distribution; confidence intervals for m."

Many entries in the present table were obtained by interpolation in Thompson's Table, using the four-point Lagrangian formula, Eq. (7) in [2].

In the middle of the distribution the interpolates agree through the third decimal with Campbell's values. In the tails of the distribution agreement is somewhat poorer, a difference of 1 or 2 units in the third decimal being usual.

J. M. Cameron

Statistical Engineering Laboratory
National Bureau of Standards
Washington 25, D. C.

1. R. A. Fisher, *Statistical Methods for Research Workers*, 11th Ed., Table III, Oliver and Boyd, Edinburgh, 1950.
2. Catherine M. Thompson, "Table of the percentage points of the χ^2-distribution," *Biometrika*, v. 32, Part II, October, 1941.
3. George A. Campbell, "Probability curves showing Poisson's exponential summation," *Bell System Tech. J.*, January, 1923.
4. J. Arthur Greenwood & H. O. Hartley, *Guide to Tables in Mathematical Statistics*, Princeton Univ., 1962.

10[L].—A. R. Curtis, *Tables of Jacobian Elliptic Functions whose Arguments are Rational Fractions of the Quarter Period*, National Physical Laboratory, *Mathematical Tables*, Vol. 7, Her Majesty's Stationery Office, London, 1964, iii + 81 p., 28 cm. Paperback. Price 15 shillings ($3.00).

Table 1 (p. 8–78) has one page for each of the 71 arguments $q = 0(0.005)0.35$, where $q = \exp(-\pi K'/K)$ is Jacobi's nome, and K, K' are the usual quarter periods. Each page gives, entirely to 20 D, values of k, K, sn (mK/n), cn (mK/n), dn (mK/n), where k is the modulus and the values of m/n form the Farey series $\mathfrak{F}_{15}$, i.e., m and n take all positive integral values for which $m < n \leqq 15$ and m/n is in its lowest terms, while the various m/n are arranged in ascending order of magnitude. This Farey series of arguments also, as it happens, has 71 members.

Table 2 (p. 80–81) gives, again for $q = 0(0.005)0.35$ and entirely to 20 D, the values of k, k', the modular angle $\theta = \sin^{-1}k$ in radians, K, K' and the period-ratio K'/K.

The tables were prepared to facilitate filter design computations, as well-known tables by the Spenceleys, which proceed by ninetieths of K, did not contain all the desired m/n arguments nor always give the desired number of decimal places. The argument q was used in order that the distribution of k-values should be dense as k approaches unity (25 of the 71 values of k exceed 0.99), and no interpolation facilities are provided in either table because in the design problem k can usually be chosen to coincide with a tabular value. The methods of computation on DEUCE are described in the introductory text.

A. F.

11[L].—CHIH-BING LING, *Tables of Values of σ_{2s} Relating to Weierstrass' Elliptic Function*, Institute of Mathematics, Academia Sinica, Taiwan, China, 1964. Ms. of 7 typewritten sheets deposited in UMT File.

These manuscript tables of coefficients σ_{2s} to 16 S, for $s = 2(1)25$, which appear in the expansion of Weierstrassian elliptic functions when $\omega = ai$ and when $\omega = \frac{1}{2} + ci$, are described in a paper of the same title by Professor Ling in this issue, where an abridgment to $s \leqq 10$ appears.

J. W. W.

12[L].—T. Y. NA & A. G. HANSEN, *Tabulation of the Hermite Function with Imaginary Arguments, $H_n(ix)$*, ms. of 2 typewritten pages + 4 computer sheets of tables, deposited in the UMT File.

In a recent similarity analysis of the flow near an oscillating plate, the authors required numerical values of the Hermite function with an imaginary argument. Herein they present tables of $i^{-1} H_m(ix)$, $m = 1(2)15$, and of $H_m(ix)$, $m = 2(2)16$, both for $x = 0(0.1)5$, to 8 S in floating-point form, as calculated on the IBM 7090 system at the University of Michigan. Previous tabulations of this function have been limited to real arguments.

J. W. W.

13[L].—ROBERT SPIRA, *Calculation of the Riemann Zeta Function*, Memo 64-1-3,16, Special Research in Numerical Analysis, Duke University, Durham, North Carolina, 16 March 1964, 12 microcards (24 sides) deposited in UMT File.

This is a reproduction on microcards of the main table of 940 pages of values of the Riemann zeta function (together with thirteen introductory tables) described previously in this journal. (See v. 18, 1964, p. 519, UMT **78.**)

The author has informed the editors that a limited number of copies of these cards are available upon request to Duke University.

J. W. W.

14[L, M].—ATHENA HARVEY, *Tables of $\int_0^x e^{-bt} I_0(t)$*, four typewritten pages deposited in UMT File.

In a brief explanatory introduction the author states that this integral appears in the analytical expression of the integrated reflecting power of X-rays for absorbing perfect crystals [1].

The tabular data were computed on an IBM 1620 and are listed to 6 S for $b = 0(0.1)1.0$ and $x = 0(0.1)10.0$. Accuracy to within 2 units in the last place is claimed.

Reference is also made to the treatise by Luke [2], which includes a discussion of this integral, and gives an equivalent expression in closed form when $b = 1$.

J. W. W.

1. NORIO KATO, "Integrated intensities of the diffracted and transmitted X-rays due to ideally perfect crystals (Laue case)," *J. Phys. Soc. Japan*, v. 10, 1955, p. 46–55.
2. YUDELL L. LUKE, *Integrals of Bessel Functions*, McGraw-Hill Book Co., 1962, p. 122 and Chapter X.

15[L, M].—N. SKOBLIÂ, *Tables for the Numerical Inversion of Laplace Transforms*, Academy of Sciences of USSR, Moscow, 1964, 44 p., 22 cm. Paperback. Price 13 kopecks.

Consider the Laplace transform pair (which we assume exists)

$$(1)\qquad p^{-s}g(p) = \int_0^\infty e^{-pt}f(t)\,dt, \qquad f(t) = (2\pi i)^{-1}\int_{c-i\infty}^{c+i\infty} p^{-s}e^{pt}g(p)\,dp$$

where $c > 0$ and c lies to the right of all singularities of $g(p)$. Suppose that $g(p)$ is known and can be represented by a polynomial in $1/p$. Then an approximation formula for $f(t)$ is readily constructed from the second formula in (1). Now it may be shown that

$$(2)\qquad \begin{aligned} &\int_{c-i\infty}^{c+i\infty} e^{p}p^{-s}P_n(p^{-1})P_m(p^{-1})\,dp = \delta_{mn}\,h_n\,, \\ &h_n = \frac{(-1)^n n!}{(2n-1+s)\Gamma(n-1+s)} \end{aligned}$$

where δ_{mn} is the Kronecker delta, and in hypergeometric notation,

$$(3)\qquad \begin{aligned} P_n(x) &= 2F_0(-n, n+s-1; x) \\ &= (2-2n-s)_n\, x^n\, {}_1F\left(-n; 2-2n-s; \frac{1}{x}\right). \end{aligned}$$

This shows that numerous properties of $P_n(x)$ follow from known results on confluent hypergeometric functions. In view of (2), we have the approximation

$$(4)\qquad f(t) \sim (2\pi i)^{-1}\sum_{k=1}^{n} A_{k,n}g(p_k)$$

where

$$(5)\qquad P_n(p_k^{-1}) = 0, \qquad k = 1, 2, \cdots, n,$$

and the weights, $A_{k,n}$ are the Christoffel numbers. Thus, the approximation is exact if indeed $g(p)$ is a polynomial in $1/p$ of degree $(2n-1)$. A convenient formula for the weights is

$$(6)\qquad A_{k,n} = \sum_{m=0}^{n-1} \{P_m(p_n^{-1})\}^2/h_m\,.$$

The pamphlet gives some properties of $P_n(x)$, though (3) and (6) are not among them. The following are tabulated to 7S: p_k, $A_{k,n}$ for $k = 1(1)n$, $n = 1(1)10$, and $s = 0.1(0.1)3.0$.

The case $s = 1$ has been treated by H. Salzer. (See "Orthogonal polynomials arising in the numerical evaluation of Laplace transforms," *MTAC*, v. 9, 1955, p. 164–177, and "Additional formulas and tables for orthogonal polynomials originating from inversion integrals," *J. Math. Phys.*, v. 40, 1961, p. 72–86.) These latter sources give the zeros and weights to 15D for $n = 1(1)15$. Note that Salzer's quadrature formula is exact if $g(p)$ is a polynomial in $1/p$ of degree $2n$ such that $g(\infty) = 0$. In the booklet under review, the quadrature formula is exact if $g(p)$ is of degree $(2n - 1)$, but $g(\infty)$ need not vanish. Thus the Christoffel numbers in Salzer's work differ from those of the present author. However, the zeros are the same. Twice the negatives of the zeros of $P_n(x)$ have been tabulated mostly to 5 D by V. N. Kublanovskaia and T. N. Smirnova. (See "Zeros of Hankel functions and some related functions," *Trudy. Mat. Inst. AN*, USSR No. 53, 1959, p. 186–192. This is also available as Electronic Research Directorate, Air Force Cambridge Research Laboratories Report AFCRL-TN 60-1128, October 1960.)

Y. L. L.

16[M, X].—E. L. Albasiny, R. J. Bell & J. R. A. Cooper, *A Table for the Evaluation of Slater Coefficients and Integrals of Triple Products of Spherical Harmonics*, National Physical Laboratory Mathematics Division Report No. 49, 1963, xi + 163 pages.

This is a table of integrals

$$\int_{-1}^{+1} \Theta_{l_1}^{m_1}(x)\Theta_{l_2}^{m_2}(x)\Theta_{l_3}^{m_3}(x)\,dx$$

where

$$\Theta_l^m(x) = (-1)^m \left[\frac{(2l+1)}{2}\frac{(l-m)!}{(l+m)!}\right]^{1/2} \frac{(1-x^2)^{m/2}}{l!2^l}\frac{d^{l+m}}{dx^{l+m}}(x^2-1)^l, \qquad 0 \leqq m \leqq l$$

is an associated Lengendre function. These integrals are closely related to integrals which occur in molecular structure calculations (see, for example, [1]).

Values of the above integrals are tabulated to 12 decimal places for (m_i, l_i integers)

$$m_1 \pm m_2 \pm m_3 = 0,$$
$$l_1 \leqq l_2 \leqq l_3,$$
$$l_1 + l_2 + l_3 \text{ even},$$
$$| l_1 - l_2 | \leqq l_3 \leqq l_1 + l_2,$$
$$l_1, l_2 \leqq 12, l_3 \leqq 24.$$

Under these conditions the integrand is a polynomial of degree $\leqq 48$, and thus can be calculated exactly, using an n-point Gauss-Legendre quadrature formula [3, p. 107–111] for $n \geqq 25$. The tables were computed using the 25-point formula tabulated by Gawlik [2] and recomputed as a check using the 26-point formula. The calculations were carried out on the ACE computer, which has a 46-bit floating-point mantissa. The tabulated values are exact to within two units in the last place.

The above integrals are also known in closed form [4]. However, the expressions for them are not as convenient for computations as the quadrature formulas.

A. H. STROUD

Computation Center
University of Kansas
Lawrence, Kansas 66045

1. A. R. EDMONDS, *Angular Momenta in Quantum Mechanics*, Princeton Univ. Press, Princeton, N. J., 1957, Chapters 3, 4.
2. H. J. GAWLIK, *Zeros of Legendre Polynomials of Orders 2-64 and Weight Coefficients of Gauss Quadrature Formulae*, Armament Research and Development Establishment Memorandum (B) 77/58, 1958. [See *Math. Comp.*, v. 14, 1960, p. 77, RMT 4.]
3. V. I. KRYLOV, *Approximate Calculation of Integrals*, Macmillan, New York, 1962.
4. J. MILLER, "Formulas for integrals of products of associated Legendre or Laguerre functions," *Math Comp.*, v. 17, 1963, p. 84–87.

17[M, X].—L. KRUGLIKOVA, *Tables for Numerical Fourier Transformations*, Academy of Sciences of USSR, Moscow, 1964, 30 p., 22 cm. Paperback. Price 13 kopecks.

This pamphlet contains Gaussian quadrature formulas of the form

$$\int_0^\infty (1 + \sin x) f(x)\, dx \cong \sum_{k=1}^{n} A_k f(x_k),$$

$$\int_0^\infty (1 + \cos x) f(x)\, dx \cong \sum_{k=1}^{n} A_k f(x_k),$$

which are exact whenever

$$f(x) = (1 + x)^{-s-i}, \qquad i = 0, 1, \cdots, 2n - 1.$$

Values of x_k and A_k are given for $n = 1, \cdots, 8$ for the following values of the parameter s:

$$s = \tfrac{5}{4}, \tfrac{4}{3}, \tfrac{3}{2}, \tfrac{5}{3}, \tfrac{7}{4}, 2, \tfrac{9}{4}, \tfrac{7}{3}, \tfrac{5}{2}, \tfrac{8}{3}, \tfrac{11}{4}, 3, \tfrac{13}{4}, \tfrac{10}{3}, \tfrac{7}{2}, \tfrac{11}{3}, \tfrac{15}{4}, 4.$$

The x_k are given to between 8 and 10 significant figures and the A_k to between 5 and 11.

These formulas can also be used to approximate integrals of the form

$$\int_0^\infty f(x) \sin \alpha x\, dx, \qquad \int_0^\infty f(x) \cos \alpha x\, dx.$$

This is done by writing these as

$$\int_0^\infty \phi(y) \sin y\, dy, \qquad \int_0^\infty \phi(y) \cos y\, dy,$$

$$\alpha x = y, \qquad \phi(y) = \frac{1}{\alpha} f\left(\frac{y}{\alpha}\right),$$

and approximating

$$\int_0^\infty \phi(y)\, dy$$

by some other method. Then, for example,

$$\int_0^\infty \phi(y) \sin y\, dy = \int_0^\infty (1 + \sin y)\phi(y)\, dy - \int_0^\infty \phi(y)\, dy.$$

A. H. Stroud

18[P, X].—L. S. Pontryagin, V. G. Boltyanskii, R. V. Gamkrelidze & E. F. Mishchenko, *The Mathematical Theory of Optimal Processes*, John Wiley & Sons, Inc., New York, 1962, viii + 360 p., 23 cm. Price $11.95.

One of the major problems of the modern mathematical theory of control processes can be posed in the following terms: "Given a vector differential equation $dx/dt = g(x, y)$, $x(0) = c$, where x represents the state of a physical system at time t, the state vector, and $y(t)$ represents the control vector, determine $y(t)$ so as to minimize a given scalar functional $J = \int_0^T h(x, y, t)\, dt$, where x and y are subject to local constraints of the form $r_i(x, y) \leqq 0$, $i = 1, 2, \cdots, N$, global constraints of the form $\int_0^T h_i(x, y)\, dt \leqq k_i$, and terminal conditions of the form $f_i(x(T), y(T), T) \leqq 0$." In some cases of importance, T itself depends upon the history of the process, $T = T(x, y)$, and, indeed, may be the quantity we wish to minimize.

The book under review represents a fine and substantial contribution to a new mathematical domain. The major theme of the work is the "maximum principle," an analytic condition which provides important information concerning the structure of extremals, in the terminology of the calculus of variations, or of optimal policies, in the parlance of dynamic programming and control theory.

Since the book is an excellent one that will be widely read and used, it is worthwhile to analyze its objectives and results carefully within the framework of the classical theory of the calculus of variations, and with the desiderata of modern control theory in mind.

In the simplest version of classical variational theory, there are no local or global constraints. The first variation yields the Euler equation, generally a nonlinear differential equation, with two-point boundary conditions. For a variety of reasons, this direct approach is seldom effective computationally. If global constraints are present, Lagrange multipliers may be used to reduce the problem to one without constraints, at the expense of further computational difficulties.

If local constraints of the type indicated above are present, as they are in a large number of the most important classes of processes, the situation is even more complex. This is due to the fact that sometimes the Euler equation holds and sometimes the constraints determine the extremal, or policy. Hence, the analytic and computational difficulties that existed before, as far as effective algorithms for the solution are concerned, are now compounded.

Nevertheless, analogues and extensions of the classical results can be obtained. The pioneering work is that of Valentine [1]. Results of Valentine were used by Hestenes in some unpublished work on constrained trajectories in 1949. In 1961

Berkovitz [2] showed how the maximum principle and results of greater generality could be obtained from Valentine's work combined with the classical calculus of variations.

The principal point of all this discussion is that the maximum principle does not provide us with any analytic approaches which we did not already possess, and does not seem to aid us in the fundamental objective of providing numerical answers to numerical questions. Unfortunately, at the present time, we possess no straightforward approach to the effective analytic solution of constrained variational problems.

This does not diminish the value of the book. Its very elegant presentation of results pertaining to extensions of classical problems and its consideration of processes involving time delays and stochastic elements will have a very stimulating effect upon research in this new field. It will serve the very useful purpose of focussing attention upon new, fascinating, and significant areas of investigation.

Let us now discuss some of the contents of the volume, and present some detailed comments. The authors present a general treatment of the control process formulated above, using the maximum principle (which is, as Berkovitz points out, a restatement of the Weierstrass condition as adapted by Valentine), and discuss some quite interesting examples in detail. In particular, they consider the "bang-bang" control process, where $dx/dt = Ax + y$, y is constrained by the conditions that its components can assume only the values ± 1, and it is desired to reach the origin in minimum time. Following this, they discuss a control process involving retardation (work of Kharatishvili), pursuit processes (work of Kelenozheridze), some interesting applications to approximation theory, problems involving constraints on state variables, and finally some stochastic control processes. The discussion of pursuit and stochastic control seems far more difficult and involved than one based upon the functional equation approach of dynamic programming, and is based upon "open loop" control rather than feedback control.

The authors indicate the intimate relation between dynamic programming and the calculus of variations, and state (p. 7): "... Thus, Bellman's considerations yield a good heuristic method, rather than a mathematical solution of the problem," and again (p. 73): "Thus, even in the simplest examples, the assumptions which must be made in order to derive Bellman's equations do not hold."

These statements provoke some further remarks. In the first place, if one refers to Berkovitz's article, it will be seen that the equations derived from the functional equation approach can be made completely rigorous in a number of cases. In those cases where lines of discontinuity, or more generally, surfaces of discontinuity exist ("switching surfaces"), we have a situation similar to the existence of shocks in hydrodynamics. The classical equations exist on both sides of the shock, and the problem is now that of continuation of the solution from one region to the other.

Perhaps even more important is the following consideration. At the moment, we intend to base computational algorithms on the use of a digital computer. Consequently, there is some merit in formulating control processes in discrete terms from the beginning. If we proceed in this fashion, all problems of existence of extremals vanish, and we face directly the fundamental problems of numerical solution and determination of the structure of optimal policies. Dynamic programming can now be applied in a uniform fashion to the study of deterministic, stochastic, and adaptive control processes. If we so desire, we can establish that various limits exist as the discrete process merges into a continuous one.

The digital computer can be used for mathematical experimentation, with the hope of discerning the structure of optimal policies from the solution of particular problems.

Let us finally note that the authors make no mention of a number of other techniques available for the study of constrained variational problems. Such alternative techniques include: function-space methods [3]; gradient techniques of the type used by Bryson and Kelley [4]; quasilinearization [5]; and techniques based on the Neyman-Pearson lemma [6].

Taking into account all that has been said, there is no question that this book is an important contribution to the theory of control processes: one that must be read by everyone working in that field. Its translation is a fitting tribute to a great mathematician and his distinguished colleagues.

RICHARD BELLMAN

The RAND Corporation
Santa Monica, California

1. F. A. VALENTINE, "The problem of Lagrange with differential inequalities as added side conditions," *Contributions to the Calculus of Variations 1933–1937*, University of Chicago Press, Chicago, Ill., 1937, p. 407–448.
2. L. D. BERKOVITZ, "Variational methods in problems of control and programming," *J. Math. Anal. Appl.*, v. 3, 1961, p. 145–169.
3. J. P. LASALLE, "Time optimal control systems," *Proc. Nat. Acad. Sci.*, v. 45, 1959, p. 573–577.
4. G. LEITMANN (Editor), *Optimization Techniques*, Academic Press, New York, 1962.
5. R. BELLMAN, H. KAGIWADA & R. KALABA, "A computational procedure for optimal system design and utilization," *Proc. Nat. Acad. Sci.*, v. 48, 1962, p. 1524–1528.
6. R. BELLMAN, I. GLICKSBERG & O. GROSS, *Some Aspects of the Mathematical Theory of Control Processes*, The RAND Corporation, Report 313, 1958.

19[P, X].—YA. Z. TSYPKIN, *Sampling Systems Theory and its Application*, translated from the Russian by R. C. Hutchison and A. Allan, edited by I. Cochrane Pergamon Press, Oxford, 1964 (two volumes), xv + 375 p., 24 cm. Price $30.00.

The avowed intent of this book is to provide methods for analyzing pulse systems and their properties, using insofar as possible techniques that are already familiar in the analysis of continuous systems. Unfortunately, the book is, in my opinion, very unsuccessful in its attempts to meet its aim.

A book of this type should have its subject matter clearly divided into three sections: mathematical material (for example, the discrete Laplace transformation and its application to linear difference equations); systems concepts, such as principles of pulse modulation and digital feedback theory; and, if desired, component descriptions. However, the present book contains a confused mixture of all three. Chapter I, which is supposed to be an introduction to pulse systems, very soon dives into complicated circuit diagrams for the control of electrical machinery, electronic circuits, temperature, and some amazingly intricate mechanical systems, with a very unsatisfactory discussion of modulation theory. Chapter II, which is intended to provide the mathematical background for the sequel, is cluttered with a great number of trivial examples and inelegant theorems; moreover, hardly any of the mathematics of modern adaptive and statistical control theory is introduced. The reader is much better served, in much shorter time, by a good book on Laplace transformations, as, for example, the treatise by Widder [1], or the first chapter of the book by Bellman and Cooke [2], or, in the engineering literat re, the first chapter in the book by Truxal [3], and the book by Ragazzini and Franklin [4].

The same plodding approach to theory, through involved and unenlightening examples, is followed through very long chapters on open-loop and closed-loop systems, replete with formulas and valuable computations and curves which would be more useful in an edited version for a reference work on particular control systems. Even there, however, I would prefer the relevant parts of the book of Gibson and Tuteur [5].

In addition to presenting the reading public with this disorganized compilation of theory, practice, and practical results, the publishers have obtained a very bad translation, set in a fashion that can only be described as a sorry example of the printer's art. The translation reads like pure Russian with (mostly) English words in which articles are inserted or omitted capriciously, commas are used in keeping with the original Russian grammar, and, more seriously, the precise English word is often neglected in favor of an easier choice. Thus, we have "law" instead of "characteristics," "image" instead of "transform," "closed-system" instead of "closed-loop system," "trapezium" when "trapezoid" is meant, and so on. Coupled with the undulations of the lines and the very poor proof-reading, the clumsiness of the translation makes the text annoying to read.

In sum, the book is a very poor text, although it may have some value thanks to the curves, formulas, and descriptions of very complicated control systems that it contains. The reader interested in an introduction to the field would be much better advised to use Chapter 9 of Truxal's book, Ragazzini and Franklin's book, or the book of J. T. Tou [6]. These books, as well as the others cited above, also present a much more understandable discussion of the underlying mathematics and system concepts.

IVAN SELIN

The RAND Corporation
Santa Monica, California

1. D. V. WIDDER, *The Laplace Transform*, Princeton Univ. Press, Princeton, N. J., 1946.
2. R. BELLMAN & K. L. COOKE, *Differential-Difference Equations*, Academic Press, New York, 1963.
3. J. G. TRUXAL, *Automatic Feedback Control System Synthesis*, McGraw-Hill Book Co., New York, 1955.
4. J. R. RAGAZZINI & G. F. FRANKLIN, *Sampled Data Control Systems*, McGraw-Hill Book Co., New York, 1958.
5. J. E. GIBSON & F. B. TUTEUR, *Control System Components*, McGraw-Hill Book Co., New York, 1958.
6. J. T. TOU, *Optimum Design of Digital Control Systems*, Academic Press, New York, 1963.

20[V].—I. A. KIBEL′, *An Introduction to the Hydrodynamical Methods of Short Period Weather Forecasting*, Pergamon Press, Ltd., Oxford, England, distributed by The Macmillan Co., New York, 1963, xiii + 383 p., 23 cm. Price $14.50.

This book is a translation by the British Air Ministry from the original volume in Russian published at Moscow in 1957. As such, the material represents the state of short range numerical weather prediction at the end of its first decade. This was the era of quasi-geostrophic, or "balanced," hydrodynamic frameworks. Since then, considerable progress has been achieved with primitive equation hydrodynamics, i.e., where the sole filtering approximation is that of quasi-hydrostatic equilibrium.

The volume's interest is therefore mainly historical. However, it will have an additional attraction to the meteorological community of the western world. Despite the author's attempt to maintain a broad perspective of contributions, it is, as one might expect, weighted toward the Soviet literature—and yet one can make a virtue of this inevitable necessary characteristic. The volume provides one with a self-critique of significant Soviet scientists and of their progress that may be difficult to glean from the large volume of Soviet journals, both in the original and translated, that is accessible to the West.

The first two chapters provide the basic Navier-Stokes hydrodynamic framework, accommodating readers with a background primarily in mathematics and physics. Here, in addition to discussing boundary layer exchange processes, Kibel′ also provides a scale-analysis justification of the hydrostatic approximation. The next chapter extends the scale argument to demonstrate the quasi-geostrophic, quasi-nondivergent and quasi-barotropic character of large-scale atmospheric motions, laying the ground work for the filtering approximations to be used in the remainder of the book. The following three chapters apply linear analytical techniques to the study of the properties of such motions and finally to the stability characteristics of baroclinic disturbances.

Chapter 7 begins the consideration of the general non-linear problem, with some emphasis on Green's function expansions. The next two chapters demonstrate the application of finite difference methods to the integration of barotropic and then to baroclinic models. Kibel′ then turns his attention to the smaller-scale frontal motions and considers the application of the balance ("quasi-solenoidal") approximation. The last two chapters touch on non-inertial influences: the introduction at the lower boundary of large-scale orographic barriers, boundary layer exchanges of momentum, heat and water vapor, the internal release of lateral heat, and radiative transfer.

The Conclusion, though short, speculates upon the use of the primitive equations of motion, and proposes the application of numerical hydrodynamical methods to the problems of the prediction of cloud and precipitation, cumulus convection, the sea-breeze, and small-scale orographic motions, all of which have become a reality since the writing of this book.

It is unfortunate that the references are not always documented beyond the author and year of publication. Also the reproductions of figures from the original are only marginally acceptable. Despite these deficiencies, no dynamic meteorology library can be considered complete without this volume.

JOSEPH SMAGORINSKY

U. S. Weather Bureau
Washington, D. C.

21[V, X].—R. W. CLAASSEN & C. J. THORNE, *Steady-State Motion of Cables in Fluids, Part 2, Tables of Cable Functions for Vertical Plane Motion*, PMR-TM-63-9, Pacific Missile Range, Point Mugu, California, 1963, 487 p., 21 cm.

In Part 1, tables [1] were given for calculating the shape and tension of a neutrally-buoyant flexible cable in a stream. The present volume, Part 2, extends the previous work to include the effect of a weight parameter.

These tables should be quite useful in the design of cable-towed systems. Their application is illustrated in the first part of the volume by a series of eleven examples. A less extensive set of tables for the same purpose is available in David Taylor Model Basin Report 687, by Leonard Pode, dated March 1951 [2], and in a supplement thereto [2].

LOUIS LANDWEBER

State University of Iowa
Iowa City, Iowa

1. CHARLES J. THORNE, GEORGE E. BLACKSHAW & RALPH W. CLAASSEN, *Steady-State Motion of Cables in Fluids, Part 1, Tables of Neutrally Buoyant Cable Functions*, NAVWEPs Report 7015, Part 1, NOTS TP 2378, China Lake, California, 1962. [For review, see *Math. Comp.*, v. 18, 1964, p. 337, RMT **55**.]
2. LEONARD PODE, *MTAC*, v. 11, 1957, p. 282–283, RMT **123**.

22[X].—*The Universal Encyclopedia of Mathematics* (with a foreword by JAMES R. NEWMAN), Simon and Schuster, New York, 1964, 715 p., 19 cm. Price $8.95.

This pretentiously entitled compendium of mathematical information is an English translation and adaptation of *Meyers Rechenduden*, published in 1960 by the Bibliographisches Institut in Mannheim.

The body of this encyclopedia is arranged in three main subdivisions: Alphabetical Encyclopedia under Subjects, Mathematical Formulae, and Mathematical Tables.

In the publisher's prefatory note we are informed that the range of mathematics covered is that "from beginning High School through College, but stopping short of a degree in mathematics." The book does not profess to be addressed to the professional mathematician, but rather to the interested layman and the technical student.

The Foreword further clarifies the extent of the subject matter by including a remark that many branches of mathematics from arithmetic through the calculus are included, but that higher branches such as group theory and algebraic topology are excluded.

This limitation in the material presented is reflected also in the numerous formulas presented, which are listed under the nine subheadings of Arithmetic, Algebra, Applications, Geometry and Trigonometry, Analytical Geometry, Special Functions, Series and Expansions in Series, Differential Calculus, and Integral Calculus.

A total of five mathematical tables, together with explanations of their use, comprise the last part of this book. Table 1 gives x^2 (exact), x^3 (3 D), x^{-1} (5 D), $x^{1/2}$ (4 D), $(10\,x)^{1/2}$ (3 D) and $x^{1/3}$ (4 D) for $x = 1(0.01)\,10$; Table 2 lists to 5D the mantissas of the common logarithms of all integers from 1000 through 10009; Table 3 gives $\sin x$, $\cos x$, $\tan x$ (each to 5 D) and $\cot x$ (6 S) for $x = 0(0.001)0.8$, as well as the sexagesimal equivalent of x to the nearest $0.01''$; Table 4 contains $\sinh x$, $\cosh x$, $\tanh x$ (each to 5 D), $\coth x$ (6 S), $\ln x$ (5 D), and $e^{\pm x}$ (5 D) for $x =$

0(0.001)1.65; and Table 5 gives sin φ to 4 D for $\varphi = 0(6')90°$, tan φ to 4 D for $\varphi = 0(6')70°$, to 3 D for $\varphi = 70°(6')\ 85°$, and to 2 D for $\varphi = 85°(6')89°54'$, and the radian equivalent of φ to 4 D. The publisher states that every tabular value was independently calculated electronically, and the tables appear to have been produced by a photo-offset process from the computer output sheets.

A number of improvements can be incorporated in a subsequent edition, in the opinion of this reviewer. For example, the discussion of Diophantine equations (p. 172–173) is restricted to a consideration of a single linear equation in two variables with integral coefficients, without any reference to the fact that this topic includes indeterminate equations of higher degree as well as of more variables. (Incidentally, the heading on p. 173 should be "Diophantine Equations" instead of "Diameter.") Moreover, on p. 231 the statement of the fundamental theorem of algebra is unnecessarily restricted to algebraic equations with *real* coefficients.

It is interesting and informative to compare this book with recent mathematical dictionaries by Karush [1] and by James and James [2]. The breadth of coverage appears to be greater in the latter two references; however, the treatments therein of certain topics in elementary mathematics such as circles and triangles and their properties are not as extensive as in the book under review.

Within the limitations described in the Foreword and referred to in this review, the present book will serve as a useful reference for the technical student, although it does not attain the pre-eminence that is implied by its ambitious title.

J. W. W.

1. William Karush, *The Crescent Dictionary of Mathematics*, The Macmillan Company, New York, 1962. (Reviewed in *Math. Comp.*, v. 17, 1963, p. 478, RMT **88**.)
2. Glenn James & Robert C. James, *Mathematics Dictionary*, 2nd ed., Van Nostrand, Princeton, N. J., 1959. (Reviewed in *MTAC*, v. 13, 1959, p. 331-332, RMT **66**.)

23[X].—Kaj L. Nielsen, *Methods in Numerical Analysis*, Second Edition, The Macmillan Company, New York, 1964, xiv + 408 p., 23 cm. Price $9.00.

Since this text is designed for a one-semester undergraduate course, no knowledge of mathematics beyond elementary calculus is assumed.

The customary topics of least-squares approximation, interpolation, numerical integration and differentiation, finite-difference methods applied to differential equations, and the solution of systems of equations (linear and nonlinear) are discussed. Material is included on such topics as multiple integration, trigonometric fitting of data, smoothing of data, autocorrelation, and a chapter on linear programming.

The book stresses the proper organization of various numerical methods for efficient use of a desk calculator or larger computer. Many examples are worked out in complete detail, with further exercises for the student at the end of each chapter. The author has also included useful tables to aid in the construction of various algorithms.

Bert E. Hubbard

University of Maryland
College Park, Maryland

24[X].—L. Fejes Tóth, *Regular Figures*, The Macmillan Company, New York, 1964, xi + 339 p., 22 cm. Price $12.00.

The discrete groups of isometries in the plane are the basis for ornamental patterns. The simplest geometrical representations for them are displayed, as well as ten beautifully colored classic designs based on them.

Their extension to three-space is seen in spherical arrangements and the classical geometrical crystal classes. Since these do not exhaust the permutations of the group parameters, it is necessary to go to hyperbolic space to complete the utilization of all the possible values of the parameters. Their groups are derived, and tesselation examples are displayed.

The regular and semi-regular polyhedra are derived and exhibited in photographs and well-drawn figures. They include the Platonic and Archimedean solids, the Kepler-Poinsot star-polyhedra, and the regular honeycombs. The convex regular polytopes and Euclidean tesselations in all higher dimensions are derived from purely combinatorial considerations.

Problems concerning the most efficient packing of congruent figures in the plane are considered, as well as the most economical covering of the plane by congruent figures. These have technical applications as well as artistic applications. The problems are generalized to the use of sets of non-congruent figures. Also, multiple coverage and corresponding problems on a sphere are considered. The results are compared with biological patterns such as those occurring in pollen grains. The problems in this field are very difficult and many are still unsolved.

Problems in three-space, which involve the properties of polyhedra, include the isoperimetric problems, covering with clouds of spheres, sphere packing, and honeycombs. Extensions to higher spaces are made. Many beautiful results are derived, but there are many promising avenues to be explored.

A six-page bibliography and a good index make the book an excellent reference work.

The excellent three-dimensional anaglyphs in the book-pocket are not mentioned in the table of contents or the index. Their proper use is not explained in the text. For best viewing, these plates should be horizontal with the near edge about a foot from the eyes. They should be viewed through the colored spectacles with the green lens before the right eye and the red lens before the left eye. The line of sight should be depressed about 45°.

A few typographical errors were noted, which the reviewer has communicated to the author. On p. 119, it is stated that Gauss proved that the only regular p-gons that can be made by Euclidean constructions are for those values of p whose odd prime factors are distinct Fermat primes. Gauss did not quite prove this. (See Archibald's note on p. 84 of his translation of *Famous Problems* by Felix Klein.)

Michael Goldberg

5823 Potomac Avenue, N.W.
Washington, D. C.

25[Z].—R. Baumann, M. Feliciano, F. L. Bauer & K. Samelson, *Introduction to ALGOL*, Prentice-Hall, Inc., Englewood Cliffs, N. J., 1964, x + 142 p., 24 cm. Price $8.00.

The ALCOR group is a cooperative association of about 25 institutions in Europe and the USA, primarily interested in the construction of ALGOL compilers and the establishment of common hardware representations. In order to promote program exchangeability the group began the development of an ALGOL manual based on courses and lectures conducted by some of the member institutions, and also on the practical experience of using and compiling ALGOL. While this book is the result of several revisions and extensions of this manual, it is no longer a manual as such. As stated in the Preface, "This book is intended to give a needed introduction to ALGOL, which should enable the nonspecialist, for whose benefit ALGOL was primarily conceived, to write clear and readable ALGOL programs from which a reasonable translator would produce efficient machine codes. . . . Emphasis is on the normal use of the language rather than on artificial examples exploiting tricky possibilities . . . ."

Without question, the book is indeed a well written, tutorial introduction to ALGOL, very well suited for self-study by readers not previously familiar with ALGOL or other algorithmic languages. This is underscored by the arrangement of the text in three distinct parts with an increasing level of sophistication.

Following a general introduction to some of the basic ideas of computer programming, Part I provides an introduction to the elements of ALGOL: The definition of the basic ALGOL symbols is followed first by a discussion of arithmetic expressions, statements and the general construction of ALGOL programs, and then of subscripted variables, loops, conditional and jump statements. At this point the reader should be familiar enough with the language to write simple, but complete, ALGOL programs.

Part II introduces the block structure of ALGOL, Boolean statements, designational expressions and, finally, procedures. Both Parts I and II present a subset of the full language corresponding to the forthcoming IFIP Subset ALGOL 60. This subset is certainly sufficient for the majority of applications, especially for the beginner. Part III covers some additional advanced concepts—namely, the use of expressions called by name and, in a brief section, the idea of recursive procedures. The defining report of full ALGOL 60 is included as an appendix, together with the officially approved IFIP corrections and amendments of the April 1962 meeting in Rome.

There are many examples supplementing the text and showing good programming techniques. In addition, a full section of exercises at the end of the book provides a source of further information about typical programming mistakes and features of the languages. The examples in the text are all taken from numerical mathematics, and all of them are excellent illustrations for good numerical algorithms. As a whole, however, one readily observes that for many readers the examples are probably on a much higher plane than the introduction to the language itself.

The book certainly provides a good introduction to ALGOL and as such it fills a need. For the users of the ALCOR compilers some appendix on hardware representations and related questions might have been desirable—but this is at the same time a questionable suggestion since the book is of course not a manual for certain ALGOL compilers but a tutorial introduction to ALGOL itself. Unfortunately, the fairly high price of this book may well preclude the desirable, widespread distribution which it deserves.

WERNER C. RHEINBOLDT

University of Maryland
College Park, Maryland

26[Z].—L. BOLLIET, N. GASTINEL & P. J. LAURENT, *Algol*, Hermann, Paris, 1964, 196 p., 23 cm. Paperbound. Price 36 Fr.

This is a tutorial discussion of the programming language ALGOL 60, of a type which was badly needed a year or two ago. In contrast to the formal style of the ALGOL 60 report [1], it presents both informal explanations of the various features of the language, and illustrations of the ways in which they may be applied. The discussion of the potentialities of the *for* statement is particularly illuminating. The informal description of the syntax is supplemented by extensive use of syntactic charts, both for individual syntactic elements, and for the entire language.

In addition, the volume contains appendices on: 1. Hardware representations; 2. Examples of Input-Out procedures; 3. Compilation processes; 4. The ALGOL 60 Report [1], as revised at the 1962 Rome meeting; 5. A chart of ALGOL basic symbols. In addition, it includes a listing of Algorithms published in Communications of the Association for Computing Machinery through August 1963, and a bibliography of 137 items on ALGOL 60 and its implementation.

Unfortunately, there are many typographical errors, some of which might confuse the novice. The printing of digits in bold face, while letters are in normal type is also distracting. The paper binding appears unusually flimsy for a volume which will have extensive use.

The present availability of a variety of tutorial presentations of ALGOL 60 in English will restrict the value of this work outside of French-speaking regions. This is particularly so because of the decision to transcribe the ALGOL basic symbols into French, so that the language is incompatible with most translators. It is of interest, however, for the information which it contains on French implementation of ALGOL, and the convenient reference material in the appendices.

HENRY C. THACHER, JR.

Reactor Physics Division
Argonne National Laboratory
Argonne, Illinois

1. P. NAUR (Ed.), "Revised report on the algorithmic language ALGOL 60," *Comm. ACM*, v. 6, 1963, p. 1–17.

27[Z].—D. G. BURNETT-HALL, L. A. G. DRESEL & P. A. SAMET, *Computer Programming and Autocodes*, Van Nostrand Company, Inc., Princeton, New Jersey, 1964, viii + 106 p., 24 cm. Price $4.50.

The preface states that "this book is intended to serve as an introduction to the programming of automatic computers." The first 24 pages present such an introduction, apparently assuming that this is the reader's first contact with a stored-program digital computer.

Part II, which makes up about 60 percent of this short volume, consists of three chapters, each of which describes a different autocode. The word "autocode" as used in England corresponds roughly to "compiler language" in the United States. The authors apparently consider an autocode to be machine-dependent, in contrast to a "universal computing language" like Algol, which is machine independent. The three autocodes discussed are for the Pegasus-Sirius, the Elliott 803, and the Ferranti Mercury. The machines themselves are not described here in any detail. They are all rather small machines, and are not of very great general interest. Unfortunately, the same is true of their autocodes. The Mercury autocode is treated at greatest length and in greatest detail. It is an interesting system, but its interest is now mostly historical, illustrating some of the early work of Brooker and his colleagues at Manchester. Most of the material in this book will be of interest only to the devoted specialist and perhaps to the historian in the field of computer languages.

A final section of the book presents a 14-page discussion of Algol. It is a good but very brief resumé of the language.

SAUL ROSEN

Purdue University
Lafayette, Indiana

28[Z].—IRVING ALLEN DODES, *IBM 1620 Programming for Science and Mathematics*, Haden Publishing Company, Inc., New York, 1963, ix + 276 p., 24 cm. Price $9.95.

This is a very thorough and carefully written text on programming the IBM 1620. The author is chairman of the mathematics department of the Bronx High School of Science, where a course in numerical analysis has been given successfully to seniors. The course includes learning to program the 1620. This text is an outgrowth of a manual used in that course. The general style is obviously influenced by the high-school audience for which it was first intended, but this should not be construed to mean that the book is limited to such an audience. Rather, one could recommend it as a text for any audience unfamiliar with the programming of modern computers and wishing to learn something about this by using the 1620 as a specific machine.

There are four parts. Part 1 is a somewhat elementary treatment of number systems and numerical methods. Part 2 is an extensive discussion of 1620 machine programming. Part 3 describes the symbolic programming system (SPS), and part 4 treats Fortran with Format.

There are numerous exercises and illustrative examples. The material is well-organized and presented with great care.

E. K. BLUM

Wesleyan University
Middletown, Connecticut

29[Z].—SEYMOUR GINSBURG, *An Introduction to Mathematical Machine Theory*, Addison-Wesley Publishing Co., Inc., Reading, Mass., 1962, ix + 148 p., 23 cm. Price $8.75.

It is surprising that a book a couple of years old in the rapidly developing area of theory of machines (or theory of automata) is still up to date. The author has accomplished this by undertaking a modest task. He has not attempted a comprehensive development of the subject, but has rather restricted himsel for the most part to a few of those areas in which he and his colleagues have actively worked. In this respect, the title may be misleading.

The first chapter deals with *complete sequential machines* from a point of view initiated by E. F. Moore in "Gedanken Experiments on Sequential Machines." This point of view is, roughly speaking, what can one learn about the contents of a "black box" capable of assuming a finite number of states by performing "experiments" (not to be confused with the technical sense of "experiment" used by the author) which utilize only the box terminals. A typical result of this character is the following. Two complete sequential machines S_i, $i = 1, 2$, each with a fixed initial state will for each input sequence give the same output sequence provided they do so for all input sequences of length $n_1 + n_2 - 1$, where n_i is the number of states of machine S_i; moreover, this number cannot in general be reduced (Theorem 1.2). An interesting problem of this kind for which a definitive solution has been found is given in Theorem 1.5. Given a machine, how long an input sequence does it take so that the corresponding output sequences uniquely determine the final state of the machine started from an unknown initial state? Find a procedure for determining such an input sequence (while observing the corresponding output sequence).

Chapter 4 deals with "recognition devices." The point of view is that initiated by S. C. Kleene (inspired in part by work of McCulloch and Pitts) in "Representation of Events in Nerve Nets and Finite Automata." Here the emphasis is on the structure of the class of "tasks" that finite automata perform rather than on the structure of the finite automata themselves. The "task" is to recognize a set of strings of symbols (called tapes). The sets recognizable by finite automata are shown to be equivalent to the regular (in the sense of Kleene) sets and other basic properties of this class of sets are developed. The two-way and two-tape automata introduced by Rabin and Scott are also discussed here.

Chapter 3 is entitled "Abstract Machines." It incorporates work which is due almost solely to the author. It attempts to answer the question "To what extent can the problems treated in Chapter 1 be given meaning in the context of an arbitrary input and output semi-group (rather than the free ones) and to what extent do their solutions generalize?"

Chapter 2 is entitled "Incomplete Sequential Machines." Its origin is more pragmatic and its development somewhat cumbersome. An incomplete sequential machine is construed as a vehicle for specifying input-output conditions which a given machine may or may not *satisfy*. The chief problem treated is that of finding a minimum state machine which satisfies a given incompletely specified machine.

The treatment is rigorous but unmotivated, concise and sometimes terse. The most serious defect in the development is the lack of uniformity and perspective. The chapters are so loosely knit that the book comes close to being a collection of four papers.

Calvin C. Elgot

Thomas J. Watson Research Center
International Business Machines Corporation
Yorktown Heights, New York

30[Z].—EDWARD F. MOORE, Editor, *Sequential Machines: Selected Papers*, Addison-Wesley Publishing Co., Inc., Reading, Mass., 1964, v + 266 p., 22 cm. Price $7.50.

This book provides an excellent selection of major papers in the area of sequential machine theory which have appeared in diverse publications. Intended as a reference for research or for a course in this area, the collection fills the need for such very well. The papers included are:

D. A. Huffman, "The synthesis of sequential switching circuits".

M. O. Rabin and Dana Scott, "Finite automata and their decision problems".

J. C. Shepherdson, "The reduction of two-way automata to one-way automata".

M. O. Rabin, "Probabilistic automata".

J. Hartmanis, "Loop-free structure of sequential machines".

D. A. Huffman, "Canonical forms for information-lossless finite-state logical machines".

R. McNaughton and H. Yamada, "Regular expressions and state graphs for automata".

Irving M. Copi, Calvin C. Elgot & Jesse B. Wright, "Realization of events by logical nets".

Arthur W. Burks & Jesse B. Wright, "Theory of logical nets".

Edward F. Moore, "The firing squad synchronization problem".

Yu. T. Medvedev, "On the class of events representable in a finite automaton".

A. K. Kutti, "On a graphical representation of the operating regime of circuits".

An extensive bibliography is included, with good intention. The computer listing format, however, renders it virtually unreadable. Publishing of such material in a book of otherwise high quality (and price) is unfortunate.

J. W. THATCHER

Thomas J. Watson Research Center
International Business Machines Corporation
Yorktown Heights, New York

31[Z].—LOUISE SCHULTZ, *Digital Processing: A System Orientation*, Prentice-Hall, Inc., Englewood Cliffs, N. J., 1963, xii + 403 p., 24 cm. Price $8.95.

In her preface, the author states that this is intended as a text for a general introductory course on digital computers. Part 1 gives general background material on number systems and arithmetic, not very thoroughly and at a very elementary level. Part 2 is a rather sketchy ad hoc description of computing devices of miscellaneous types. After this, Part 3 is something of a surprise, discussing, as it does at varying length, vacuum tubes, transistors, diodes, tunnel diodes, and phase-locked oscillators, not to mention flip-flops, cores, NRZ magnetic recording, Williams tubes, and a variety of other kinds of hardware. There is also a chapter on logical design and two chapters on computer programming.

Since the level and the degree of detail vary widely in the presentation of these diverse topics, the use of this book as an introductory text is open to some question. Although the author disclaims it, it would appear that her book might better serve as an introductory reference to the hardware aspects of computers.

E. K. BLUM

32[Z].—Philip M. Sherman, *Programming and Coding the IBM 709-7090-7094 Computers*, John Wiley & Sons, Inc., New York, 1963, xiv + 137 p., 24 cm. Price $1.95.

This is a small booklet to be used as a "workplan" in learning the material in the author's book *Programming and Coding Digital Computers*, and relating it to the IBM 709-7090-7094 computers. Chapter headings are as follows: Basic Operations, Symbolic Coding, Program Loops, Index Registers, Sequencing in Memory Subroutines, Input-Output Operations, Numerical Problems, Algebraic Languages, Non-numerical Problems, Data Processing Macro-instructions, Interpreters and Simulation, Program Debugging and Testing.

E. K. Blum

33[Z].—Marvin L. Stein & William D. Munro, *Computer Programming: A Mixed Language Approach*, Academic Press, New York, 1964, xiv + 459 p., 24 cm. Price $11.50.

With the market so richly supplied with books on computer programming, each new entry in this field must, first, display a unique or arresting feature in order to capture attention and, second, meet exacting standards of excellence in order to gain acceptance. The book under consideration successfully negotiates the first hurdle and, in the opinion of the reviewer, possesses the necessary merit to surmount the second.

The feature that sets Stein and Munro's *Computer Programming* apart is its use of the Control Data 1604 as the model in terms of which machine language is discussed. It takes courage to eschew the easier path of inventing a fictitious machine on which to carry out the indispensable process of illustration. The authors have elected to meet the problem head on by going to a machine actually in use, thereby limiting their appeal to a highly selective market and also risking obsolescence of the book as the computer is inevitably superseded. These built-in obstacles to widespread circulation are partially offset by the obvious care that has gone into all phases of the production of the book.

The first two chapters, together about 70 pages in length, deal with number systems and the organization of a computer. The treatment of number systems, in particular, is exceptionally thorough and lucid. Programming proper, using the "mixed-language approach," begins in Chapter III. As suggested by the authors' determination to base their exposition on a flesh-and-blood machine, their point of view places them in the camp of those who believe that an initial grounding in machine code is essential for the student of computer programming, whether he is destined to be a professional or only a casual user. The progression is from machine code through symbolic machine code to FORTRAN. The step-by-step unfolding of programming, both as an attitude toward problem solving and as a corpus of techniques, occupies the remainder of the book. There are many carefully worked-out examples, some extending over several sections in episodal form. Numerous exercises, mostly drills for solidifying technical skill, are found at the end of each chapter. Special mention should be made of the end-papers, which contain a most convenient listing of the 1604 instruction repertoire.

The book was written to become a text at the upper-division level. One may assume that the dozen or so universities that have 1604's have already been made aware of its potential usefulness in this role. It can be recommended also for the library of any non-academic computing center that has this machine. The big question is, of course, whether it would be suitable as a textbook at those institutions that do not have a 1604. The authors have attempted to some extent to make an affirmative answer possible by separating material that can be treated in general terms from that which is heavily 1604-dependent. However, its value would seem to be greatest as a supplementary reference rather than a text if a 1604 is not available.

R. N. Goss

Navy Electronics Laboratory
San Diego, California

34[Z].—R. Wooldridge & J. F. Ractliffe, *An Introduction to ALGOL Programming*, Van Nostrand Company, Inc., Princeton, N. J., 1964, x + 131 p., 21 cm. Price $4.20.

This book is an addition to the rapidly growing list of introductory and tutorial texts on ALGOL programming. At the beginning, the reader is made familiar with a simple paper-tape oriented computer system and some basic notions relating to it; such as program, memory, input and output. In the following chapter, ALGOL is introduced by means of a short sample program. The sequel proceeds through the explanations of numbers, identifiers, expressions, arrays, and loops. A special chapter is inserted on input and output. The last chapters are devoted to the ALGOL block-structure and to procedures. A summary of the "main features of ALGOL" concludes the text.

The book is heavily oriented towards the ALGOL system implemented on the Elliott computers. This allows for the addition of a chapter on input-output, which discusses in detail the *read* and *print* operators of the Elliott-ALGOL system. The book is therefore particularly valuable to users of Elliott computers. For the reader interested in ALGOL 60, or implementations of ALGOL on other machines, this strong orientation is of questionable value, although the authors promise to annotate any "deviation of ALGOL 60" from Elliott ALGOL. For example, in the reviewer's opinion, the mandatory occurrence of *switch* declarations with the purpose of acting as label declarations ought to have been mentioned as a peculiarity of the Elliott system, and not as the general rule, which ALGOL 60 allows to be disregarded.

Also, example 7 does not seem to yield the expected results.

N. Wirth

Stanford University
Stanford, California