35[A, C].—W. E. Mansell, edited by A. J. Thompson, *Tables of Natural and Common Logarithms to* 110 *Decimals*, Royal Society Mathematical Tables, Volume 8, Cambridge University Press, New York, 1964, xviii + 95 pp., 28 cm. Price $7.50.

Two of the four main tables here give the natural and common logarithms of the integers 1 to 1000 to 110 decimals. The other two are the radix tables:

$$\log 1.0^k x \quad \text{for} \quad x = 1(1)9 \quad \text{and} \quad k = 4(1)11,$$

again to both bases, e and 10, and again to 110D. There is a fifth table of 25 constants: e, π, γ, etc., also to 110D.

The four main tables were computed by hand by William Ernest Mansell, a retired accountant, sometime during the 1930's. He left a provision in his will to make thier publication possible. The extensive editing was done by A. J. Thompson, who checked the tables by comparing sums with $\log_e n!$ and $\log_{10} n!$ for $n = 100$, 500 and 1000. The latter were computed by Stirling's formula, and are included in the table of constants.

The introduction indicates several (desk-computer) techniques of computing the logarithms of larger integers or irrational numbers by the use of these tables together with factor tables and the Taylor series. For example,

$$3\,141593 = 13 \cdot 437 \cdot 553 \cdot 10^{-6}$$

and

$$\pi = (1 - x)\, 3.141593/1.0^6 1 \cdot 1.0^7 1 \cdot 1.0^9 2 \cdot 1.0^{10} 6 \cdot 1.0^{11} 5,$$

where $x = 0.0^{12}79 \cdots$. From a few terms of the series for $\log (1 - x)$ one therefore obtains $\log \pi$ to 40D.

No previously published table has the same combination of range and accuracy, although even more accurate tables exist for smaller ranges. See [1] for a complete listing of such tables. It is clear, of course, that modern machines can easily surpass these very extensive hand computations of Mansell.

The four main tables are reproduced photographically, and while the printing is much better than that of many tables so reproduced, it does not have the elegance usually found in the Royal Society Tables.

D. S.

1. A. Fletcher, J. C. P. Miller, L. Rosenhead & L. J. Comrie, *An Index of Mathematical Tables*, Addison-Wesley, Reading, Mass., 1962.

36[F].—M. L. Stein & P. R. Stein, *Tables of the Number of Binary Decompositions of All Even Numbers* $0 < 2n < 200{,}000$ *into Prime Numbers and Lucky Numbers*, Volumes I & II, Los Alamos Scientific Laboratory Report LA-3106, 1964. Vol. I, 442 pp.; Vol. II, 426 pp., 28 cm. Price $5.00 each. Available from Office of Technical Services, U. S. Department of Commerce, Washington 25, D. C

The main table in Volume I (400 pages long) gives the number of solutions ν_{2n} of $2n = p_1 + p_2$, where p_1 and p_2 are primes, and $2n$ is an even number less than

200,000. Two conventions must be noted: the first (which is curiously old-fashioned) allows 1 to be considered as a prime, and the second (in distinction to some treatments) considers the decompositions $p_1 + p_2$ and $p_2 + p_1$ to be identical. Thus, $\nu_{14} = 3$, since

$$14 = 1 + 13 = 3 + 11 = 7 + 7.$$

In the introduction there is given a heuristic formula $E_{2n}(\nu)$ that is meant to estimate ν_{2n}. We will not reproduce its rather complicated definition here, but we do note that infinitely many approximations of $E_{2n}(\nu)$ are defined, depending upon an integral parameter k. The authors computed estimates based upon $k = 5$, for convenience, and compared these estimates with the actual counts ν_{2n}. They find that their estimates are usually a little too high. The average of the *absolute value* of the relative errors for the 85,000 cases: $30{,}000 \leqq 2n < 200{,}000$ is 2.60 %. The worst single disagreement in this range is between $\nu_{33,038} = 224$ and $E_{33,038}(\nu) = 254.78$. But this error of 13.74 % is rather exceptional, and in over 54,000 cases the error is less than 3 %.

No reference is given to the classical Hardy-Littlewood conjecture for the number of such Goldbach decompositions, cf. Schinzel [1, conjecture C for $a = b = 1$], and possibly the authors did not realize that their approximations become asymptotic to the Hardy-Littlewood formula when their parameter k goes to infinity. Thus, Hardy and Littlewood give

$$P(2n) \sim 2c_2 \prod_{p \mid 2n} \left(\frac{p-1}{p-2}\right) \int_2^{2n} \frac{dx}{\log^2 x}, \tag{1}$$

where the product shown is taken over all odd primes that divide $2n$, and where

$$c_2 = \prod_{p=3}^{\infty} \left(1 - \frac{1}{(p-1)^2}\right) = 0.660162.$$

There is a factor of 2 in (1) because $p_1 + p_2$ and $p_2 + p_1$ are considered distinct there. Clearly, whether one allows 1 as a prime or not does not affect the asymptotic law. Aside from these two differences in the conventions adopted, the Stein-Stein estimate, for $k = 5$, is rather close to (1), except that the constant c_2 there is replaced (in effect) by

$$\prod_{p=3}^{p=11} \left(1 - \frac{1}{(p-1)^2}\right) = 0.676758.$$

Now, this latter constant is 2.51 % larger than c_2, so if one allows for the fact that the 2.60 % mentioned above is based upon the absolute value of the error, one concludes that the Hardy-Littlewood formula fits the empirical data very well.

The second table in Volume I (20 pages long) gives an inverse function: the number of values of $2n$ for which there are exactly k decompositions: $2n = p_1 + p_2$. For every $k \leqq 3931$ they give the smallest value of $2n$, the largest value of $2n$ (up to $2n = 200{,}000$), and the number of such values of $2n$. They discover that for all values of $k \leqq 2428$ there is at least one value of $2n$, and that for $k > 8$ the smallest value of $2n$ is always divisible by 6.

There are also several small statistical tables, which we will not describe.

In Volume II we have completely analogous tables where the primes have been replaced by the "lucky numbers." That is fitting, since this sequence was invented by the Los Alamos school of number theory. No good heuristic estimate was found for the number of "lucky" decompositions.

D. S.

1. A. Schinzel, "A remark on a paper of Bateman and Horn," *Math. Comp.*, v. 17, 1963, pp. 445–447, especially p. 446.

37[F].—Thomas R. Parkin & Leon J. Lander, *Abundant Numbers*, Aerospace Corporation, Los Angeles, 1964, 119 unnumbered pages, 28 cm. Copy deposited in UMT File.

Leo Moser had shown [1] that every integer $> 83{,}160 = 88 \cdot 945$ can be expressed as the sum of two abundant numbers. This proof is first improved here to include all integers $> 28{,}121$. This is done by showing that every odd $N \geqq 28{,}123 = 89 \cdot 315 + 88$ can be written as $N = M \cdot 315 + B \cdot 88$ with $3 \leqq M \leqq 89$ and $B \geqq 1$. But $M \cdot 315$ and $B \cdot 88$ are both abundant. Further, it is easily shown that all even numbers > 46 can be written in the required manner [2].

The smallest odd N so representable is clearly 957, since 945 and 12 are the smallest odd and even abundant numbers, respectively. To examine the odd numbers between 957 and 28,123, the authors use two methods: (a) covering sets; and (b) trial and error based upon lists of abundant numbers. They thus find that 20,161 is, in fact, the largest integer not so decomposable. This had been previously found by John L. Selfridge.

The main table here (90 pages) gives a decomposition, if one exists, for every odd N satisfying $941 \leqq N \leqq 28{,}999$. There are, all in all, only 1455 integers not decomposable into a sum of two abundant numbers.

In their discussion of method (a) mentioned above, the authors erroneously state that a prime multiple of a perfect number is a *primitive abundant* number, where that is defined to be an abundant number that has no abundant proper divisor. A counterexample is $84 = 3 \cdot 28$, since this has the abundant number 12 as a divisor.

In connection with these computations (on a CDC 160A) a table of $\sigma(N)$ was computed up to $N = 29{,}000$ by the use of Euler's pentagonal number recurrence relationship. This table is reproduced up to $N = 1000$ in Appendix C. The authors planned to extend this table (on tape) up to 10^5 or 10^6, but believe that the use of the canonical factorization of the integers will be faster than Euler's method. Presumably that is because of the limited high-speed memory in the small computer which was being used.

D. S.

1. Leo Moser, *Amer. Math. Monthly*, v. 56, 1949, p. 478, Problem E848.
2. F. A. E. Pirani, *Amer. Math. Monthly*, v. 57, 1950, pp. 561–562, Problem E903.

38[F].—Karl K. Norton, "Remarks on the number of factors of an odd perfect number," *Acta Arith.*, v. 6, 1961, pp. 372–373. Table in Section IV.

Let $\alpha(n)$ be defined by

$$\prod_{r=n}^{n+\alpha(n)-2} \frac{p_r}{p_r - 1} < 2 < \prod_{r=n}^{n+\alpha(n)-1} \frac{p_r}{p_r - 1},$$

where p_r is the rth prime. If an odd perfect number N has p_n as its smallest prime divisor, it follows that it has at least $\alpha(n)$ prime divisors. A table of $\alpha(n)$ for $n = 2(1)100$ was computed on the ILLIAC and is presented here; e.g., $\alpha(2) = 3$ and $\alpha(100) = 26308$. These may be compared with a theoretical formula:

$$\alpha(n) = \tfrac{1}{2}n^2 \log n + \tfrac{1}{2}n^2 \log \log n - \cdots .$$

Up to $n = 11$ and $n = 24$, the author could have used existing tables of $\prod_{p<x} (1 - 1/p)$ due to Legendre and Glaisher, respectively, instead of the ILLIAC, but he makes no mention of this. The later, and much more extensive table of Appel and Rosser [1] was not completely printed, and allows us only to determine such bounds as

$$5{,}730{,}105 < \alpha(1217) < 5{,}760{,}003.$$

D. S.

1. KENNETH I. APPEL & J. BARKLEY ROSSER, *Table for Estimating Functions of Primes*, IDA-CRD Technical Report Number 4, 1961; reviewed in *Math. Comp.*, v. 16, 1962, pp. 500–501, RMT **55**.

39[G].—MARSHALL HALL, JR. & JAMES K. SENIOR, *The Groups of Order* 2^n $(n \leqq 6)$, The Macmillan Company, New York, 1964, 225 pp., 36 cm. Price $15.00.

From the preface: "No single presentation of a group or list of groups can be expected to yield all the information which a reader might desire. Here, each group is presented in three different ways: (1) by generators and defining relations; (2) by generating permutations; and (3) by its lattice of normal subgroups, together with the identification of every such subgroup and its factor group. In this lattice the characteristic subgroups are distinguished.

"For each group, additional information is given. Here are included the order of the group of automorphisms and the number of elements of each possible order 2, 4, 8, 16, 32, and 64.....All the groups are divided into twenty-seven families, following Philip Hall's theory of isotopy.

"Chapters 3 and 4 give the theoretical background for the construction of the tables. But these chapters are not necessary for the use of those tables; for that purpose Chapter 2 is adequate. Chapter 5 draws attention to a number of the more interesting individual groups."

The preparation of these tables was begun by the "senior" author way back in 1935. For a while Philip Hall was directly involved, and though he later withdrew as a co-author, the classification used is still based largely upon his ideas.

The outsize pages ($17'' \times 14''$) were necessary because of the lattice diagrams. Each of the 340 individual groups, for orders 2^n with $1 \leqq n \leqq 6$, is represented by such a lattice, and the more complicated diagrams require an entire page. The diagrams, and portions of the tables, will be understandable and of interest to a reader with even a causal knowledge of finite groups. Other portions of the tables and the theory underlying their construction require a much deeper understanding to appreciate. One value of the volume, indeed, is that it provides a vast amount of illustrative material that can be examined in the course of a study of these deeper aspects of the theory. It is probable that the tables will prove stimulating to many readers, and this may even lead to new developments.

As an example of such stimulation, consider the 14 groups of order 16 that are

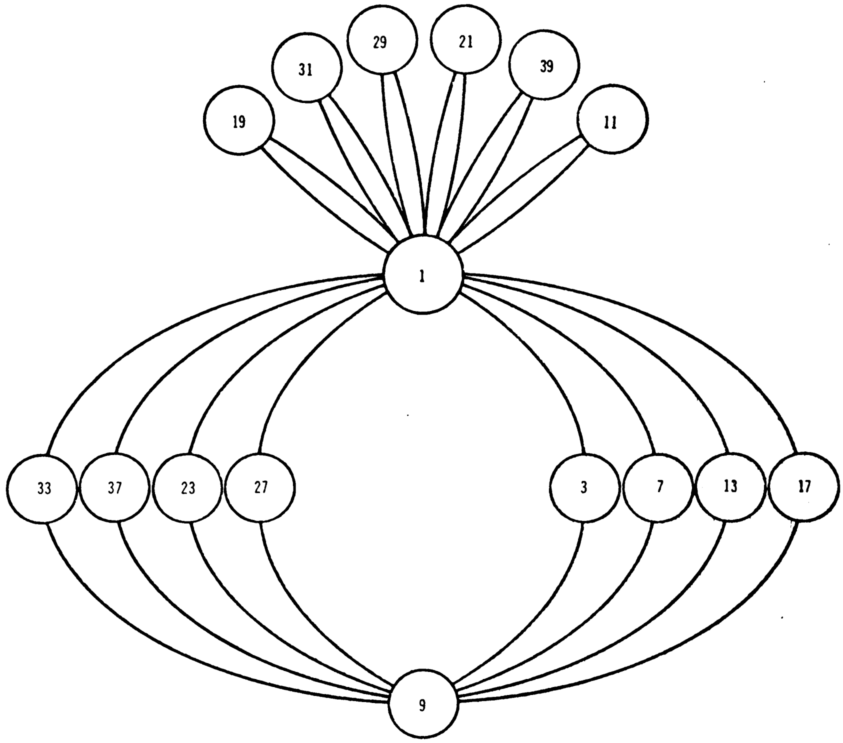

FIGURE 1. Cycle Graph of $\mathfrak{M}_{40}$

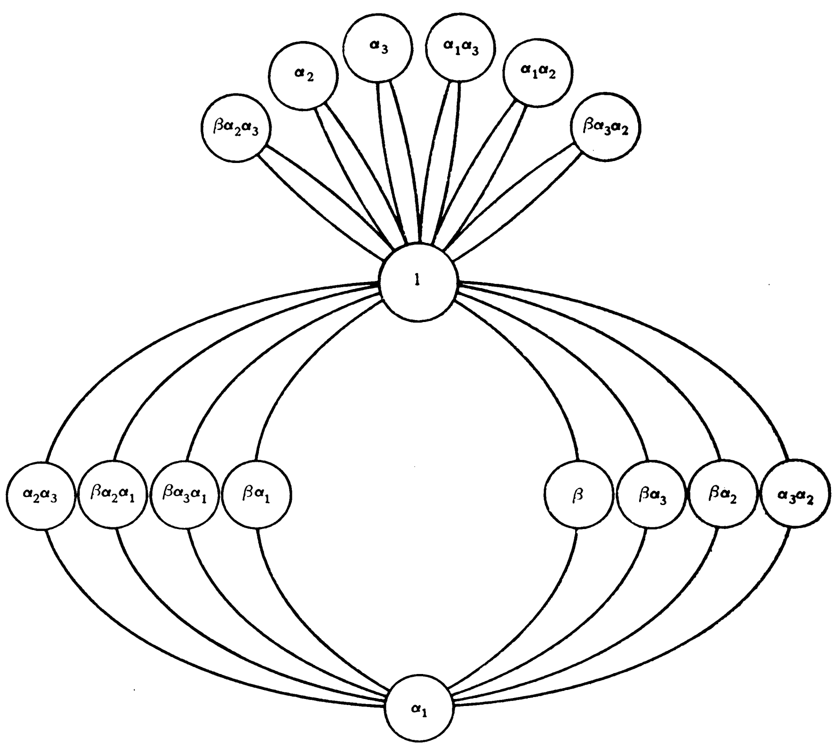

FIGURE 2. Cycle Graph of 16 $\Gamma_2 b$

tabulated here on pages 37, 39, and 45. Two of them, namely those designated as (31) and 16 $\Gamma_2 d$, are *conformal* to each other, that is, they contain an equal number of elements of each order. Likewise, the three groups (2^2), 16 $\Gamma_2 a_2$, and 16 $\Gamma_2 c_2$ are conformal, and so are the three groups (21^2), 16 $\Gamma_2 b$, and 16 $\Gamma_2 c_1$. The remaining 6 groups are conformal to no group.

By examination, we now note that although (31) and 16 $\Gamma_2 d$ are not isomorphic, they do have a lesser degree of similarity that we may call *isopotent.* We say that two groups are isopotent if their elements may be put into 1–1 correspondence: $a \leftrightarrow \alpha$, in such a way that all powers are also in correspondence: $a^n \leftrightarrow \alpha^n$. In distinction to this pair of isopotent groups, no two of the three conformal groups: (2^2), 16 $\Gamma_2 a_2$, and 16 $\Gamma_2 c_2$ are isopotent. In the second set of three groups, (21^2) is isopotent to 16 $\Gamma_2 b$, but they are not isopotent to 16 $\Gamma_2 c_1$.

It is clear that isopotence implies conformality, but not conversely. The exact relationship between the two concepts is not known to the reviewer at this time. If two groups are isopotent they have the same *cycle graph* [1]. We illustrate this in Figures 1 and 2. The group $\mathfrak{M}_{40}$ represents the 16 residue classes prime to 40 under multiplication modulo 40. It is isomorphic to (21^2). The nonabelian group 16 $\Gamma_2 b$ is generated by the permutations:

$$\begin{aligned} \beta &= (abcd)(efgh) \\ \alpha_2 &= (eg)(fh) \\ \alpha_3 &= (ae)(bf)(cg)(dh) \end{aligned}$$

with $\alpha_1 = \beta^2$. An isopotent correspondence is that indicated diagrammatically: $3 \leftrightarrow \beta$, $31 \leftrightarrow \alpha_2$, etc.

The concept of isopotence may already be known, and it may, or may not, be of significance. The reviewer has not examined these questions, but they are not relevant here, since we merely wished to indicate that the tables can be stimulating.

The tables are nicely printed. The lattice diagrams, however, were not drawn by a professional draftsman, and exhibit much shaky lettering and uneven inking. This economy on the part of the publisher is somewhat regrettable, especially since the groups will be with us forever. Nonetheless, the diagrams are legible, and their interest and value are not negated by their lack of artistic perfection.

Apparently the tables were constructed entirely by hand. It would be an interesting challenge to an experienced programmer with the requisite algebraic knowledge and interest to attempt to reproduce and extend these tables with a computer.

Only one typographical error was noted. It is recorded on page 362 of this issue of *Mathematics of Computation.*

D. S.

1. DANIEL SHANKS, *Solved and Unsolved Problems in Number Theory*, Vol. 1, 1962, Spartan, Washington, pp. 83–103, 112–115, 206–208.

40[G, X].—L. FOX, *An Introduction to Numerical Linear Algebra*, Clarendon Press, Oxford, 1964, xi + 295 pp., 24 cm. Price $8.00.

This is a welcome addition to the growing number of textbooks on matrix computation. It should be quite accessible to students at the junior-senior level, although as a textbook it suffers from having no exercises. There are, however, numerous illustrative examples.

The first two chapters provide a short introduction to numerical analysis, computing and matrix algebra up to, but not including, the Jordan canonical form.

The next six chapters, which comprise the bulk of the book, deal with the linear equation and inverse problems. The usual elimination methods associated with the names of Gauss, Gauss-Jordan, Crout, Cholesky, and Aitken are described in detail, along with methods based on orthogonalization.

There is a comparison of methods and a good elementary introduction to error analysis, which includes such topics as conditioning, rounding error, correction procedures, and effects of perturbations. The section on linear equations ends with a short chapter on iterative methods, including those of Jacobi and Gauss-Seidel.

The last three chapters, about a fourth of the book, are devoted to the eigenvalue problem. The power and inverse power methods as well as the methods of Jacobi, Givens, Householder, and Lanczos are discussed in some detail, and the LR and QR transformations are introduced. There is also a short treatment of error analysis.

At the end of each chapter is an annotated bibliography, which supplements the text with notes on more advanced topics.

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41[G, X].—R. Zurmühl, *Matrizen und ihre technischen Anwendungen*, Springer-Verlag, Berlin, 1964, xii + 452 pp., 23 cm. Price DM 36.

This is the fourth edition. The first appeared in 1950, and was given a justifiably enthusiastic review in *MTAC* in 1951 by Olga Taussky. At that time it gave by far the best existing account of numerical methods for inverting matrices and finding proper values and vectors.

Of the 446 pages of text in the present edition, most of the "technical applications" are to be found in the final chapter of 90 pages. The major emphasis is upon theory and upon numerical methods. The theory is well and concisely presented, and successive editions have included the latest in techniques. Innovations appearing in the present edition but not to be found in the third (1960) are Rutishauser's LR transformation and some discussion of vector and matrix norms. Also there is introduced the "Hadamard condition number" of a matrix, which is the ratio of the modulus of the determinant of the matrix to the product of the Euclidean norms of the rows. ALGOL algorithms are introduced in a few places.

The general organization, and the style, remain about the same. There are a number of numerical examples worked out for illustration. The exposition is uniformly good, and very little is presupposed in the way of background. The book should be very useful either as a text or for reference.

A. S. H.

42[K].—Tito A. Mijares, *Percentage Points of the Sum $V_1^{(s)}$ of s Roots* ($s = 1$–50), *A Unified Table for Tests of Significance in Various Univariate and Multivariate Hypotheses*, The Statistical Center, University of the Philippines, Manila, 1964, vii + 241 pp., 27 cm. Price $8.00.

Tables are given for percentage points of the distribution of the sum of the roots θ_i for multivariate populations of from $s = 1$ to 50 variates, the roots having a distribution

$$k \prod_{i=1}^{s} \theta_i^{Q/2}(1 - \theta_i)^{R/2} \prod_{i>j} (\theta_i - \theta_j) \prod_{i=1}^{s} d\theta_i .$$

The tables have been calculated by use of the first four moments of the distribution. A further set of tables gives the beta parameters for a beta-distribution approximation. Explanations and illustrations of the use of the tables are included.

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43[K].—R. LOWELL WINE, *Statistics for Scientists and Engineers*, Prentice-Hall, Inc., Englewood Cliffs, N. J., 1964, xvi + 671 pp., 24 cm. Price $12.00.

This book is designed as a beginning one-year textbook in modern statistics, with elementary calculus as a prerequisite. Topics covered include frequency distributions, probability, sampling and sampling distributions, sampling from normal populations, analysis of variance, factorial experiments, regression, analysis of counted data, and distribution-free methods. The book contains ten tables: ordinates of the normal density function, cumulative normal distribution, confidence belts for proportions, percentage points of the χ^2 distribution, percentage points of the χ^2/ν distribution, percentage points of the t distribution, percentage points of the F distribution, power of the analysis of variance F test, percentage points of the Studentized range, and confidence belts for the correlation coefficient ρ.

The book is presented as one that may be used as a text for either a theoretical or applied course in statistics. It is the reviewer's opinion that such an approach is not satisfactory for a textbook, which should be one or the other, but not both. Although the book is fairly well written, it reads at times like a lecture rather than a text on which to base a lecture. The book contains many examples and problems, a good feature. It also seems to be reasonably free of misprints. The book should be useful to anyone learning the problems of numerical analysis in experimentation or planned investigations.

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44[K, P, Z].—LEON LEVINE, *Methods for Solving Engineering Problems Using Analog Computers*, McGraw-Hill Book Co., Inc., New York, 1964, xiii + 485 pp., 23 cm. Price $14.50.

This book describes how an engineer or a scientist may use the analog computer as a tool in solving engineering problems. It contains very little information in electronic circuitry and computing components, but it presents the necessary mathematical background and problem-solving techniques. The book consists of eleven chapters, in addition to thirteen appendices. The contents of the first six chapters are relatively well known, but the last five chapters present material which is rather unusual.

The first chapter discusses some general concepts of analog computation. Chapter 2 briefly reviews the theory of ordinary differential equations. It brings out those properties which are important to computer users. Chapter 3, Programming of Differential Equations, describes how one draws a computer diagram (i.e., set-up diagram using block symbols of operational amplifier, potentiometer, multiplier, etc.) from the given differential equations, with emphasis on initial conditions, normalized equations, and adjoint equations. Chapter 4 describes the techniques of block diagram manipulation and of scaling, and shows how operational amplifiers are used in problem solving. Chapters 5 and 6 examine the problems of explicit and implicit function generations, respectively.

Chapter 7, Error-Reduction Techniques, treats several cases where the effects of component errors can be reduced. Chapter 8, Optimization Techniques: Gradient Methods for Finding Maxima and Minima, describes how the computer may be used to optimize solutions. It mainly presents a mathematical and heuristical approach of gradient optimization procedures, and will be of interest to control engineers and systems designers.

The last three chapters, together with most of the appendices, provide an extensive discussion of statistics and of computer implementation of statistical problems. Chapter 9, Estimation and Test of Hypotheses, discusses fundamental statistical concepts and shows how one may use statistical properties to solve engineering problems. Chapter 10, Experimental Design and Detection of Errors, considers how many data are necessary to meet the accuracy requirements of a statistical problem, and discusses techniques useful in detecting computer malfunctions. (These two chapters were contributed by Arnold Levine.) Chapter 11, Application of Statistics to Computer Operations, treats methods for simulating statistical problems on the computer.

Although this book does not discuss methods of solving problems, using finite-difference networks and using continuous field analogs, it does uniquely present methods of solving problems involving noise processes rather extensively, as well as some techniques on error-reduction and optimization. It is quite readable and well written. It should serve as an excellent text for a course on solving engineering problems by analog methods.

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45[K, X].—J. M. HAMMERSLEY & D. C. HANDSCOMB, *Monte Carlo Methods*, John Wiley & Sons, Inc., New York; Methuen & Co., Ltd., London, 1964, vii + 178 pp., 19 cm. Price $4.75.

This book is an exceptionally clear and stimulating survey of applications of the Monte Carlo method. It is not a text; very few derivations are given. It is a guide to what has been done with the Monte Carlo method, to how the method should be applied, and to what should be done with this method in future research and applications.

The Monte Carlo method associates with a given problem a statistical problem to which the answer provides an answer to the original problem. The associated statistical problem is solved by a combination of analysis and computer simulation with pseudo-random numbers.

The book surveys applications to pure mathematics, to nuclear physics, to statistical mechanics, to mathematical statistics, and to theoretical chemistry. Many original results are presented. An original discussion is presented of percolation processes, which involve deterministic flows in random media.

Through many examples the book stresses that the most obvious formulation of a Monte Carlo problem is not always the best. The required answer is a statistical expected value. To attain a given accuracy with a specified probability, the number of times an unbiased statistic must be computed is proportional to the variance of the statistic. Different Monte Carlo formulations, all of which yield the same expected value, are shown to produce very different variances.

The chapters entitled "Conditional Monte Carlo" and "Percolation Processes" are particularly fascinating. In problems involving conditional probabilities, the proper embedding in a larger measure space may make the difference between practical solvability and unsolvability. In certain complex statistical problems, such as percolation processes, direct simulation is impractical. But these problems may be shown analytically to be equivalent to other statistical problems which are solvable by direct simulation with presently available computers.

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46[L].—A. R. CURTIS, *Coulomb Wave Functions*, Royal Society Mathematical Tables, Volume 11, Cambridge University Press, New York, 1964, xxxv + 209 pp., 28 cm. Price $15.00.

Schrödinger's equation for a hydrogen-like atom or ion in a potential field takes the form

$$\frac{h^2}{2\mu}\frac{d^2R}{dr^2} + \left\{W + \frac{Ze^2}{r} - \frac{h^2}{2\mu}\frac{L(L+1)}{r^2}\right\} R = 0,$$

on separating in polar co-ordinates. Here $-e$ denotes the charge and W the total energy of the electron, and Ze denotes the nuclear charge. The most extensive existing tables of solutions of this differential equation are those of the National Bureau of Standards [1], [2]. In these tables the independent variable used is $(2\mu W)^{1/2} h^{-1} r$. In the present tables the independent variable is $x = \mu e^2 h^{-2} Zr$; with this choice, wave functions for different values of W can be compared directly for constant values of r, rather than for constant values of $rW^{1/2}$. Thus the standard form adopted for the differential equation is

$$\frac{d^2y}{dx^2} + \left\{a + \frac{2}{x} - \frac{L(L+1)}{x^2}\right\} y = 0,$$

in which x is positive, $a\ (= 2Wh^2Z^{-2}\mu^{-1}e^{-4})$ is real, and L is zero or a positive integer.

The tables cover the ranges $a = -2(0.2)2$; $L = 0, 1, 2$; $x = 0(0.1)10$ and $1/x = 0(0.002 \text{ or } 0.005)0.1$. The accuracy of the tabulated values is six decimals throughout, corresponding generally to six or seven significant figures in the actual solutions and their first x-derivatives. Modified second differences, and, where necessary, modified fourth differences, are provided for interpolation in the x-direction. The tables were reproduced photographically from copy prepared on a card-operated typewriter and the printing is quite clear.

One of the difficulties of this project, described in the Introduction to the tables, was the choice of standard solutions of the differential equation. In addition to the usual criteria for numerically satisfactory solutions of linear differential equations in exponential or oscillatory regions, a further physical desideratum is that interpolation should be feasible in the a-direction. This last requirement could be fulfilled only partially.

The Introduction also describes the computation of the tables at the National Physical Laboratory, and includes worked examples of x-wise and a-wise interpolation. Furthermore, many mathematical properties of the chosen solutions are derived, including expressions in terms of confluent hypergeometric functions and the N. B. S. Coulomb wave functions; recurrence relations; convergent expansions in series of powers of x and series of Bessel functions; asymptotic expansions for large x, large $|a|$, small $|a|$, and large L.

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1. NATIONAL BUREAU OF STANDARD, *Tables of Coulomb Wave Functions*, Vol. I, Applied Mathematics Series **17**, U. S. Government Printing Office, Washington, D.C., 1952.
2. NATIONAL BUREAU OF STANDARDS, *Handbook of Mathematical Functions*, Chapter 14, Applied Mathematics Series **55**, U. S. Government Printing Office, Washington, D.C., 1964.

47[L].—M. A. FISHERKELLER & W. J. CODY, *Tables of the Complete Elliptic Integrals K, K', E, and E'*, Argonne National Laboratory, Argonne, Ill., ms. of 14 typewritten pages deposited in the UMT File.

The authors tabulate K, E, K', and E' to 17S for $k = 0(0.005)1$ and for $k^2 = 0(0.005)1$. In a two-page introduction we are informed that the underlying computations were performed on a CDC-3600, using 25S, and the results were checked to at least 20S by means of Legendre's relation, before rounding to 17S. Accordingly, the tabulated values are believed to be accurate to within one-half a unit in the least significant figure.

Reference is made to some specialized tables of Airey [1] relating to values of K and E to 12 and 13D when k^2 approaches 1, and to the tables of Spenceley and Spenceley [2], wherein the argument is the modular angle. It seems appropriate to mention here that the WPA Project for the Computation of Mathematical Tables [3] in 1942 prepared manuscript tables of the Jacobi elliptic functions, which included as an auxiliary table values of K to 17S for $k^2 = 0(0.01)1$, that is, at twice the subinterval in k^2 appearing in these tables.

J. W. W.

1. J. R. AIREY, "Toroidal functions and complete elliptic integrals," *Philos. Mag.* (7), v. 19, 1935, pp. 177–188.
2. G. W. SPENCELEY & R. M. SPENCELEY, *Smithsonian Elliptic Functions Tables*, Smithsonian Institution, Washington, D. C., 1947. [See *MTAC*, v. 3, 1948/1949, pp. 89–92, RMT **485**.]
3. *MTAC*, v. 1, 1943/1945, pp. 125–126, UMT **12**.

48[L].—A. H. Heatley, *Tables of the Confluent Hypergeometric Function and the Toronto Function*, University of Waterloo, Waterloo, Ontario, Canada, October 1964, one typewritten sheet and four computer sheets deposited in UMT File.

In these tables the functions $e^{-x}M(\alpha, \gamma, x)$ and $T(m, n, r)$ are tabulated to 9S in floating-point form, based upon calculations performed to at least 12S on an IBM 1620 system [1].

For the confluent hypergeometric function, the ranges of parameters are:

$$\alpha = \tfrac{1}{4}(\tfrac{1}{4})1, \quad \gamma = -\tfrac{1}{2}, \tfrac{1}{2}(\tfrac{1}{2})3; \quad \alpha = \tfrac{9}{4}(\tfrac{1}{4})3, \quad \gamma = 3; \quad \alpha = \tfrac{5}{4}(\tfrac{1}{4})2, \quad \gamma = 2.$$

The values of x are such that $x^{1/2} = 0(0.2)4, 5$; except that when $\alpha = 1, \gamma = -\frac{1}{2}$, $\frac{1}{2}(\frac{1}{2})3$, we find $x^{1/2} = 0(0.1)3$.

For the Toronto function, the corresponding ranges are:

$$m = -\tfrac{1}{2}(\tfrac{1}{2})\tfrac{1}{2}, \quad n = -2(\tfrac{1}{2})2, \quad r = 0(0.2)4(1)6, 10, 25, 50; \quad \text{and}$$

$$m = 1, \quad n = -2(\tfrac{1}{2})2, \quad r = 0(0.1)3.$$

J. W. W.

1. *Math. Comp.*, v. 18, 1964, pp. 687–688, MTE **361**.

49[L].—J. R. Johnston, *Tables of Values and Zeros of the Confluent Hypergeometric Function*, Report 31901, Aircraft Division, Douglas Aircraft Company, Inc., Long Beach, Calif., August 1964, 4 pp., 28 cm.

This report briefly describes the computational procedure followed in evaluating the confluent hypergeometric function $_1F_1(A, B, X)$ and its zeros by a FORTRAN IV program prepared for use on an IBM 7094 system.

Computation of the function and its zeros was carried to 7D precision for $A = -5(0.25) - 0.25$, $B = 0.25(0.25)4$, $X = 0.05, 0.1(0.1)1(0.2)10(0.5)20$, and for $A = -20(0.5)1$, $B = 2$, $X = 0.05, 0.1(0.1)1(0.2)20(0.5)50$. These internally computed values for each of these two ranges were rounded to 4S and written as separate files on a special output tape, of which a copy can be obtained upon request from the Technical Library, Aircraft Division, Dept. C-250, Douglas Aircraft Company, Inc.

No printed output is available except for an abbreviated table of zeros to 4S that appears on the last page of this report. The range represented therein is $A = -20(1) - 1$, $B = 2$, and, with a few exceptions corresponding to $A = -4(1) - 1$, the first five positive zeros are tabulated.

For a list of related tables the reader is referred to the publication of Slater [1].

J. W. W.

1. L. J. Slater, *Confluent Hypergeometric Functions*, Cambridge Univ. Press, New York, 1960. [See *Math. Comp.*, v. 15, 1961, pp. 98–99, RMT **22**.]

50[L].—Siu-kay Luke & Stanley Weissman, *Bessel Functions of Imaginary Order and Imaginary Argument*, University of Maryland, Institute for Molecular Physics, Report DA-ARO(D)-31-124-G466 No. 1, 1964, College Park, Md.

This report gives a rather extensive tabulation of

$$G_q(v) = K_{iq}(v) = \frac{1}{2}\pi \frac{I_{iq}(v) - I_{-iq}(v)}{\sinh q\pi}, \qquad v = e^x,$$

where $I_\mu(z)$ and $K_\mu(z)$ are the modified Bessel functions of the first and second kinds, respectively. The range on q and x varies. For example, $q = 0.2(0.2)10$, $x = 1.0(0.01)2.22$; $q = 0.4(0.2)10$, $x = 2.23(0.01)2.29$; $q = 1.2(0.2)10$, $x = 2.30(0.01)2.39$; $q = 1.6(0.2)10$, $x = 2.40(0.01)2.49$. Roughly speaking, we have data for $q = 0.2(0.2)50$, where the tables were "cut at an x value for each set of q's where the oscillating amplitude appears to be a constant." When $x > \ln q$, the tables were "cut at its first zero after it passed the turning point." The entries were found by numerical integration of the differential equation. The authors expect the data to be good to at least 5S for $q < 40$ and to 4S for higher q. The only other tables of this kind known to us are by S. P. Morgan. [See *MTAC* v. 3, 1948–1949, pp. 105–107, RMT **504.**] There is some overlap.

Y. L. L.

51[L].—M. M. Stuckey & L. L. Layton, *Numerical Determination of Spheroidal Wave Function Eigenvalues and Expansion Coefficients*, AML Report 164, David Taylor Model Basin, Washington, D. C., 1964, 186 pp., 26 cm.

Spheroidal wave functions result when the scalar Helmholtz equation is separated in spheroidal coordinates, either prolate or oblate. The angular prolate spheroidal wave functions, for example, satisfy a differential equation of the form

$$\frac{d}{dz}\left[(1 - z^2)\frac{du}{dz}\right] + \left(\lambda_{mn} - c^2z^2 - \frac{m^2}{1 - z^2}\right)u = 0.$$

The solutions of this equation are much more complicated than either Bessel or Legendre functions, in which, in fact, series solutions of the spheroidal functions are most often expanded. The complexity arises from the fact that the spheroidal differential equation has an irregular singular point at ∞ and two regular ones at $z = \pm 1$, in contrast to the three regular ones of the Legendre equation and to the one regular and one irregular singularity of the Bessel equation.

The construction of tables of spheroidal wave functions involves the calculation of the eigenvalues λ_{mn} of the differential equation, that is, those values of λ for which there are solutions that are finite at $z = \pm 1$, and the calculation of the coefficients in expansions in terms of either Legendre or spherical Bessel functions. In the past, such calculations have been, for the most part, sporadic and in many cases not very accurate.

The tables of the spheroidal eigenvalues and expansion coefficients in this report from the David Taylor Model Basin are the most complete that have been made available so far. Values of λ_{mn} are given to 11S, in floating-point form, for $m = 0(1)9$, $n = m(1)m + 9$, for $c = 0.25(0.25)10(1)20$. Values of the expansion coefficients d_r^{mn} are given for $c = 0.25(0.25)10$, $m = 0$ and 1, $n = m(1)10$, $r = 1(2)29$ for $n - m$ odd, and $r = 0(2)28$ for $n - m$ even.

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52[L, M].—Andrew Young & Alan Kirk, *Bessel Functions. Part* IV, *Kelvin Functions*, Royal Society Mathematical Tables, Volume 10, Cambridge University Press, New York, 1964, xxiii + 97 pp., 27 cm. Price $11.50.

As noted in the title, this volume is the fourth in a series of tables devoted to Bessel functions that was initiated by the Mathematical Tables Committee of the British Association for the Advancement of Science and has been continued by its successor, the Mathematical Tables Committee of the Royal Society.

The first three parts of this tabulation of Bessel functions, which dealt with those functions of the first two kinds of real and of pure imaginary arguments, have been previously reviewed in this journal (*MTAC*, v. 1, 1943–1945, pp. 361–363; *ibid.*, v. 7, 1953, pp. 97–98; *Math. Comp.*, v. 15, 1961, pp. 214–215).

The present tables are concerned with Kelvin functions of integer orders and real arguments. These functions are defined in terms of Bessel functions of the first two kinds by the relations

$$\operatorname{ber}_n x + i \operatorname{bei}_n x = J_n(xe^{3i\pi/4})$$

and

$$\operatorname{ker}_n x + i \operatorname{kei}_n x = \frac{i\pi}{2} \{J_n(xe^{3i\pi/4}) + iY_n(xe^{3i\pi/4})\}.$$

Table I presents 15D values of the Kelvin functions of orders 0 and 1 for $x = 0(0.1)10$. Table II consists of 7S and 8S values of these functions and of their polar forms for $n = 0(1)2$ and $x = 0(0.01)2.5$. The main table is Table III, which generally gives 6S and 7S values of the functions and of their polar forms for $n = 0(1)10$ and $x = 0(0.1)10$, together with second and fourth central differences. The second differences are modified when appropriate and are so designated.

It should be remarked that Table III is especially useful and important because in all previously published tables of this kind, the orders of the functions have not exceeded 5.

Interpolation in certain parts of Tables II and III is not readily accomplished; there the user is referred to Table IV, where auxiliary functions are tabulated for $\operatorname{ker}_n x$ and $\operatorname{kei}_n x$ when $n = 0$ and 1, and where the modified functions $x^{-n} \operatorname{ber}_n x$, $x^{-n} \operatorname{bei}_n x$, $x^n \operatorname{ker}_n x$, $x^n \operatorname{kei}_n x$, and the similarly modified moduli, $x^{-n}M_n(x)$ and $x^nN_n(x)$, are uniquely tabulated, generally to 7S and 8S, for $n = 3(1)5$, $x = 0(0.1)2.5$; $n = 6(1)10$, $x = 0(0.1)5$. For $n = 2$, only $x^n \operatorname{ker}_n x$, $x^n \operatorname{kei}_n x$, and $x^nN_n(x)$ are given; these appear to 7D for $x = 0(0.01)0.2$. Throughout Table IV second central differences and fourth central differences, where required, are shown.

The numerical tables are preceded by a section of 12 pages entitled Functions and Formulae, wherein appear basic definitions and properties of the Kelvin functions, as well as their various expansions in series. Also listed therein are indefinite integrals involving these functions, after which there is a detailed description of the preparation of the tables.

The authors state that, prior to printing, all the tabular entries, originally computed on desk calculators, were recomputed, using double- and triple-precision arithmetical routines when necessary, on the DEUCE computer in Liverpool University. Because of these elaborate precautions to insure accuracy, it is claimed that each tabular value is correct to within a unit in the least significant recorded figure.

A bibliography, consisting of 34 titles, serves to round out this valuable and attractive addition to the tabular information on Bessel functions.

J. W. W.

53[Q, V, X].—A. H. Taub, General Editor, *John von Neumann, Collected Works, Volume* VI, Macmillan Co., New York, 1963, viii + 538 pp., 25 cm. Price **$14.00.**

This is the sixth and last volume of the collected works of John von Neumann, published under the general editorship of A. H. Taub, a close associate of von Neumann. This volume alone is sufficient to remind those of us, who had the good fortune to know von Neumann personally, of the breadth of his scientific interests and achievements and of the fundamental contributions which he had made in so many diverse fields. The sixth volume contains papers in the theory of games, astrophysics, hydrodynamics, and meteorology. In each of these fields he not only made significant contributions but initiated, by his work, new areas of research which will be pursued by scientists for decades to come. Thus, he may be considered as the father of the modern theory of games as well as of the numerical prediction of weather by the solution of the governing hydrodynamic equations.

The writer is most familiar with his work on the interaction of shock waves, which is covered in this volume. This is another field in which his contributions have become the basis for a major area of scientific endeavor both by his contemporaries and by future investigators. The readers of this journal may be interested to know that von Neumann was not only a prime mover in the development of modern digital computer systems, but that he was perhaps the most outstanding human computer of his generation. The paper entitled "The Mach effect and height of burst," by F. Reines and John von Neumann (pages 309–347) reminds the reviewer of an incident which occured during World War II, just prior to the detonation of the first atomic device over Japan. Von Neumann had arrived in Washington to attend a special meeting at which the possible use of this new weapon was discussed. On the train from Princeton, he hurriedly carried out a very lengthy and complex computation in order to determine the height at which the burst should take place in order to attain maximum blast damage. He did this with a pencil on a piece of scratch paper, without the aid of any mathematical tables, formulas, or any modern computer devices. Upon his arrival he handed the paper to me and asked me to please obtain an accurate solution and to call him at the conference as soon as possible. Using all the mathematical tables at my disposal and an electrically operated Friden calculator, I proceeded to recalculate the optimum height of burst as rapidly and accurately as I could. I obtained my result only after four hours of painstaking work and found to my surprise and some chagrin that his result and mine agreed to four decimal places.

H. P.

54[S].—Dietrich Hahn, et al., *Seven-Place Tables of the Planck Function for the Visible Spectrum,* Academic Press, New York, 1964, xxi + 135 pp., 21 cm. Price $5.50.

Let

$$S = \lambda^{-5}[-1 + \exp(c_2/\lambda T)]^{-1}; \qquad H = SV(\lambda);$$

$$\Sigma S = \frac{1}{\Delta\lambda}\int_{350}^{845\,\mathrm{nm}} S\,d\lambda; \qquad \Sigma H = \frac{1}{\Delta\lambda}\int_{385}^{780\,\mathrm{nm}} SV(\lambda)\,d\lambda.$$

In the above, $V(\lambda)$ is the relative spectral luminosity—an empirical function which is given in Table 1, page 3, to 3S for $\lambda = 385(5)780$ nm [1,000 nm $= 1$ μm]. Two values of c_2 are used; namely 14380 and 14420 m°K. Values of S and H corresponding to the first and second value of c_2 are termed S_1, H_1 and S_2, H_2, respectively. The main table, consisting of 120 pages, gives S_1, S_2, H_1, H_2 to 7S for $\lambda = 350(5)$ 845 nm, for each of 60 values of T, ranging between 973.15°K and 15,000°K. The decimal point has been omitted; it belongs "in the first gap" of the tabular entry, as stated in the Introduction. Since H depends on $V(\lambda)$, it is not known to more than three significant figures. The inclusion of seven digits (and the omission of the decimal point) must be ascribed to the needs of mass-production handling. Two-page tables of ΣS_i and ΣH_i, where $i = 1, 2$, are given, for the values of T of the main table; another two pages contain their common logarithms. Table 5 (the last) is a one-page tabulation of $B = 60 \int_0^\infty H(\lambda, T)\, d\lambda / \int_0^\infty H(\lambda, T_{pt})\, d\lambda$ for $c_2 = 14380$ and $T_{pt} = 2042.15$ (units the same as before), for 24 values of T.

The introductory material is given in both German and English. The table proper is a clear, legible reproduction of IBM tabulations. The calculations of S appear to be correct. However, the values of ΣS_i are reliable to only three significant figures, according to tests made by the reviewer.

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55[W, X, Z].—SAUL I. GASS, *Linear Programming, Methods and Applications*, McGraw-Hill Book Company, New York, 1964, xii + 280 pp., 24 cm. Price \$8.95.

This is the second edition of this work, which originally appeared in 1958 and was reviewed in *MTAC*, v. 13, 1959, pp. 60–61. The book is intended as a text for a rather complete first course in linear programming at an upper-undergraduate or graduate level. Care has been given to make it especially understandable and useful for the non-mathematics major.

Other than the correction of typographical errors, the reviewer could find no significant alteration of the original text, but several additions enhance its usefulness. The number of exercises at the end of each chapter has been increased. A number of the added exercises are of a rather simple computational nature to help the less mature reader. Several explanatory footnotes and short paragraphs have been scattered throughout the volume. The results of an iteration missing in the first edition have been added to the example on page 109.

The more significant changes from the original text are as follows: the Survey of Linear-Programming Applications has been moved from Chapter II to the Introduction, where it belongs. A paragraph has been added on page 70 to note how the determinants of the bases used in the simplex procedure can be readily obtained as a by-product of the computation. A short statement about slack variables has been expanded to a full section on pages 76 and 77.

A valuable nine-page section on sensitivity analysis has been added to the end of the chapter on parametric linear programming. Full sections on integer linear programming and the decomposition algorithm of Dantzig and Wolfe to reduce computation in solving large-scale systems have been added to the chapter entitled Additional Computational Techniques.

The listing of available digital computer codes has been expanded. The original list referred to 10 machines of four manufacturers; the new list covers 28 machines made by 11 firms. One doubts the wisdom of including such a list in a basic text of this type, since it will be necessarily dated and incomplete. Furthermore, a reader with a problem will have a very limited number of machines available to him on a practical basis, and will query those installations for available codes, anyway.

The excellent, complete bibliography has been brought up to date.

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56[X].—J. C. Butcher, *Tables of Coefficients for Implicit Runge-Kutta Processes*, ms. of 9 sheets deposited in the UMT File.

In a paper (*J. Austral. Math. Soc.*, v. 3, 1963, pp. 185–201) Butcher generalized the idea of Runge-Kutta integration to include implicit as well as explicit problems. At the same time he cleared up a number of theoretical points related to the use of Runge-Kutta methods for systems of differential equations. In two subsequent papers (*Math. Comp.*, v. 18, 1964, pp. 50–64 and pp. 233–244) Butcher considered various aspects of implicit Runge-Kutta processes. In particular, he derived formulas for the weights and parameters of Runge-Kutta methods based on the abscissas of the Legendre-Gauss, Radau, and Lobatto quadrature formulas.

In the tables being reviewed here values of these weights and parameters are given for values of m, the number of terms in the Runge-Kutta sum, ranging from 3 to 10. The order of accuracy of the Runge-Kutta methods are $2m$ in the Legendre-Gauss case, $2m - 1$ in the Radau case and $2m - 2$ in the Lobatto case. All quantities in the tables are claimed to be in error by less than 10^{-20}.

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57[X].—Irving Allen Dodes & Samuel L. Greitzer, *Numerical Analysis*, Hayden Book Company, Inc., New York, 1964, 390 pp., 23 cm. Price $9.95.

This is an elementary book on numerical analysis based on a course that has been taught at the Bronx High School of Science since 1955. Table of Contents: Desk Calculator Arithmetic, Chapter 1; Iterative Techniques, Chapter 2; Statistical Analysis: Condensation, Chapter 3; Comparing Two Distributions for Similarity, Chapter 4; Comparison for Difference, Chapter 5; The Problem of Prediction, Chapter 6; Writing a Research Paper in the Sciences, Chapter 7; Solving an Equation by Iteration, Chapter 8; Determinants and Matrices, Chapter 9; Linear Programming, Chapter 10; Dimensional Analysis, Chapter 11; Getting About on the Earth, Chapter 12; Mathematics of Astronomy, Chapter 13; Empirical Formulas and Interpolation, Chapter 14.

P. J. D.

58[X].—L. È. ÈL'SGOL'C, *Qualitative Methods in Mathematical Analysis*, Volume 12, Translations of Mathematical Monographs, American Mathematical Society, Providence, R. I., 1964, vii + 250 pp., 23 cm. Price $14.60.

The preface of this interesting and comprehensive book has the "Russian" quality of disclosing to the reader at the very outset some of the "secrets" of the trade! A careful distinction is made very early between the purely qualitative methods, pertaining to geometry and topology, which are considered in the first three chapters, and the more nearly quantitative processes from the domain of analysis, which are treated in the last three chapters. The reviewer believes that the dividing line between the two categories is even more blurrable by means of high-speed computers.

The six chapters deal with the following topics: I. Extremal problems, notably the Poincaré-Morse type number theory. II. Application of complex variables. III. Fixed points: theorems of Brouwer, Lefschetz, and Schauder. IV, V, VI. Differential equations: the index theory of Poincaré and all manners of questions concerning existence theorems, approximations, oscillations, and systems with retarding or advancing argument.

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59[X].—ERNST PESCHL & KARL WILHELM BAUER, *Über eine nichtlineare Differentialgleichung 2. Ordnung, die bei einem gewissen Abschätzungverfahren eine besondere Rolle Spielt*, Forschungsberichte des Landes Nordrhein-Westfalen, No. 1306, Westdeutscher Verlag, Opladen, 1964, 59 pp., 24 cm. Price DM 43.50. (Paperback.)

The first four chapters of this monograph treat in detail solutions of the equation

$$ff'' - f'^2 + 3Lf' - 2f + 2L'f + 2L^2 = 0,$$
$$f = f(\alpha), \qquad L = -1 - \epsilon e^{2\alpha}.$$

A complete summary of the solutions is presented in Chapter 5. The solution of the above equation with $2L^2$ replaced by $-2L^2$ is discussed in Chapter 6. Graphical representations of the solutions for $\epsilon = 0$ and $\epsilon = \pm 1$ are provided.

Y. L. L.

60[X].—V. I. ZUBOV, *Methods of A. M. Lyapunov and Their Application*, P. Noordhoff, Ltd., Groningen, 1964, xvi + 263 pp., 22 cm. Price $9.50.

This is an excellent translation of a monograph devoted to recent work by Yerugin, Barbashin, Krasovsky, Nemytsky, Sobolev, and Zubov himself, on the application of Lyapunov's first and second methods to the study of the stability and periodicity of solutions of nonlinear ordinary and partial differential equations. The book is recommended to all those interested in the modern theory of ordinary differential equations and control processes.

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61[X, Z].—N. METROPOLIS, ET AL., editors, *Experimental Arithmetic, High Speed Computing and Mathematics.* Proceedings of Symposia in Applied Mathematics, Volume 15, American Mathematical Society, Providence, R. I., 1963, ix + 396 pp., 26 cm. Price $9.10.

This interesting collection includes all but two papers which were given at two symposia: *Experimental Arithmetic* at Chicago, and *Interactions between Mathematical Research and High-Speed Computing* at Atlantic City, both in April, 1962. The papers here are:

Purposeful and unpurposeful computing, by Harvey Cohn.

Eliminating the irrelevant from mechanical proofs, by Martin Davis.

The mechanization of mathematical arguments, by Hao Wang.

Towards more versatile mechanical translators, by E. T. Irons.

Information theory and decoding computations, by Peter Elias.

Adaptive neutral networks as brain models, by H. D. Block.

Computer investigation of orthogonal Latin squares of order ten, by E. T. Parker.

Determination of division algebra with 32 elements, by R. J. Walker.

How programming difficulties can lead to theoretical advances, by E C. Dade and H. Zassenhaus.

Methods of successive restrictions in computational problems involving discrete variables, by C. B. Tompkins.

An experimental study of the simplex method, by Harold W. Kuhn and Richard E. Quandt.

Large and nonconvex problems in linear programming, by R. E. Gomory.

Some high speed logic, by D. H. Lehmer.

Stability questions for some numerical methods for ordinary differential equations, by Germund G. Dahlquist.

Some applications of the quotient-difference algorithm, by Peter Henrici.

Plane-rotations in floating-point arithmetic, by J. H. Wilkinson.

New aspects in numerical quadrature, by F. L. Bauer, H. Rutishauser, and E. Stiefel.

On Jacobi rotation patterns, by H. Rutishauser.

Automatic numerical integration of ordinary differential equations, by Arnold Nordsieck.

Survey of stability of different schemes for solving initial value problems for hyperbolic equations, by Peter D. Lax.

Unexpected dividends in the theory of prime numbers, by J. Barkley Rosser.

The particle-in-cell method for numerical solution of problems in fluid dynamics, by Francis H. Harlow.

Numerical experiments in atmospheric hydrodynamics, by J. G. Charney.

The oscillations of the earth and of the atmosphere, by Gordon J. F. MacDonald.

Few particle experiments in statistical mechanics, by Berni J. Alder.

An approach to the Ising problem using a large scale fast digital computer, by Chen-Ping Yang.

Applied mathematics as used in theoretical chemistry, by Joseph O. Hirschfelder.

The mechanization of science, by R. W. Hamming.

The editors have combined the papers of both symposia, and have ordered them here without regard to the particular symposium at which they were given, and with no indication of that symposium. The title of the volume similarly reflects this loss of precision. In extenuation, it must be said that the resultant fuzzing of two admirably precise and important subjects stems mostly from the fact that more than one of the speakers did not really speak to the point. They presented "papers." Perhaps it could even be said that these were good papers. Still, the willingness to disregard the title of the symposium seems to this reviewer an attitude that should be corrected.

Consider the subject of the second symposium. That seems clear enough. Ideally, a paper here would examine a chain of theoretical and computational problems that led from one to the other and then back at a higher level. Such chains are surely known. Had every speaker presented material of this type, it could then be hoped that the commentators would attempt to generalize these experiences and formulate a resulting scientific methodology. The symposium was of value, but was not that successful. Surely, though, this is a subject of great scientific importance. If mathematicians are to learn to consistently use computers as scientific tools (say, in the way Ernest Rutherford used physical equipment) and not merely to obtain scattered results, a study of such interactions remains a prime necessity.

Some speakers at the second symposium, such as Zassenhaus and Lehmer, did speak directly to the point. But others did not. Likewise, Rosser and Charney spoke directly to the point of the first symposium.

For all that, the volume is certainly of value.

D. S.

62[Z].—Franz L. Alt & Morris Rubinoff, editors, *Advances in Computers*, Volume 3, Academic Press, New York, 1962, xiii + 361 pp., 23 cm. Price $12.00.

This third annual volume of *Advances in Computers* serves well to cover a number of additional areas in the computer field. Subjects include the Computation of Satellite Orbit Trajectories, Alternating Direction Implicit Methods, Recent Developments in Nonlinear Programming, Multiprogramming, Combined Analog-Digital Techniques in Simulation, and Information Technology and the Law.

S. D. Conte's contribution on the calculation of satellite orbit trajectories considers the problems of both predicting and determining such orbits. He presents a lucid survey and evaluation of methods of numerically integrating the equations of motion for a satellite moving under the influence of a central body force and subject to various perturbative forces. There is a careful discussion of various accuracy tests, including methods of estimating truncation and roundoff error accumulation. Results of a numerical study are presented which compare the computational efficiency of various methods. The problem of determining satellite orbits based on data received during launching and subsequent tracking is considered. In addition, building blocks for a comprehensive orbit prediction and determination computer program package are outlined.

Alternating-direction implicit methods constitute in many cases the best methods available to us today for solving large systems of elliptic and parabolic partial difference equations. G. Birkhoff, R. S. Varga, and D. Young present an excellent review of the status of the problem of providing a rational explanation of their effectiveness for the solution of systems of linear elliptic difference equations in the plane. Included is a survey of theoretical results comparing the effectiveness of alternating direction methods with systematic overrelaxation methods. In addition, results of some systematic numerical experiments performed to test the comparative effectiveness of the various methods are presented.

P. Wolfe's paper, Recent Developments in Nonlinear Programming, surveys methods of solving programming problems which involve maximizing a concave objective function subject to a set of constraints specified by inequalities which taken together specify a convex set. The lack of information concerning the relative computational efficiency of many of the methods is pointed out.

E. F. Codd defines multiprogramming as "the technology associated with the concurrent execution of instructions which are not restricted to being immediate neighbors in any instruction string." He discusses the role which multiprogramming can play in various environments. A system for "batch multiprogramming" on a STRETCH-like system with one processing unit is outlined. Problems associated with scheduling runs and allocating parts of the computer system to various programs in order to minimize the running time for a set of runs are considered. Finally, multiprogramming system requirements are reviewed in the case where two or more processing units are involved, and the implementation of multiprogramming systems in this case is discussed.

H. K. Skramstad, in a chapter entitled Combined Analog-Digital Techniques in Simulation, surveys a number of interconnected analog-digital systems. He indicates some areas of applying such systems as well as some methods of combining them.

R. C. Lawler's stated purpose in a report on Information Technology and the Law is "to provide an overview of the activity occurring at the interface between information technology and law and to suggest which problems may be most significant from the standpoint of the lawyer, the courts, and the public as a whole." A broad range of subjects is included from the use of computers in the retrieval of legal information and in predicting court decisions to some of the legal problems posed by the use of information in a computer, such as possible copyright infringements, for example.

The editors are to be commended in their continuing successful efforts to bring together well written surveys of representative areas from the broad field of computer science and technology "as an antidote to specialization."

E. Cuthill

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63[Z].—C. Andersen, *An Introduction to ALGOL* 60, Addison-Wesley, Reading, Mass., 1964, vi + 57 pp., 28 cm., plus foldout ALGOL Syntactical Chart. Price $1.75.

This book is a well organized introduction to ALGOL: it deals progressively with the elements and constructions of the language and with the uses to which it may be put. Instructive examples are placed at strategic points in the text, which is eminently suited to the teaching methods currently in use in American universities.

The author has adopted the hardware conventions associated with the machine (GIER) with which he is most familiar; furthermore, he devotes one chapter to certain additions to ALGOL which increase the efficiency of the programs produced by the GIER compiler. The instructor who uses this book as a text may well wish to make certain modifications concerning these points. He may also wish to place slightly more emphasis upon such matters as conditional statements in arithmetic expressions—which are relegated to an appendix, on recursive procedures—to which only fleeting reference is made, and to own variables which are dealt with not at all.

There are a few typographical errors (for example on page 3 two signs, $\neq$ and $=$, should be interchanged, and on page 53 there is a redundant open bracket), though mistakes of this nature occurring in the ALGOL texts will doubtless either be recognized by anomalous functioning of the program or be picked up by the ALGOL monitor.

The publishers are to be congratulated on their enlightenment in offering this book at such a moderate price.

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64[Z].—KARL-HEINZ BÖHLING, *Zur Strukturtheorie sequentieller Automaten*, Forschungsberichte des Landes Nordrhein-Westfalen, No. 1279, Westdeutscher Verlag, Opladen, 1964, 73 pp., 23 cm. Price DM 45. (Paperback)

The author defines a *sequential system* as a triple $\langle \epsilon, G, F \rangle$, where ϵ is the union of disjoint sets θ (comprising input and output alphabets) and S (the set of states), and where $G \subseteq \theta \times S \times S$ and $F \subseteq \theta \times S \times \theta$ are ternary relations on ϵ corresponding, respectively, to the transition and output functions of a conventional deterministic sequential machine. The apparent purpose of this monograph is to show that sequential systems are sufficiently general to embrace all of the principle models current in automata theory, including incompletely specified machines, nondeterministic machines, Rabin-Scott machines, and abstract (Ginsburg) machines among others—a conclusion that is hardly surprising. Aside from this, a tedious attempt is made to develop a formalism for distinguishing among the various types of machines, considered as sequential systems.

As the author grants in his introduction, no attempt is made to generalize, unify, or even present the existing theories, though he promises to deduce some consequences in a subsequent publication. At least until this program is carried out and the sequential system is shown to be a fruitful generalization, one must regard the present work as virtually useless, either as a text or a reference book.

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65[Z].—ROY DUBISCH, *Lattices to Logic*, Blaisdell Publishing Co., New York, 1964, vii + 88 pp., 20 cm. Paperback. Price $1.65.

This treatment of lattices, sets, switching circuits and logic is written primarily for mathematical beginners. Partially ordered systems and lattices are introduced through Hasse diagrams of assorted shapes and structures, but after this general introduction, except in one short chapter on convex sets, attention centers chiefly on Boolean algebras. It is shown through examples that the equalities of set theory have analogues in the equivalences of switching circuits and of propositional logic. The discussion proceeds at the notational level, no attempt being made to locate the source of the analogies. Numerous exercises are included, along with complete solutions.

There are some errors. To correct these, it should be pointed out to prospective young readers that (p. 23) not all lattices have universal elements, that (p. 46) the second illustrative example on switching circuits is incorrectly worked out, that (pp. 58–59) the remarks and usages concerning logical and material implication are misleading, that (p. 59) the exercise in which the reader is asked to prove that a conditional statement is logically equivalent to its converse contains an unfortunate misprint, and that (p. 60) C. S. Peirce's name is repeatedly misspelled.

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66[Z].—L. I. Gutenmakher, *Electronic Information-Logic Machines*, John Wiley & Sons, Inc., New York, 1963, x + 170 pp., 23 cm. Price $8.00.

The term "information-logic machines" describes systems capable of dealing with data presented in written form. These machines are considered to be able to perceive, to store, and to manipulate such data. Their purpose is to mechanize much of the intellectual work of mankind, just as earlier machines reduced the need for man to perform physical labor.

Electronic Information-Logic Machines contains, basically, two kinds of material; on the one hand, concise and practical descriptions of hardware implementations in use today; on the other, some preliminary calculations and speculation as to the work such machines may eventually be expected to perform.

Professor Gutenmakher intends for his systems to go well beyond what is normally understood by "information retrieval." For example, it would be possible to store the 100,000,000 or so titles thus far accumulated by man, and, then, by clever programming, by "specialized algorithms," to synthesize new information from the old.

In Chapter 6 he discusses at great length the problems in indexing, classifying, and translating to a common *machine-oriented* language the literature in the physical sciences. Again, he does not stop there but continues on to show how his information-logic machine will propose ways to, say, synthesize new substances based on what is stored in the chemistry literature. The machine will, in effect, be "teaching" and "self-instructing" in the sense of up-dating its stored files with results of experiments run to confirm its "suggestions." [On p. 156 the translators appear to be carried away by Professor Gutenmakher's futuristic mechanical brain and have it do its own experimental testing as well!]

In summary, Professor Gutenmakher is highly informative in his discussions of hardware and in the calculations to justify the future use of "info-logic" machines; and also quite entertaining in the "scientific science-fiction" approach to the ultimate capability of these systems, programmed, but no longer limited, by humans! His citation of U. S. and other Western literature is generous, and perhaps his book is too strongly influenced by some of our own "automation addicts" whose philosophies are: "if one can count the bits involved, one can mechanize the process" or "it's just a matter of 'zeros' and 'ones', what could be simpler?" In Russia, as in the U. S., the tendency to confuse some of the simpler facts of information theory with the far less understood theory of knowledge and brain functions is thus apparent. The book, then, is useful, not only for what it says explicitly but also for what it implies—that the blue sky knows no iron curtain!

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67[Z].—THEODORE E. HARRIS, *The Theory of Branching Processes*, Prentice-Hall, Inc., Englewood Cliffs, N. J., 1963, xiv + 230 pp., 24 cm. Price $9.00.

This book presents a systematic and thorough treatment of a class of Markov processes called "branching processes." The simplest example is the Galton-Watson process $\mathfrak{Z} = \{Z_n : n = 0, 1, 2, \cdots\}$, where $Z_0 = 1$ and the conditional distribution of Z_n, given $Z_{n-1} = k$, is that of the sum of k independent, identically distributed non-negative integer-valued random variables. In the classical interpretation, Z_n is the number of descendants in the nth generation of the progenitor ($Z_0 = 1$). The chapter headings are:

Chapter I.	The Galton-Watson branching process
Chapter II.	Processes with a finite number of types
Chapter III.	The general branching process
Chapter IV.	Neutron branching processes (one-group theory, isotropic case)
Chapter V.	Markov branching processes (continuous time)
Chapter VI.	Age-dependent branching processes
Chapter VII.	Branching processes in the theory of cosmic rays (electron-photon cascades)

The mathematical level required to read this book is about that of Feller, although there is frequent use of material that is found in books such as those by Doob and Loève. A large number of theorems, remarks, and examples are given without proof. Since there are no problems, these "loose ends" provide a perfect opportunity for the reader to check his comprehension of the material.

This book is highly recommended as an authorative and well written exposition by a significant contributor to this field.

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68[Z].—H. D. HUSKEY & G. A. KORN, editors, *Computer Handbook*, McGraw-Hill Book Company, Inc., New York, 1962, xviii + 21 (individually numbered) sections, 24 cm. Price $25.00.

The *Computer Handbook* presents the general principles of the design and utilization of both analog and digital computers. Sufficient detail is presented in both fields so that a computer could be constructed or, if one had a computer, it could be used efficiently.

Sections 1 through 9 cover analog computers, while sections 10 through 21 are concerned with digital computers.

Covered under the subject of analog computers are terminology, design of analog-computer building blocks and systems, and the organization and maintenance of analog computers. Also covered are the applications of analog computers to control systems (such as process-control), random-process studies, and their application to mathematical solutions (such as algebraic-equation solvers), the solution of partial differential equations, and linear programming methods.

There is also a section devoted to solid-state analog computer components, which describes many new solid-state circuits.

The author describes many techniques that are not too familiar to many engineers, such as network-type analogies for fields and structures. These are illustrated by mechanical, electro-mechanical, hydrodynamic, and heat-transfer problems.

The remainder of the book deals with digital computers, the solid-state components used with computers and typical circuits such as emitter followers, shift registers, and adders. Also, input-output devices such as magnetic drums and tape handlers are covered thoroughly.

The only omission noted in the book was in the section dealing with logical elements. The use of stroke functions [1] in the reduction of logical systems was not mentioned as a means of determining minimal nets corresponding to a given set of logic functions.

The section covering programming and coding is thorough—indeed, probably too detailed in its description of ALGOL-60, in contrast to its abbreviated discussion of other languages.

The last section covers special-purpose computers, with particular emphasis on the digital differential analyzer, which the author states can be substituted in many cases for an analog computer in the solution of ordinary differential equations. The former provides a higher order of accuracy, since the precision of operations is limited only by a register size rather than by the tolerance of components of an analog computer. This type of special-purpose computer could be used to generate and supply continuous-control variables or other functions to a general-purpose computer.

In the opinion of the reviewer, this handbook offers much valuable information, even to the most experienced computer personnel.

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1. N. T. Grisamore, "Logical design using stroke functions," IRE *Transactions on Electronic Computers*, Vol. EC-7, No. 2, June 1958.

69[Z].—Allen Newell, et al., *Information Processing Language-V Manual*, Second Edition, Prentice-Hall, Inc., Englewood Cliffs, N. J., 1964, xxxvi + 267 pp., 27 cm. Price $7.95 (paperbound).

The size of this book needs to be somewhat discounted because most of it is set in pica type, at four lines per vertical inch, and the pages are provided with generous margins. It is well laid out, clearly written, and has a bibliography of 62 pertinent references. It consists of two parts; namely, a learning manual and a reference manual. The learning manual does not require prior knowledge of any form of computer programming. It contains a section, "Organizing Complete Tasks," which would be of value to persons learning other forms of programming. The learning manual contains exercises and solutions for some of them and is suitable for self-teaching. There is an index, which applies to the reference manual only.

For those already acquainted with IPL-V, the differences between the first and second editions are:

1. Conventions and definitions in the initial manual have been changed at only six minor points, all associated with loading and monitoring. These may impose minor modifications to existing programs. About 30 basic processes have been added. All the changes and extensions are listed in four pages of the manual and are also incorporated in their appropriate places in Part Two, the Reference Manual.

2. Part One, the section for those learning the language, has been largely rewritten. Exercises, with sample solutions for some of them, have been added.

For those meeting IPL-V for the first time, if you have wondered about such expressions as list-processing, list structures, Newell-Shaw-Simon, push-down, pop-up, and recursion, this is a good source of relevant information. IPL-V, which was developed mainly by the RAND Corporation, is designed for non-arithmetic processing of symbolic information, especially information which can be organized in the form of lists. A list element consists of a data portion and the location of the next list element. Programs for computers having a "plus-one" instruction format, such as IBM 650, are lists of this type. The introduction gives the developments in programming which have led to IPL, explains the basic idea of lists, and gives some areas in which IPL has been used. Theorem proving, game playing, natural language processing, and artificial intelligence are examples thereof.

IPL-V is one of several list-processing languages now in use. The "V" indicates the fifth version in its series. IPL-V is a pseudo-code; that is, it is similar to a machine code, and it is processed by an interpretive program, which can be considered as an IPL-V computer. IPL-V is a lower level language than a symbolic assembly is, in some respects. For example, IPL symbolic names consist of only one alphabetic character followed by up to four numbers, allowing very slight mnemonic possibilities. The power of the language lies in the large number of basic processes or subroutines which are available, processes which are useful in list-processing and in organizing and debugging a program. There are provisions for tracing, dumping, saving for restart, and dividing a program into more than one memory load.

Thirteen IPL-V implementations for 12 different computers are listed as available at the time of publication. Input-output conventions are peculiar to the different machine systems and are not covered in the IPL-V manual. They would have to be looked up before actual running of an IPL program. System questions, such as how to add new processes, are also peculiar to the particular implementation and are not covered in this manual.

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70[Z].—Karl Nickel, *ALGOL Praktikum. Eine Einführung in das Programmieren*, Verlag G. Braun, Karlsruhe, W. Germany, 1964, vii + 220 pp. + separately bound "ALGOL-Wörterbuch" of 52 pp., 24 cm. Price DM 56.

If there was one thing that a numerical analyst acquired twenty years ago, it was a feeling for numerical calculation. For hours every day he observed such things as the propagation of rounding errors, the loss of accuracy due to cancellation, and from time to time the collapse of a badly formulated computational problem. Since the advent of electronic computers large numbers of people have become involved in numerical computation who have become increasingly out of touch with the elements of the subject; this has resulted in a growing inability to appreciate the value of competently produced programs, and is the cause of an extensive misuse of computers. Of textbooks on computing methods there have been more than enough, nor in recent times has there been any lack of literature on computers and languages for their instruction, but there has arisen no book specifically designed to impart to the student some of the awareness of what goes on in numerical processes that he would formerly have gained by experience. As the years have gone by and the dependence of the engineering and physical sciences on digital computation has increased, the need for such a book has become critical. In Professor Nickel's *Algol Practical: An Introduction to Programming* an attempt has been made to meet this need.

It is for the most part a collection of forty carefully contrived examples designed to illustrate the course of a numerical computation: these (particularly those that end in total failure; for, as the author remarks, more is often to be learned from things that go wrong than otherwise) are most instructive. Each problem is analyzed, the necessary numerical method is discussed, a block diagram is constructed, an ALGOL program is given, and finally some numerical results are displayed. There is an extensive introductory section dealing with the correct appreciation of what is being required from the calculation, and the need for formulating a computational problem correctly. This book is not an introduction to ALGOL: this language is used as a pedagogic weapon. It is presumed that the student will pick up what he wants to know as he goes along; in cases of uncertainty a detachable ALGOL dictionary is available. The writing is lively throughout and informed by an emphatic German wit.

If one may presume to offer adverse criticisms they are these: first, there is the usual run of typographical errors—an inverted letter on p. 26, three open brackets on p. 190, where by implication there should be seven, a wrong formula on p. 166, and so on. More seriously, since, whether the author likes it or not, his important book will be read with interest by numerical analysts throughout the world, he might have spared them a thought. It would not appear to be a matter of general concern that a particular problem took a certain length of time to run on the Z-22, though indications to this effect occur throughout the book: furthermore the Z-22 has been programmed in such a way as to print out numbers to nine decimal figures and work with slightly less. Thus, for example, the arguments $0 \cdot 8$, $0 \cdot 5$ and $-0 \cdot 7$ occur in the text as $8.00000001_{10} - 1$, $5.00000002_{10} - 1$ and $-6.99999999_{10} - 1$. It would not seem too much trouble to have generalized the writing procedures so as to have permitted the rounding off of the above and similar numbers. Admittedly the programs would not then have conformed to the ALCOR conventions, but these, at least for the purposes of this book, need not have been regarded as Holy Writ. Lastly, although the programs and numerical results are printed in colour, and quite clearly enormous trouble has been taken in preparation, in offering a book of 220 pages at DM 56 (i.e., *at least* \$14.00 in the United States) the publishers have come very near pricing the book clean out of the student's market.

It is clear that Professor Nickel has initiated a new *genre* in works on numerical analysis: his book should be read by anyone who is, or proposes to be in any way, associated with numerical computations, and studied thoroughly by any person intending to specialize in the subject.

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71[Z].—GORDON RAISBECK, *Information Theory, an Introduction for Scientists and Engineers*, The M. I. T. Press, Cambridge, Mass., 1964, x + 105 pp., 21 cm. Price \$4.00.

Information Theory, an Introduction for Scientists and Engineers discusses the fundamental ideas of Information Theory and their applications to signal transmission and detection. It is addressed to the scientist or engineer with no specialized knowledge of Information Theory, but with some facility in mathematics. The liberal use of mathematics enables the author to include a great deal of substance. It is not, however, overloaded with mathematical detail and reads quite well.

The first chapter treats the problem of assigning a quantitative measure of "information." The second and third chapters are addressed to the noiseless and noisy coding problems, respectively, including Shannon's fundamental theorems on noiseless and noisy coding. The last two chapters discuss detection problems. The book concludes with a descriptive bibliography for the benefit of those who wish to do further reading on this subject.

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