107[A–F, K–N, Q].—Robert C. Weast, Samuel M. Selby & Charles D. Hodgman, Editors, *Handbook of Mathematical Tables*, Second edition, The Chemical Rubber Co., Cleveland, Ohio, 1964, x + 680 p., 24 cm. Price $7.50.

This second edition is 100 pages longer than the previous one, which was reviewed here in [1]. The largest addition is the 52-page section on the numerical solution of ordinary and partial differential equations. This topic is a new one for this handbook, but not for some of its competitors, e.g. [2]. In addition, the section on finite differences is expanded 15 pages, but in another location; this disjointedness being a persistent feature of this handbook. Vector analysis is expanded by 9 pages, and tables concerning octal-decimal conversion (7 pages) and solid angles (17 pages) have been added. The latter is excerpted from a larger table previously reviewed here [3]. A one-page table of Bernoulli and Euler numbers has been dropped, no reason being given, although it does make the increase in pages exactly 100. Happily, the price has remained the same, but unhappily, the quality of the paper has not.

Some of the table-consuming public will appreciate the additions here available, but perhaps owners of previous editions may feel some annoyance with the publisher's policy of constant modification. Future editions are already promised. It would seem that less frequent, and more thorough and careful revisions might be more appropriate. In their preface the publishers deny, by implication, any commercial motivation, and suggest instead that they feel subject to a compulsive sociological imperative. They write:

"Society can no longer afford the time of waiting for the explosive growth of knowledge to make its way gradually into the schools' curricula and industrial change-over. Improvements must be made with instantaneous speed in academic instruction, as well as the appreciation of new principles and techniques for technological development.

"This scientific environment has created the motivating influences responsible for a re-examination of the information contained in the first edition. Numerical analysis with all its ramifications has become a necessary tool for research, irrespective of what discipline in science is involved."

The implication here that the need for these additions developed sometime between 1962 and 1964 could certainly be contested, assuming that it was meant to be taken seriously.

As a procedure for designing a fine collection of tables this policy of constant patching does have serious drawbacks. There are disturbing variations in typography, duplications (e.g., in $1/n!$ on p. 209 and p. 268), disorder in the sequencing of the tables (discussed previously in the review of [1]), and even contradictions (e.g., use of $\underline{|n}$ for $n!$ on p. 463, and admonition against such a use on p. 651). Quantity replaces quality.

D. S.

1. Samuel M. Selby, Robert C. Weast, Robert S. Shankland & Charles D. Hodgman, Editors, *Handbook of Mathematical Tables*, Chemical Rubber Publishing Company, Cleveland, Ohio, 1962. (Reviewed in *Math. Comp.*, v. 17, 1963, pp. 303–304, RMT **34**.)

2. Milton Abramowitz & Irene A. Stegun, Editors, *Handbook of Mathematical Func-*

tions with Formulas, Graphs, and Mathematical Tables, National Bureau of Standards Applied Mathematics Series, No. 55, U. S. Government Printing Office, Washington, D. C., 1964. (Reviewed in *Math. Comp.*, v. 19, 1965, pp. 147–149, RMT **1**.)

3. A. V. H. Masket & W. C. Rodgers, *Tables of Solid Angles: I. Solid Angle Subtended by a Circular Disc; II. Solid Angle Subtended by the Lateral Surface of a Right Circular Cylinder*, Office of Technical Services, Washington, D. C., 1962. (Reviewed in *Math. Comp.*, v. 17, 1963, pp. 207–208, RMT **25**.)

108[B, I, L].—(a) D. S. Mitrinović & D. Ž. Djoković, *Specijalne Funkcije* (*Special Functions*), Gradjevinska Knjiga, Belgrade, 1964, 267 p., 24 cm. (b) D. S. Mitrinović (Editor), *Zbornik Matematičkih Problema* (*Collection of Mathematical Problems*), three volumes, three editions, three publishers (see below), Belgrade, 1957–62, 24 cm.

One sometimes encounters a collection of tables which contains little original or very extensive, yet is worth noting as a collection. The tables contained in the works under review are, broadly speaking, of this character. The texts of the works are printed in the Latin alphabet of the Serbo-Croat language. *Specijalne Funkcije* (hereafter called *S.F.*) is a concise exposition of the field of special functions, while *Zbornik* (hereafter *Zb.*) is a collection of problems (some solved, some to be solved) from a wide field of mathematics; both works are designed for students at universities and similar institutions. All volumes contain numerical tables, mostly grouped together near the end.

As far as tabular matter is concerned, *S.F.* gives a moderately wide coverage of Legendre polynomials $P_n(x)$ and Legendre coefficients $P_n(\cos\theta)$, Bessel functions $J_n(x)$, $N_n(x)$, $I_n(x)$, $K_n(x)$, Kelvin functions ber x, bei x, Laguerre polynomials $L_n(x)$, Hermite polynomials, both $H_n(x)$ based on $\exp(-x^2)$ and $H_n^*(x)$ based on $\exp(-\frac{1}{2}x^2)$, Chebyshev polynomials $T_n(x)$ and Chebyshev functions $U_n(x)$. For some of these, the information given includes all of (i) explicit analytical expressions, (ii) numerical values of functions, (iii) numerical values of zeros, and (iv) graphs. For example, to take a case in which *S.F.* may well be found convenient (because of the comparative paucity of other sources), the following information is tabulated for the Laguerre polynomials $L_n(x)$: explicit algebraic expressions for $n = 0(1)10$ on p. 70, 6D zeros for $n = 1(1)15$ on p. 222, 4D values for $n = 2(1)7$, $x = 0(0.1)10(0.2)20$ on pp. 226–228, and graphs of $\exp(-\frac{1}{2}x)L_n(x)/n!$ on p. 262.

Explicit expressions for $P_n(x)$ and for $P_n(\cos\theta)$ as Fourier series are quoted for $n = 0(1)20$ on pp. 25–26 and pp. 29–30 respectively from the 1936 tables of the Egersdörfers. *S.F.* also contains exact factorials up to 60! on p. 213, and complete and incomplete elliptic integrals of the first and second kinds, also period ratios and log q, on pp. 237–248. On pp. 250–263 is a set of graphs by D. V. Slavić. It is no doubt a sign of the times that young Yugoslav mathematicians have available, for use in science, engineering, technology and so on, as handsome a set of graphs of the more usual higher functions as the reviewer can recall seeing anywhere.

A review of *S.F.* implies mention of *Zb.*, which contains, among other tables, a number in common with *S.F.* Using roman numerals for volumes and suffixes for editions, the reviewer has had available *Zb.* I_1 1957, I_2 1958, I_3 1962, II_1 1958 and III_1 1960; II_2 1960 has not been available. I_1 and I_2 were published by Nolit, and II_1 by Naučna Knjiga; I_3 and III_1 were published by Zavod za izdavanje Udžbenika N.R.S., which now presumably publishes all three volumes. The variations between editions are very great.

Several of the tables in *S.F.* are also given in *Zb.* II and/or III. With unimportant exceptions, the tables in *S.F.*, when not identical, are fuller than those in *Zb.* Among many tables (some small) in *Zb.*, one may mention (excluding any also given in *S.F.*) the following, where numbers in brackets are page numbers: sums of the kth powers of the first n natural numbers, even numbers, and odd numbers, for $n = 1(1)12$, $k = 1(1)12$, in *Zb.* I_1 (216), I_2 (292), preceded by general expressions; some exact Stirling numbers in *Zb.* I_1 (230), I_2 (309); the first 36 Bernoulli numbers as exact fractions in *Zb.* I_2 (348), I_3 (500); error integral and ordinate, in the exp $(-\frac{1}{2}z^2)$ form, in *Zb.* II_1 (329); values, zeros, etc. of Kelvin functions in *Zb.* III_1 (314); exact binomial coefficients up to $n = 60$ in *Zb.* III_1 (319); and exact powers n^p for $n = 2(1)83$, $p = 1(1)10$ in *Zb.* III_1 (325). One may also note, as rather unusual, that the years of birth and death of more than 170 mathematicians are listed in *Zb.* I_2 (xv), I_3 (501).

A. F.

109[F].—Albert H. Beiler, *Recreations in the Theory of Numbers—The Queen of Mathematics Entertains*, Dover Publications, Inc., New York, 1964, xvi + 349 pp., 22 cm. Price $2.00 (paperbound).

This book addresses itself primarily to the amateur, and its tone, as indicated, is one of recreation. It deals in perfect and amicable numbers, Fermat's theorem and its converse, Wilson's theorem, digit properties, repeating decimals, primitive roots, Pythagorean numbers, Pell's equation, primes, etc. The author was clearly fond of his task, since he has lovingly and industriously compiled long bibliographies after each chapter, 103 tables, 33 pages of answers to the problems, and an 11-page index. There is little, or no attempt to give proofs, and when these are sketched, they are almost never rigourous. In at least one case there is outright fallacy: on page 16 it is stated that if $p \mid a^n - 1$, with p prime and $n < p$, then $n \mid p - 1$. Not so, since $31 \mid 2^{20} - 1$. There are also scattered errors in terminology, judgment, or fact: Uhler's "perfect numbers" on p. 18; an assessment of Wilson's theorem on p. 49; and the claim, on p. 292, that Gauss was unassuming, gentle and naïve. But these blemishes do little harm to the author's main purpose.

The author's style is exceedingly rich. Chapter XX begins: "Inseparably woven into the fabric of number theory, nay, the very weft of the cloth, are the ubiquitous primes. Almost every investigation includes them; they are the elementary building blocks of our number edifice. From the humble 2, the only even prime, and 1, the smallest of the odd primes, they rise in an unending succession aloof and irrefrangible." Chapter XV begins: "There is something about a square! Note its perfection and symmetry. All its sides are equal, its angles are neither stupidly obtuse nor dangerously acute. They are just right. The square has many beautiful geometric properties." It is not clear here whether the author merely means to thus convey his enthusiasm, or whether this is intended to add to the book's recreational value.

For an amateur the book is a real grab-bag, but even a professional may derive some information from the many tables, bibliographies, and occasional curiosities and odds-and-ends that he may not have previously encountered.

D. S.

110[F].—JU. V. LINNIK, *The Dispersion Method in Binary Additive Problems*, Translations of Mathematical Monographs, Vol. 4, American Mathematical Society, Providence, Rhode Island, 1963, x + 186 pp., 23 cm. Price $12.30.

In recent years the author has had some spectacular successes with problems in the additive theory of numbers that seem to lie just beyond the reach of the now classical Hardy-Littlewood-Vinogradov method. He achieved these remarkable results by combining the methods of analytic number theory with some elementary tools from probability theory, specifically, the concepts of dispersion and covariance and the Chebyshev inequality. The present book is devoted to a systematic exposition of this work. Since the problems involved are old and difficult ones, the detailed proofs require elaborate computations, and are by no means easy to read.

An example of the results obtained in this book is an asymptotic formula for the number $Q(n)$ of solutions of $x^2 + y^2 + p = n$, x and y integers, p prime. Linnik proves that for large n we have

$$Q(n) = \pi A h(n) \frac{n}{\log n} + O\left(\frac{n}{(\log n)^{1.028}}\right),$$

where

$$A = \prod_p \left\{1 + \frac{\chi(p)}{p(p-1)}\right\}, \qquad h(n) = \prod_{p \mid n} \left\{\frac{(p-1)(p-\chi(p))}{p^2 - p + \chi(p)}\right\},$$

the products being taken over the odd primes and $\chi(p)$ being an abbreviation for $(-1)^{(p-1)/2}$. This asymptotic formula, along with many similar assertions, was conjectured by Hardy and Littlewood in their paper, "Some problems of partitio numerorum III: On the expression of a number as a sum of primes," *Acta Math.*, v. 44, 1923, pp. 1–70. A systematic tabulation of the present status of the many interesting conjectures made in this famous paper may be found in a recent note by A. Schinzel, "A remark on a paper of Bateman and Horn," *Math. Comp.*, v. 17, 1963, pp. 445–447.

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111[F].—ALBERT LEON WHITEMAN, Editor, *Theory of Numbers*, Proceedings of Symposia in Pure Mathematics, Volume VIII, American Mathematical Society, Providence, R. I., 1965, vii + 214 pp., 26 cm. Price $7.10.

There are given here 22 of the lectures presented at an AMS number theory symposium at the California Institute of Technology on November 21–22, 1963. The lectures will be of much interest to many readers of this journal, especially since a number of them touch upon, or refer directly, to work that has appeared here in recent years. See, for example, the papers of Bateman, Carlitz, Mills, and Cohn. The paper of Bateman and Horn is discussed, at length, in the following review.

An unforgettable episode at the symposium was the disruption and termination of its third session upon receipt of the news that President Kennedy had been killed.

The papers here are by Atle Selberg, R. R. Laxton and D. J. Lewis, W. J. LeVeque, Marshall Hall, Jr., Albert Leon Whiteman, Basil Gordon and E. G. Straus, Kenkichi Iwasawa, Morris Newman, E. C. Dade and O. Taussky, E. T. Parker, Gordon Pall, B. J. Birch, N. C. Ankeny, Paul T. Bateman and Roger A. Horn, Tom M. Apostol, S. Chowla and H. Walum, L. Carlitz, W. H. Mills, Leo Moser, P. Erdös, Harvey Cohn, and J. Lehner.

D. S.

112[F].—Paul T. Bateman & Roger A. Horn, "Primes represented by irreducible polynomials in one variable," *Theory of Numbers* (see previous review), pp. 119–132, in particular Tables II–V.

This paper is concerned with further development of a topic previously examined in this journal in references [1], [2]. The question is to estimate the number of integers n between 1 and N for which $f_i(n)$ $(i = 1, 2, \cdots, k)$ are simultaneously primes, where the f_i are distinct, irreducible polynomials. Under broad conditions, Bateman has conjectured that this number $P(N)$ satisfies

$$P(N) \sim c \frac{N}{(\log N)^k} \tag{1}$$

where the constant c is given by an explicit slowly convergent infinite product.

In a series of papers, [3]–[8], the reviewer had developed techniques of accurately computing these constants c for, say, $k = 1$ and $f_1 = n^4 + 1$ or $f_1 = n^2 + a$, and for $k = 2$ and $f_{1,2} = (n \pm 1)^2 + 1$. Bateman points out here that in all these cases the f_i are *abelian* polynomials, and he gives a general approach to the problem for any abelian polynomials. This general attack, like the specific ones mentioned, uses certain Dirichlet series, but it does not attain the degree of convergence which had been obtained in those special cases.

The authors also examine here (among others) eight *non-abelian* cases: $x^3 \mp 2$, $2x^3 \mp 1$, $x^3 \mp 3$, $3x^3 \mp 1$, and they give empirical counts of such primes for $x <$ 14000, 6000, 14000, and 8000, respectively. But for these non-abelian cases *no accurate way of computing the constants is known.* For example, the number of primes of either form $n^3 \mp 2$ is conjectured to satisfy

$$P(N) \sim \frac{1}{3} A \int_2^N \frac{dn}{\log n} \tag{2}$$

where

$$A = \prod_p \frac{p - \alpha(p)}{p - 1}, \tag{3}$$

the product being taken over all primes $p = 6m + 1$ with $\alpha(p) = 3$, or 0, according as p is, or is not, expressible as $a^2 + 27b^2$. The sequence of partial products here not only converges very slowly, but has an annoying, irregular "drifting" character that frustrates any standard acceleration technique. In the limit, there are twice as many primes p with $\alpha(p) = 0$ as with $\alpha(p) = 3$, (that is why the product converges), but the two types of p occur in a "random" manner, and this causes the sequence to drift up and down in a way that defies the instinct of any numerical analyst. Presumably, a Césaro sum would help some, but that is not very satisfactory. The authors tentatively suggest $A = 1.29$, which they obtain from (3) with $p < 1000$.

Since there are relatively few convergent sequences that arise naturally for which someone has not found an effective acceleration device, the evaluation of A must be considered a real and worthwhile challenge to analysts and number theorists. It is natural to hope that an analogue of the Gauss sums could be found for the appropriate algebraic integers and Dirichlet series present here, and with this one could proceed as before. Indeed, in the lecture by Birch mentioned in the previous review (on p. 107), there is a similar problem that has been solved by Birch and Davenport. But this is a difficult and incompletely presented paper, and the reviewer is unable to say whether similar techniques will work here. A *quite different* approach is to find a *theorem*, not a *conjecture*, in which the constant A enters. Then one could logically estimate A by empirical studies. The reviewer has, in fact, found [9] the following theorem. The number of positive odd numbers $<x$ of the form $4u^2 + 2uv + 7v^2$ is given by

$$\frac{\frac{7}{12} b_3 x}{\sqrt{\log x}} \left[1 - \frac{d\sqrt[3]{\mathcal{L}(1)}}{\sqrt[3]{\log x}} + O\left(\frac{1}{\log x}\right)\right] \tag{4}$$

where b_3 is a known constant, [9] or [8, p. 136], where

$$d = \frac{18}{7} \frac{\sqrt[6]{\pi}}{\Gamma(\frac{1}{6})} (b_3)^{2/3}, \tag{5}$$

and where

$$\mathcal{L}(s) = \prod_p (1 - p^{-s})^{1-\alpha(p)}, \tag{6}$$

using the notation of (3). Thus $\mathcal{L}(1)$ could be estimated empirically from (4). Now

$$A = \frac{1}{\mathcal{L}(1)} \prod_p \left(1 - \frac{3}{p}\right)\left(1 - \frac{1}{p}\right)^{-3},$$

where this product is taken over all primes $p = a^2 + 27b^2$. The new product converges more rapidly, and monotonically. The error may therefore be bounded.

The counts of primes $x^3 \mp 2$, etc., were obtained here by a straight-forward trial-and-error method on a CDC 1604. No previously published tables of such counts are known to the reviewer, although in [10] W. A. Golubew indicated that he has made counts for $x^3 + 2$. It should be added that not only are the constants c much more difficult to compute in these cases, but that the counts of the primes themselves are also much more cumbersome. For primes of the form $x^2 + a$, for example, there is available the very efficient sieve method based upon the p-adic square roots of $-a$ wherein there is no trial-and-error whatsoever. Such an algorithm is not possible in these cubic cases, cf. [5].

D. S.

1. Paul T. Bateman & Roger A. Horn, "A heuristic asymptotic formula concerning the distribution of prime numbers," *Math. Comp.*, v. 16, 1962, pp. 363–367.

2. A. Schinzel, "A remark on a paper of Bateman and Horn," *Math. Comp.*, v. 17, 1963, pp. 445–447.

3. Daniel Shanks, "A note on Gaussian twin primes," *Math. Comp.*, v. 14, 1960, pp. 201–203.

4. DANIEL SHANKS, "On the conjecture of Hardy and Littlewood concerning the number of primes of the form $n^2 + a$," *Math. Comp.*, v. 14, 1960, pp. 321–332.
5. DANIEL SHANKS, "On numbers of the form $n^4 + 1$," *Math. Comp.*, v. 15, 1961, pp. 186–189; Corrigendum, *ibid.*, v. 16, 1962, p. 513.
6. DANIEL SHANKS, "Supplementary data and remarks concerning a Hardy-Littlewood conjecture," *Math. Comp.*, v. 17, 1963, pp. 188–193.
7. DANIEL SHANKS, "Polylogarithms, Dirichlet series, and certain constants," *Math. Comp.*, v. 18, 1964, pp. 322–324.
8. DANIEL SHANKS & JOHN W. WRENCH, JR., "The calculation of certain Dirichlet series," *Math. Comp.*, v. 17, 1963, pp. 136–154; Corrigenda, *ibid.*, p. 488.
9. DANIEL SHANKS & LARRY P. SCHMID, "Variations on a theorem of Landau," (to appear).
10. W. A. GOLUBEW, "Primzahlen der Form $x^2 + 3$," *Österreich. Akad. Wiss. Math.-Nat. Kl.*, 1958, Nr. 11, pp. 168–172.

113[F].—SIDNEY KRAVITZ & JOSEPH S. MADACHY, *Divisors of Mersenne Numbers*, $20,000 < p < 100,000$, ms. of 2 typewritten pages + 48 computer sheets, deposited in the UMT File.

The authors computed all prime factors $q < 2^{25}$ of all Mersenne numbers $M_p = 2^p - 1$ for all primes p such that $20,000 < p < 100,000$. The computation took about one-half an hour on an IBM 7090. There are 2864 such factors q. These are listed on 48 sheets of computer printout in the abbreviated form: k vs. p, where $q = 2pk + 1$. A reader interested in statistical theories of such factors may wish to examine the following summary that the reviewer has tallied from these lists. Out of the 7330 primes p in this range, M_p has 0, 1, 2, 3, or 4 prime divisors $q < 2^{25}$, according to the following table

0	1	2	3	4
4920	2006	356	46	2

The two values of p with four such factors are $p = 26,681$ and $68,279$.

The authors do not indicate whether or not any of these factors q is a multiple factor, that is, whether $q^2 \mid M_p$. Heuristically, the probability of a multiple factor here is quite low. Such a q has not been previously found [1], but, on the other hand, no convincing heuristic argument has ever been offered for the conjecture [1] that they do not exist. The alleged proof given in [2] is certainly fallacious, and for the quite closely analogous ternary numbers $\frac{1}{2}(3^p - 1)$ one finds a counterexample almost at once.

For earlier tables of factors of M_p see [1], [3] and the references cited there.

D. S.

1. JOHN BRILLHART, "On the factors of certain Mersenne numbers. II," *Math. Comp.*, v. 18, 1964, pp. 87–92.
2. E. KARST, "Faktorenzerlegung Mersennescher Zahlen mittels programmgesteuerter Rechengeräte," *Numer. Math.*, v. 3, 1961, pp. 79–86, esp. p. 80.
3. DONALD B. GILLIES, "Three new Mersenne primes and a statistical theory," *Math. Comp.*, v. 18, 1964, pp. 93–97.

114[F].—H. C. WILLIAMS, R. A. GERMAN & C. R. ZARNKE, *Solution of the Cattle Problem of Archimedes*, copy of the number T, 42 computer sheets, deposited in the UMT File.

There is deposited here the number T, the total number of cattle in Archimedes' problem, the computation of which is discussed elsewhere in this issue. This enormous integer comprises 206,545 decimal digits and is nicely printed on 42 computer sheets. Lessing's version of Archimedes' "epigram", in which the problem is given, may also be found in the convenient reference [1], while for interesting historical commentary the reader should examine [2].

Although the authors' title proudly suggests that the problem is solved, we must add, in candor, that they have merely given the number T. The breakdown of this into the numbers of white bulls, black bulls, spotted cows, yellow cows, etc. is not given, although by the authors' own statement this constitutes part of the problem. Perhaps, though, they conceive of this as an exercise which is left to the reader. Actually, it would appear that there are 1.397 bulls for each cow, a ratio that could lead to serious difficulties, particularly under such crowded conditions.

The calculation was done on IBM computers in the English part of Canada. Conceivably, had the computation been done in Quebec instead, the investigators may have been more inclined to use the machines of L'Compagnie Bull, a choice which, in one way, might seem more appropriate.

D. S.

1. James Newman, *The World of Mathematics*, Vol. 1, Simon and Schuster, New York, 1956, pp. 197–198.
2. T. L. Heath, *The Works of Archimedes*, Dover (reprint), New York, undated, pp. xxxiv–xxxv, pp. 319–326.

115[G].—P. S. Alexandroff, *Introduction à la Théorie des Groupes*, Dunod, Paris, 1965, xi + 128 pp., 22 cm. Price 13 francs (paperbound).

This is an introductory book for the use of first-year college or last-year high school students. It is a clear and straightforward account of the basic facts of group theory, illustrated mainly by a few permutation groups and the groups of the regular polyhedra. For the investigation of these, the imagination is supported by a fair number of good drawings. Lagrange's Theorem (but not Cauchy's Theorem) is proved, and the book proper ends with the first theorem of homomorphism. A first appendix gives an outline of the elementary theory of sets, and a second one contains a proof of the simplicity of the alternating groups on more than four symbols, which follows the one given in Van der Waerden's *Algebra*.

There is a lucid introduction which presents group theory as part of the science (or art) of calculating.

On the whole, applications or unusual examples are absent. Apart from the use of the additive (instead of the multiplicative) notation for the composition of group elements, the text and the material are pretty much standard. The corollary to the homomorphism theorem (p. 109) is somewhat misleading. That a homomorphism is an isomorphism if and only if the kernel is trivial follows from the very definition of isomorphism and without the homomorphism theorem. As it stands, the corollary may give the reader the erroneous idea that the homomorphic image of a group G can be isomorphic to G if and only if the kernel is trivial.

As a well-written truly elementary introduction to group theory, the book may be expected to be very welcome to many people.

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116[G].—H. S. M. Coxeter & W. O. J. Moser, *Generators and Relations for Discrete Groups*, Second edition, Ergebnisse der Mathematik und ihrer Grenzgebiete, Band 14, Springer-Verlag, New York, 1965, ix + 161 pp., 24 cm. Price $8.00.

This is a revised version of the first edition, which appeared in 1957. For a brief review of the latter see that by G. Higman in *Math. Reviews*, v. 19, 1958, p. 527. The changes here are relatively small, but there is inclusion of further results on binary polyhedral groups, the groups $GL(2, p)$ and $PGL(2, p)$, and the Mathieu groups M_{11} and M_{12}. There is also mention of recent work on the Burnside problem and of some studies on electronic computers. It would appear that, so far, the use of computers has not changed the subject very significantly.

The 12 tables in the back of the book on non-Abelian groups, point groups, space groups, crystallographic groups, symmetric groups, reflexible maps, finite maps, regular maps, etc. are carried over unchanged from the first edition except that some of them, like the bibliography that follows, have been reset in a more spacious format.

The book remains, as before, the definitive work on the subject, and with its further improved and corrected text, and the new hard cover in which it is bedecked, it is one that the student of group theory will want to possess.

The reviewer agrees with the opinion in Higman's review that the study and knowledge of many specific groups forms a valuable basis for insight and inspiration concerning the general theory. The modern style is, of course, usually more abstract. It would be of value if some student of the psychology of mathematical invention would undertake a serious, quantitative study of the relative effectiveness of these two approaches.

D. S.

117[G, X].—L. Fox, *An Introduction to Numerical Linear Algebra*, Oxford University Press, New York, 1965, xi + 327 pp., 24 cm. Price $8.50.

This American edition differs from the earlier British edition [cf. the review by Ortega, *Math. Comp.*, v. 19, 1965, pp. 337–338] only in that 32 pages of exercises have been added. Some of these call attention to errors and ambiguities in the text. Mostly, however, they include a number of numerical examples, with small matrices usually of integer elements, and exercises providing commentaries on or extensions of the theory. All are fairly straightforward, and considerably enhance the value of the book, whether for self-study, or as a classroom text.

The Oxford University Press, Oxford, England has available copies of the exercises that can be obtained on request by those owning the British edition.

A. S. H.

118[H].—Herbert E. Salzer, Charles H. Richards & Isabelle Arsham, *Table for the Solution of Cubic Equations*, McGraw-Hill Book Co., Inc., New York, 1958, xv + 161 pp., 21 cm. Price $4.50. Paperback (1963), $2.25.

The cubic equation $ax^3 + bx^2 + cx + d = 0$, by the substitution $x = y - b/3a$, is transformed to (1) $a'y^3 + b'y + c' = 0$, where $a' = a$, $b' = c - b^2/3a$ and $c' = d - bc/3a + 2b^3/27a^2$. The three roots of (1) are given by $(-c'/b')f_i(\theta)$, $i = 1, 2, 3$, where $\theta = a'c'^2/b'^3$ and

$$f_1(\theta) = \frac{[1 + \sqrt{1 + (4/27\theta)}]^{1/3} + [1 - \sqrt{1 + (4/27\theta)}]^{1/3}}{(2\theta)^{1/3}},$$

$$f_2(\theta), f_3(\theta) = -\frac{f_1(\theta)}{2} \pm \sqrt{\frac{f_1(\theta)^2}{4} - \frac{1}{\theta f_1(\theta)}}.$$

For $-4/27 < \theta < 0$, all roots are real; for $\theta > 0$ or $\theta < -4/27$, there is one real root and a pair of complex roots. By suitably different choices of the phase angle for the one-third powers, the function $f_1(\theta)$ is always made to be real (even though discontinuous at $\theta = -4/27$).

The present table gives $f_i(\theta)$, $i = 1, 2, 3$, for $1/\theta = -0.001(-0.001) - 1$, $\theta = -1(0.001)1$, $1/\theta = 1(-0.001)0.001$, to 7D everywhere except for $f_2(\theta)$ and $f_3(\theta)$, for $\theta = -0.148(0.001) - 0.001$, which are given to 7S. The accuracy is to within about a unit in the last place. First differences Δ are tabulated everywhere, and second differences Δ^2 wherever they are $\geqq 4$ units in the last place.

The introductory text contains the following material: comparison of (1) with the form (2) $y^3 + py + q = 0$, obtained by dividing through by a' (see below for an amendment of that section); discussion of related tables for solving cubics, with particular attention to those of H. A. Nogrady, B. M. Shumiagskii, and A. Zavrotsky; method of interpolation; illustrations of the use of the table, consisting of four examples worked out in complete detail; method of computation, which was on the Univac Scientific Computer (ERA 1103) for $\theta > 0$ and $\theta < -4/27$, and which was by desk calculator for $-4/27 < \theta < 0$.

For an efficient way to use this table to help to solve any quartic with real coefficients, by desk calculation, see H. E. Salzer, "A note on the solution of quartic equations," *Math. Comp.*, v. 14, 1960, pp. 279–281.

In 1958 the author-reviewer notified the publisher about some half-dozen minor printing defects, which are still present in the paperback edition (e.g., the absence of a page numbered 1). But just recently the author-reviewer noted the following misleading material in the paragraph on pp. vi–vii: It is stated there that when in (1) the a' is much smaller than b' or c' and given to much fewer significant figures, since p and q in (2) will be given to around the same relative accuracy as a', say e, the corresponding $\theta = q^2/p^3$ might have a relative error as large as $5e$ instead of the relative error of approximately e in $\theta = a'c'^2/b'^3$ corresponding to (1). It is also stated there that the factor $-c'/b'$ in $(-c'/b')f_i(\theta)$ leads to greater accuracy than the factor $-q/p$ corresponding to (2) because q and p have much greater relative errors than c' and b'. Now both these statements are true only if we retain about the same number of figures in p and q as occurs in a'. However, if in the division $p = b'/a'$, $q = c'/a'$, we retain the same, or one more, number of digits in the quotients p and q as are in b' and c' respectively, even though the p and q still have about the same large relative error e from the a', the computation of $\theta = q^2/p^3$ leads to the same accuracy as $\theta = a'c'^2/b'^3$, and the computation $-q/p$ will give the same accuracy as $-c'/b'$. Thus, if one takes the precaution of retaining sufficient numbers of figures in the divisions, the implication in that paragraph that (1) yields greater accuracy than (2) no longer holds.

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119[H].—T. A. Ciriani & A. L. Frisiani, *Tabulation of Solutions of the Cubic Equation* $z^3 + Az - A = 0$, IBM Italia and Instituto di Elettrotecnica Università di Genova, Genova, Italy, undated ms. of 10 typewritten pp. + a block diagram + 32 pp. of tables, deposited in UMT File.

The equation (1) $x^3 + ax^2 + bx + c = 0$, under the transformation $y = x + (a/3)$, becomes (2) $y^3 + py + q = 0$, where $p = (3b - a^2)/3$ and $q = (2a^3 - 9ab + 27c)/27$. Setting $z = -py/q$, (2) becomes (3) $z^3 + Az - A = 0$, where $A = p^3/q^2$. If z_1 is a root of (3), the other two roots are given by

$$z_{2,3} = -\frac{z_1}{2} \pm \sqrt{\left(-A - \frac{3z_1^2}{4}\right)}. \tag{4}$$

For $A \leqq -6.75$, equation (3) has three real roots; for $A > -6.75$, it has one real and two complex conjugate roots.

The tables give all three roots for $\pm A = 0.0001(0.0001)0.01(0.001)0.1(0.005)0.5(0.01)1(0.05)10(0.1)20(1)100(5)500$, to 8S. No aids to interpolation are tabulated. In the text it is stated that extensive checks were performed (not described) and that the roots were found accurate to 8S except in the neighborhood of $A = -6.75$ (accuracy there not specified).

The computations were performed on an IBM 1401, using 12S. First a real root z_1 was computed by a method of successive approximations which about halved the error at each step. For $A < -6.75$, the other two real roots were obtained from (4). For $A > -6.75$, a first approximation to the complex pair, $C_0 \pm jD_0$, was obtained from (4) and successively improved, using J. A. Ward's downhill method [1], which appears to about halve the error at each stage.

For A outside the range of the table, namely for $A < -500$, $|A| < 0.0001$ and $A > 500$, first approximations to z_i, $i = 1, 2, 3$, are given in terms of A, with bounds for the relative error that range from $1.6 \cdot 10^{-2}$ down to $7 \cdot 10^{-4}$, together with a function γ, expressed in terms of A, such that a better approximation may be obtained by multiplying the first approximation by $1 + \gamma$.

On p. 8 the statement is made that the only previous tabulation of this form known to the authors extends over a smaller range and gives only the value of a real root. Apparently the authors are unaware of the fact that in H. E. Salzer, C. H. Richards & I. Arsham, *Table for the Solution of Cubic Equations*, McGraw-Hill, New York, 1958, there are similar tables for obtaining all three roots, as functions of an argument $\theta = 1/A$ corresponding to the complete range of A.

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1. J. A. Ward, "The down-hill method of solving $f(z) = 0$," *J. Assoc. Comput. Mach.*, v. 4, 1957, pp. 148–150.

120[I].—D. S. Mitrinović & R. S. Mitrinović, *Tableaux d'une classe de nombres reliés aux nombres de Stirling*, (a) IV: Belgrade, Mat. Inst., *Posebna izdanja*, Knjiga 4 (*Editions spéciales*, 4), 1964, 115 pp., 24 cm., (b) V: *Publ. Fac. Elect. Univ. Belgrade* (*Série: Math. et Phys.*), No. 132, 1965, 22 pp., 24 cm.

The first three installments of these tables were reviewed in *Math. Comp.*, v. 17, 1963, p. 311 and v. 19, 1965, pp. 151–152 (in the latter review, *for* ${}^{p}P_n{}^{+}$, *read* ${}^{p}P_n{}^{r}$ in two places, *for* x^{+}, *read* x^{r}, and *for* Instituto, *read* Istituto).

The fourth and fifth parts continue the tabulation of the integers ${}^{\nu}S_n^{\,k}$, where

$$t(t-1)\cdots(t-\nu+1)(t-\nu-1)\cdots(t-n+1)=\sum_{k=1}^{n-1}{}^{\nu}S_n^{\,k}t^{n-k}.$$

In the fifth part, at the end of equation (2), for ${}^{\nu}S_n^{\,n-1}$ read ${}^{\nu}S_n^{\,n-1}t$. The values of ${}^{\nu}S_n^{\,k}$, already listed in the third part for $n = 3(1)26$, are now given in the fourth part for $n = 27(1)35$ and in the fifth for $n = 36$. As before, the other arguments are $\nu = 1(1)n - 2$ and $k = 1(1)n - 1$, and all tabulated values are exact; for $n = 36$ they involve up to a maximum of 41 digits. The tables were calculated by Ružica S. Mitrinović under the direction of D. S. Mitrinović. Further extensions of the tables are in progress.

A. F.

121[K].—B. M. Bennett & C. Horst, *Tables for Testing Significance in a* 2×2 *Contingency Table: Extension to Cases* $A = 41(1)50$, University of Washington, Seattle, Washington. Ms. of 55 computer sheets + 3 pages of typewritten text deposited in UMT File.

These manuscript tables constitute an extension of Table 2 in the published tables of Finney, Latscha, Bennett, and Hsu [1]. According to the explanatory text, the underlying calculations were performed on an IBM 7094 system, using a program originally developed by Hsu in 1960. For a discussion of the accuracy of this extension as well as the various statistical applications, the user is directed by the authors to the Introduction to the published tables cited.

J. W. W.

1. D. J. Finney, R. Latscha, B. M. Bennett & P. Hsu, *Tables for Testing Significance in a* 2×2 *Contingency Table*, Cambridge University Press, New York, 1963.

122[L].—H. T. Dougherty & M. E. Johnson, *A Tabulation of Airy Functions*, National Bureau of Standards Technical Note 228, U. S. Government Printing Office, Washington, D. C., 1964, 20 pp., 27 cm. Price $0.20.

These tables give numerical values for Wait's formulation [1] of the Airy function and its first derivative.

Although Miller's tables [2] are mentioned, the authors seem to have missed the very close connection between Wait's functions and those tabulated by Miller. In fact, the functions now tabulated are

$$\begin{aligned} u(t) &= \sqrt{\pi}\,Bi(t) & u'(t) &= \sqrt{\pi}\,Bi'(t)\\ v(t) &= \sqrt{\pi}\,Ai(t) & v'(t) &= \sqrt{\pi}\,Ai'(t)\\ |W(t)| &= \sqrt{\pi}\,F(t) & |W'(t)| &= \sqrt{\pi}\,G(t)\\ \theta(t) &= \chi(t) & \theta'(t) &= \psi(t) \end{aligned}$$

These are all given to 8S (or 8D at most), with $\theta(t)$ and $\theta'(t)$ in degrees to 5D, for $t = -6(0.1)6$.

Thus, the only range for which [2] is not at least as extensive is for $t = -6(0.1) -2.5$, where logarithms of $Ai(t)$ and $Bi(t)$ and logarithmic derivatives are given instead.

It is difficult to understand why these tables were prepared and issued, and why they were computed as they were.

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1. K. P. Spies & J. R. Wait, *Mode Calculations for VLF Propagation in the Earth-Ionosphere Waveguide*, NBS Technical Note No. 114, U. S. Government Printing Office, Washington, D. C., 1961.
2. J. C. P. Miller, *The Airy Integral, giving Tables of Solutions of the Differential Equation* $y'' = xy$, British Association Mathematical Tables, Pt.-Vol. B, Cambridge University Press, Cambridge, 1946.

123[L].—M. I. Z͡hurina & L. N. Karmazina, *Tables of the Legendre Functions, Part* 2, Pergamon Press Mathematical Tables Series, Volume 38, The Macmillan Company, New York, 1965, xiii + 409 pp., 26 cm. Price $16.75.

This volume is an English translation by Prasenjit Basu of the Russian book entitled *Tabli͡tsy funk͡tsii Lez͡handra* $P_{-1/2+i\tau}(x)$, Tom II published by Akad. Nauk SSSR, Moscow in 1962, and reviewed in this journal (v. 18, 1964, pp. 521–522, RMT 79).

The Russian edition of Part 1, which was reviewed herein (v. 16, 1962, pp. 253–254, RMT 22), has also been published in an English translation by Pergamon Press as Volume 22 of their Mathematical Tables Series.

J. W. W.

124[L].—M. Atoji & F. L. Clark, *Tables of the Generalized Riemann Zeta Functions*, ms. of 120 computer sheets deposited in UMT File.

These manuscript tables consist of 7D approximations to $\zeta_N(s, a)$ for $s = 1, 2$, $a = 0.01(0.01)1$, $N = 1(1)200$, and thus form an elaboration of the 4D published tables by the same authors, described in the following review.

J. W. W.

125[L, S].—M. Atoji & F. L. Clark, *The Generalized Riemann Zeta Functions and their Applications in the Calculations of Neutron Cross Sections*, Report ANL-6970, Argonne National Laboratory, Argonne, Illinois, December 1964, 55 pp., 28 cm. Available from the Clearinghouse for Federal Scientific and Technical Information, National Bureau of Standards, U. S. Department of Commerce, Springfield, Virginia. Price $3.00.

The generalized incomplete Riemann zeta function is defined by the equation

$$\zeta_N(s, a) = \sum_{n=0}^{N} (a + n)^{-s}$$

for $s > 1$, where n and N are nonnegative integers.

This report contains two tables. Table 1 gives 4D values of $\zeta_N{}^1(1, a) = \zeta_N(1, a) - a^{-1}$ and $\zeta_N{}^1(2, a) = \zeta_N(2, a) - a^{-2}$ for $a = 0.01(0.01)0.5(0.02)1$, $N = 1(1)100$ and $N = 1(1)50$, respectively. Table 2 gives 4D values of

$$\zeta(2, a) = \sum_{n=0}^{\infty} (a + n)^{-2}$$

for $a = 0.01(0.0005)0.5(0.001)1$. The FORTRAN programs used in performing the underlying calculations on a CDC 3600 are given as prefaces to the tables.

The authors include a preliminary section describing the formulas used in the calculations. The body of the report concludes with a discussion of applications of the tables, particularly in the evaluation of the unmeasured resonance-level contribution in calculations of neutron cross sections and amplitudes.

The appended list of 16 references should be augmented by a citation of the pertinent paper of E. R. Hansen and M. L. Patrick [1].

J. W. W.

1. E. R. Hansen & M. L. Patrick, "Some relations and values for the generalized Riemann zeta function," *Math. Comp.*, v. 16, 1962, pp. 265–274.

126[L, M].—F. M. Arscott, *Periodic Differential Equations*, The Macmillan Company, New York, 1964, x + 283 pp., 22 cm. Price $9.50.

This volume deals with the group of special functions, namely, Mathieu functions, Lamé functions, spheroidal and ellipsoidal wave functions, which have the common property that they satisfy a second-order linear differential equation with periodic coefficients. The functions are of considerable importance in applied problems. The book is intended for both the pure mathematician who is interested in the theory of these functions and for the applied worker who desires to use them. The volume is suitable as a text on the graduate level, and each chapter gives examples, along with appropriate references.

The book assumes added stature because it is well written and because there are few books devoted entirely to the subject. The most recent books which deal to some extent with these topics are those by J. Meixner and F. W. Schäfke (*Mathieusche Funktionen und Sphäroidfunktionen*, Springer, 1954), by A. Erdélyi, W. Magnus, F. Oberhettinger, and F. G. Tricomi (*Higher Transcendental Functions*, Vol. III, Chapters XV and XVI, McGraw-Hill, New York, 1955), and by C. Flammer (*Spheroidal Wave Functions*, Stanford Univ. Press, Stanford, Calif., 1957).

In the last decade, new material has appeared in journals, and the present volume serves to codify much of this information.

Chapter 1 shows how the differential equations satisfied by the functions noted arise from the separation of the wave equation in various coordinate systems. Let us write Mathieu's equation as $w'' + (a - 2q \cos 2z)w = 0$. Chapter 2 studies properties of the solution of this equation which can be deduced from the differential equation itself without recourse to actual construction of the solutions. Chapters 3–5 deal with solutions of Mathieu's equation and their properties, when q is given and a is selected, so that the solution is periodic. Analysis of the solutions when both q and a are given is the subject of Chapter 6. Hill's generalization of Mathieu's equation is taken up in Chapter 7. Chapters 8, 9, and 10 are concerned with the spheroidal wave equation, Lamé's equation, and the ellipsoidal wave equation, respectively.

There are three appendices giving some properties of Bessel functions, Legendre functions, etc., which are needed for the development in the volume proper. Also included is a section summarizing results obtained or published while the book was in press.

Y. L. L.

127[L, M].—V. M. BELYAKOV, R. I. KRAVTSOVA & M. G. RAPPOPORT, *Tables of Elliptical Integrals*, Part 1, translated by PRASENJIT BASU, Pergamon Press, Ltd., Oxford, England, distributed by the Macmillan Company, New York, 1965, xiii + 647 pp., 26 cm. Price $20.00.

This is the English edition of the Russian table previously reviewed in *Math. Comp.*, v. 18, 1964, RMT 93, p. 676–677. For technical details see that review. In preparation for the photographic reproduction the known typographical errors in the Russian edition were corrected, and the frequently imperfect ruling there was mended. The present binding is stronger, but is not very attractive.

The translator's rendition of the title is a little curious; normally it would be translated as *Tables of Elliptic Integrals, Volume 1.* Even more gauche, but not without a certain rhythmical quality, is the designation for $K(k^2)$ and $E(k^2)$ in Table VI as *Total Elliptical Integrals.* On the other hand, the Russian price of 5 rubles, 14 kopecks is translated, with an admirable, no-nonsense attitude, as $20.00.

For a description of another recent table of these functions, see the review of the table by Fettis and Caslin, *Math. Comp.*, v. 19, 1965, RMT 81, p. 509.

D. S.

128[L, M].—K. A. KARPOV, *Tables of the Function* $w(z) = e^{-z^2} \int_0^z e^{x^2}\,dx$ *in the Complex Domain*, The Macmillan Company, New York, 1965, xxi + 519 pp. + 1 insert, 27 cm. Price $19.75.

This book, which is volume 27 of the Pergamon Press Mathematical Tables Series, is an English translation of the Russian *Tablit͡sy funkt͡siĭ* $w(z) = e^{-z^2} \int_0^z e^{x^2}\,dx$ *v kompleksnoi oblasti*, published in 1954 by the Academy of Sciences, U.S.S.R. The Russian edition has been previously reviewed in this journal (MTAC, v. 12, 1958, pp. 304–305).

The translation by D. E. Brown of the introduction is excellent, and the typography is uniformly good.

J. W. W.

129[L, M].—N. V. SMIRNOV, Editor, *Tables of the Normal Probability Integral, the Normal Density, and its Normalized Derivatives*, The Macmillan Company, New York, 1965, xvi + 125 pp., 28 cm. Price $7.50.

This set of tables, constituting Volume 32 of the Mathematical Tables Series of Pergamon Press, is a translation by D. E. Brown of *Tablit͡sy normal'nogo integrala veroyatnostei, normal'noi plotnosti i yeye normirovannyk͡h proizvodnyk͡h*, published in 1960 by the Academy of Sciences of the U.S.S.R.

Table I consists of 7D approximations to the values of the normal probability integral

$$\Phi_0(x) = (2\pi)^{-1/2} \int_0^x e^{-(1/2)t^2}\,dt$$

and its derivative, the normal probability density function, for $x = 0(0.001)2.5(0.002)3.4(0.005)4(0.01)4.5$, together with first differences.

Table II gives the same functions to 10D, without differences, for $x = 4.5(0.01)6$.

Table III consists of 5D values of $-\log [\frac{1}{2} - \Phi_0(x)]$, for $x = 5(1)50(10)100(50)$-500.

Table IV, comprising nearly two-thirds of the book, gives 7D values, without differences, of the tetrachoric functions

$$\tau_s(x) = \frac{(-1)^{s-1}}{\sqrt{s!}} \frac{d^{s-1}}{dx^{s-1}} \left(\frac{1}{\sqrt{2\pi}} e^{-(1/2)x^2} \right) = \frac{H_{s-1}(x)}{\sqrt{s!}} \frac{e^{-(1/2)x^2}}{\sqrt{2\pi}}$$

for $s = 2(1)21$, $x = 0(0.002)4$. The entries in this table were calculated from the recurrence relation

$$\tau_s(x) = xp_s\tau_{s-1}(x) - q_s\tau_{s-2}(x)$$

where $\tau_0(x) = \frac{1}{2} - \Phi_0(x)$ and $\tau_1(x) = \Phi_0'(x)$. The corresponding values of the coefficients p_s and q_s are given to 10D in Table V. The recurrence formula for the Hermite polynomials, $H_m(x)$, enables one to deduce that $p_s = 1/\sqrt{s}$ and $q_s = (s-2)/\sqrt{(s(s-1))}$. The reviewer has thereby discovered three minor errors in this table; namely, the terminal digits in the tabulated values of p_{20}, q_5, and q_{19} should each be decreased by a unit.

Table VI gives, in floating-point form, 10S values of the normalizing factor $\lambda_s = \sqrt{(s!)}$, for $s = 1(1)25$. Here, again, we find terminal-digit errors; namely, the tabulated values of λ_s corresponding to $s = 4, 7, 9, 14, 20, 22$ should be increased by a unit in the least significant figure, while those corresponding to $s = 18, 21, 24$ should be decreased by a like amount.

A critical table of coefficients to 3D for Bessel quadratic interpolation is appended for use with Table II. On the other hand, it is shown in the Introduction that linear interpolation suffices throughout Table I.

Acknowledgment is made of the use of the corresponding 15D NBS tables [1] as the basis for Table I. Furthermore, it is stated that Tables II and III were taken from statistical tables of Pearson and Hartley [2].

A significant contribution to tabular literature is to be found in Table IV. This represents the most extensive tabulation of the tetrachoric functions published to date. The various applications of these functions, particularly in mathematical statistics, are discussed and illustrated in the informative Introduction.

J. W. W.

1. NEW YORK W. P. A. MATHEMATICAL TABLES PROJECT, *Tables of the Probability Functions*, Volume II, New York, 1942. Reissued with corrections as *Tables of Normal Probability Functions*, NBS Applied Mathematics Series, No. 23, U. S. Government Printing Office, Washington, D. C., 1953.

2. E. S. PEARSON & H. O. HARTLEY, *Biometrika Tables for Statisticians*, Volume I, Cambridge University Press, Cambridge, 1954.

130[L, V].—OTTO EMERSLEBEN, *Die Strömungsbereiche bei zentrischer Überlagerung zweier Grundfunktionen doppeltperiodischer Parallelströmungen*, Anwendungen der Mathematik Nr. 11, Institut für Angewandte Mathematik der Universität Greifswald, Greifswald, 1964, 21 pp., 30 cm.

For a viscous flow in the z-direction, the velocity $v(x, y)$ satisfies the conditions:

$$\Delta(x, y) = -C, \qquad v(x, y) = 0 \quad \text{on boundaries.} \tag{1}$$

The author considers the particular solutions of (1), given by

$$v(x, y) = \frac{C}{4\pi^2} \cdot [Z_0 - R_k(x, y)], \quad \text{if} \geqq 0 \tag{2}$$

$$= 0, \quad \text{otherwise}$$

with

$$R_k(x, y) = \frac{1}{1+k} \left\{ Z \begin{vmatrix} 0 & 0 \\ x & y \end{vmatrix} (2) + k \cdot Z \begin{vmatrix} 0 & 0 \\ x + \frac{1}{2} & y + \frac{1}{2} \end{vmatrix} (2) \right\}.$$

Here $Z \begin{vmatrix} 0 & 0 \\ x & y \end{vmatrix} (2)$ is the Epstein zeta function of the second order. The solutions (2) are periodic in x and y, with period 1. The contour lines of $R_k(x, y)$ are drawn for $k = 2(1)9$ and 5.098. Choosing as Z_0 in (2) the value of a contour line, the domain of flow (the region where $v > 0$) can be obtained from the graphs. This domain of flow can consist of several homeomorphic components. The connectivity of the component depends on k and ϵ. Here ϵ, the porosity, is the ratio of the area of domain of flow to the total area. This connectivity is shown for $0 \leqq \epsilon \leqq 1$ and $1 \leqq k \leqq 9$; it is either 1, 2 or ∞.

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131[L, X].—HAROLD T. DAVIS, *Tables of the Mathematical Functions*, The Principia Press of Trinity University, San Antonio, Texas, 1963, Vol. I, xiii + 401 pp.; Vol. II, xiv + 391 pp., 26 cm. Price $8.75 each.

These two volumes constitute a revised edition of the work originally entitled *Tables of the Higher Mathematical Functions*, which was published in two volumes in 1933 and 1935, respectively. A third volume [1], which first appeared in 1962, has been reviewed in this journal.

The first volume has now been revised and enlarged by the inclusion of two tables (12A and 12B) giving, respectively, log $\Gamma(x)$ to 12D for $x = 100(1)3100$ and $1/\Gamma(x)$ to 25D or 25S for $x = 1(1)100$. In the table of contents (p. viii) the range of the first of these tables is erroneously given as identical with that of the second.

A valuable feature of this work is the inclusion in Volume I of an elaborate introductory section of 172 pages, entitled Tables and Table Making, which contains detailed information on: the classification and history of mathematical tables; modern mathematical instruments of calculation (such as, Taylor's theorem, analytic continuation, Laurent series, asymptotic series, methods of saddle points and of steepest descent); and interpolation (including tables of interpolation coefficients and derivative coefficients, generally to 10D). A selected bibliography of more than 300 titles concludes this section of the book.

The remainder of the first volume is devoted to a detailed discussion of the properties of the gamma function and its logarithmic derivative, the psi function, together with extensive tables of these functions. The 12 tables in the original edition have been retained, with a number of known errors corrected. Herein $\Gamma(x)$ and its common logarithm are tabulated to from 10D to 20D over the interval

$-10 \leqq x \leqq 101$ at subintervals varying from 10^{-4} to 10^{-1}, and $\psi(x)$ and $\log |\psi(x)|$ are given to from 10D to 18D over the interval $-10 \leqq x \leqq 450$ at subintervals varying from 10^{-4} to 0.5. Furthermore, the real and imaginary parts of $1/\Gamma(re^{i\theta})$ are given to 12D for $r = -1(0.1)1$ and $\theta = 0°, 30°, 45°, 60°, 90°, 120°, 135°, 150°$.

Volume II remains virtually unaltered in the present edition. A supplementary bibliography of nearly 70 titles has been retained. In his preface Professor Davis acknowledges the current inadequacy of his bibliography, and refers the reader to the FMRC Index [2] and to his index [3], compiled in collaboration with Vera J. Fisher.

In this second volume the first four derivatives of the psi function are tabulated in a series of 16 tables to from 10D to 19D over the interval $-10 \leqq x \leqq 100$, at subintervals varying from 0.01 to 0.1. The next two sets of tables relate to the Bernoulli and Euler polynomials. These 11 tables include: values of $B_n(x)$ and $E_n(x)$ to 10D for $n = 2(1)8$, $x = 0(0.01)1$; numerators and denominators of the first 90 Bernoulli numbers, B_n, and the first 62 as repeating decimals; $\log B_n$ to 10D and B_n to 9S, for $n = 1(1)250$; exact values of the first 50 Euler numbers, E_n; $\log E_n$ to 12D and E_n to 10S, for $n = 1(1)250$; tables of the sums S_n to 32D of the reciprocals of powers of the positive integers, for $n = 2(1)100$, and sums of related series; 24D values of $\log S_n$ and $\sum_n$, the sum of the reciprocals of the nth powers of the primes, for $n = 2(1)80$; exact values of $S_n(p)$, the sum of the nth powers of the first p positive integers, for $n = 1(1)10$, $p = 1(1)100$, and $n = 1(1)3$, $p = 101(1)1000$; 12D values of the coefficients $A_n(r)$ in Lubbock's summation formula, for $n = 2(1)7$, $r = 2(1)100$.

This volume is concluded with a discussion of Gram polynomials and two sets of tables of coefficients to 10S for fitting polynomials to equally spaced data by the method of least squares.

Although a number of known errors in the first edition have been corrected in this one, there remain several reported errors that have escaped the attention of the author. Principal among these are two corrections announced in this journal [4]; namely, in Volume I, on p. 201 the value of $\Gamma(1.0255)$ should read 0.98590 94917 instead of 0.98590 26815, and on p. 250 the value of $\log \Gamma(22.7)$ should read 20.6459 . . . instead of 20.5459 . . . . On pages 805 and 806 of reference [2] there appear lists of errors in the first edition of these tables. The error noted in $\log \psi'(0.01)$ persists: *for* 4.00002 69776, *read* 4.00007 04027. Other errors noted therein that remain uncorrected occur in $\Gamma(1.664)$ and its common logarithm, in $\psi(1.017)$, and in $\log |\psi(x)|$ for $x = 1.299, 1.451, 1.458, 1.473$, and 1.475. Furthermore, the final digit of $\Gamma(1.564)$ has been erroneously changed to 4 instead of 5, and a similar last-place error appears in $\Gamma(1.986)$, where the ending digit should be 0 instead of 1. A more serious error in correction occurs in $\log \Gamma(85.9)$, where the eighth decimal place should be 4 instead of 7.

These relatively few errors remaining in the new edition should not significantly detract from the value of this impressive work, which thirty years after its initial appearance still contains the most extensive published tables of the gamma and polygamma functions.

J. W. W.

1. H. T. Davis & Vera J. Fisher, *Tables of the Mathematical Functions: Arithmetical Tables*, Volume III, Principia Press, San Antonio, Texas, 1962. See *Math. Comp.*, v. 17, 1963, pp. 459–461, RMT **68.**
2. A. Fletcher, J. C. P. Miller, L. Rosenhead & L. J. Comrie, *An Index of Mathematical Tables*, second edition, Addison-Wesley Publishing Co., Reading, Massachusetts, 1962.
3. H. T. Davis & Vera Fisher, *A Bibliography and Index of Mathematical Tables*, Northwestern University, Evanston, Illinois, 1949.
4. *MTAC*, v. 10, 1956, p. 180, MTE **248.**

132[M].—Max Morris & Orley E. Brown, *Differential Equations*, Fourth Edition, Prentice-Hall, Inc., Englewood Cliffs, New Jersey, 1964, vi + 366 pp., 22 cm. Price $11.35.

The first edition of this book was published in 1933. At that time it was stated in the preface that the text had been designed for use in colleges and engineering schools for students with a background of only a first-year course in calculus. The present edition differs from the original only in the addition or deletion of certain topics. A short section on Laplace transforms has been added and considerably more emphasis is placed on numerical solution of both ordinary and partial differential equations. Throughout the text the manipulatory aspects of differential equations are stressed. There is little emphasis on proving theorems of any kind. A number of the most important theorems are stated without proof.

It is possible to get a general idea of the coverage of this book from the list of the chapter titles. Thus, we have Chapter 1, "Introduction," Chapter 2, "Differential Equations of the First Order and First Degree," Chapter 3, "Equations of the First Order But Not of the First Degree," Chapter 4, "Linear Differential Equations," Chapter 5, "Numerical Methods for Ordinary Differential Equations," Chapter 6, "Integration in Series," Chapter 7, "Linear Partial Differential Equations with Constant Coefficients," and Chapter 8, "Numerical Solutions of Partial Differential Equations." The treatment of the various topics in the different chapters is similar to that contained in many of the older or more elementary text books on differential equations.

The topics introduced in this edition have been chosen with an eye to modernizing the text book. This has not been wholly successful. For example, the treatment of the Laplace transform is purely formal and hardly gives the student sufficient material to make use of it. The additional material on the numerical solution of ordinary and partial differential equations has been more successfully introduced. It represents very useful and important material. Several items involving the formal solution of partial differential equations in terms of arbitrary functions have been deleted from the present edition. The reviewer feels that this represents a distinct improvement in the text. Much of this formal material does not represent the approach to differential equations usually taken in more modern texts. In addition, it does not aid the student in any way when he is forced to approach the solution of a practical problem by using numerical techniques.

The outstanding feature of this particular text has been retained through all the editions. This is the large number of carefully selected problems together with answers. Any student who works through this large group of problems will certainly be able to produce formal solutions of many types of ordinary differential equations. This particular feature of the book would recommend its adoption over other texts which present the same material in a similar manner but do not include so many problems. This text should still be very useful for engineering and technical students. Its old-fashioned approach to differential equations, however, will find little favor with the present modern approach taken in more purely mathematical texts.

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133[M].—G. SANSONE & R. CONTI, *Non-linear Differential Equations*, The Macmillan Company, New York, 1965, xiii + 535 pp., 24 cm. Price $15.00.

This is a superb book devoted to the classical and modern theory of linear and nonlinear ordinary differential equations. It covers existence and uniqueness theorems, stability theory, perturbation techniques, asymptotic behavior, periodic solutions, and Briot-Bouquet theory, with encyclopedic thoroughness and in careful detail. Perhaps most valuable is the way in which ideas and concepts are illustrated by means of specific examples. An almost complete set of references to important papers in the field is given.

Students in mathematics, engineering, and physics will find this book of great value, and it will be equally useful to research workers. The authors have written a beautiful and lucid exposition of this area of analysis which can be used as a basis for a variety of different courses. It is unreservedly recommended.

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134[X].—DIETER GAIER, *Konstruktive Methoden der konformen abbildung*, Springer-Verlag, Berlin, 1964, xiii + 294 pp., 23 cm. Price DM 68.

This book is a very thorough survey of those aspects of the theory of conformal mapping which relate to the numerical computation of conformal maps. Theory (and occasionally proof) are followed in close order by numerical techniques and, whenever available, the results of numerical experiment. This book is an absolute "must" for every computer lab; but because of the wealth of material it contains, it will also be of considerable use to people whose interest is purely theoretical. There is a bibliography of 480 items.

The five chapters are entitled, respectively: The Conformal Mapping of Simply Connected Domains by means of Integral Equations with a Neumann Kernel; The Method of Theodorsen for the Conformal Mapping of $|z| < 1$ on a Region; Approximation of Conformal Maps by means of Polynomials with Extremal Properties; Additional Methods for the Conformal Mapping of Simply Connected Regions; Conformal Mapping of Multiply Connected Regions on Canonical Regions.

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135[X].—RUFUS ISAACS, *Differential Games*, John Wiley & Sons, Inc., New York, 1965, xxii + 384 pp., 24 cm. Price $15.00.

Although the theory and application of differential games (and control theory) has received much attention since 1950, I feel it will be worthwhile to begin with a short description of the types of problems dealt with in these fields, and in particular in this book.

In a quite general case, the evolution of a system, whose state we assume is determined by the state variables $x = (x^1, \cdots, x^n)$, is governed by a differential equation (1) $\dot{x} = F(x, \phi, \psi)$ with the initial condition $x(0) = x_0$, where $\phi = (\phi^1, \cdots, \phi^s)$ and $\psi = (\psi^1, \cdots, \psi^r)$ are termed controls. There will be two players, one controlling the function ϕ, which may have values in a given set in Euclidean s-dimensional space (E^s); the other controlling ψ within a given set in E^r. There is also given a manifold C, the terminating manifold, such that the game ends when a trajectory of (1) enters C, and a payoff functional defined on the space of trajectories of (1). The game proceeds with one player controlling ϕ in such a way as to generate a trajectory which maximizes payoff, the other player utilizing ψ to try to minimize payoff.

At this point one should notice that if at the offset each player knows that the other will play optimally, the functions ϕ and ψ can be computed as functions of *time* alone, and the value of the game becomes merely a function of the starting position x_0. However, if a player may err, this is no longer the case. In order that the opposing player may use the error to his benefit, he must be able to perceive it, i.e., to make measurements on the state of the system as the game progresses. With the exception of Chapter 12, the author assumes that at each instant of time during the course of play, both players have complete knowledge of the present state of the system; he defines a *strategy* as a determination of the controls ϕ and ψ as functions of the state. Since it is assumed that the players do not err, but play optimally, the determination of a strategy implies that the game has been solved for arbitrary initial positioning x_0.

I feel that these important distinctions in possible types of games have been passed over somewhat lightly. For instance, the casual reader may not notice that a proposed method of play in a game of two players (example 8.1.1, p. 202) requires one player to have "memory", i.e., the value of the control depends on past history of the state.

As the author remarks, the theory of differential games grew from solving problems, and this is the approach taken in the book. Little time or effort is spent on theorem proving, instead many diversified types of problems are formulated, often completely solved, and a theory introduced which stems from the method of solution.

The book begins at a leisurely pace, with the first three chapters being accessible to a person with little mathematical background. Chapters 1 and 2 are of an introductory nature while Chapter 3 deals with discrete games. The variety of fascinating problems formulated, and often solved, in these chapters alone should delight a wide audience of readers.

In Chapter 2, pages 41–43, the need for having the vectogram (the set of values $\{F(x, \phi, \psi)\}$ as ϕ, ψ take on all admissible values) convex for each x in order that a solution exist, is illustrated by example. Here the author has led the reader in a very natural way to the basis of the deep existence theorems of control theory.

The mathematical theory begins in Chapter 4. The approach taken is to show that the payoff, or value $V(x)$, of a game starting at the arbitrary initial state x, satisfies what has become known as the "Bellman equation" which somewhat resembles a partial differential equation of Hamilton-Jacobi type. The solutions of such equations often exhibit extreme changes in neighborhoods of certain surfaces in state space called singular surfaces by the author, or switching surfaces in control theory. The majority of the remainder of the book is concerned with showing, mainly by example, the types of behavior which solutions may exhibit near these surfaces, in classifying the surfaces and in solving problems using the concepts introduced.

On the whole, the printing of the book is very good. There are only occasional minor errors, e.g., the rightmost vectogram for the player P in Figure 3.3:1, page 51, is in error and will not yield the shown solution.

The wide range of possible applications of differential games is exemplified in the many examples discussed and solved throughout the text. While obtaining solutions to these intriguing problems, the author has done an excellent job in providing insight into the deep mathematical theories which exist and the difficulties which must still be overcome.

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136[X].—BEN NOBLE, *Numerical Methods*, Oliver & Boyd Ltd., Edinburgh, Scotland, 1964. Volume 1, *Iteration, Programming and Algebraic Equations*, xii + 156 pp., 19 cm. Price \$2.75. Volume 2, *Differences, Integration and Differential Equations*, viii + 372 pp., 19 cm. Price \$3.00.

A reasonable knowledge of numerical analysis should be possessed by every engineer, scientist or applied mathematician. A great many books have been recently published in an attempt to fill the demand for this knowledge, particularly at an elementary level. Many of these books combine numerical analysis and computer programming, using a problem-oriented language like FORTRAN. These books are frequently disappointing, particularly if you have read the publisher's claims on the dust jacket before you read the author's preface.

It is a pleasure to report that these two volumes under review have accomplished their stated purpose and constitute an excellent elementary introduction to the most commonly used numerical methods. The first chapter of Vol. I sets the general level of the work by presenting a clear concise account of several topics such as round off, absolute and relative errors, error analysis and control, etc. The treatment is both practical and elementary. This is followed with chapters on iteration methods, elementary programming, linear equations, and matrix methods. The emphasis is placed on presenting a few methods in some detail. The chapter on programming uses a problem-oriented language, but does not try to teach FORTRAN or ALGOL. The language is used to illustrate how a source language is used without going into the vast amount of detail necessary to present an existing source language.

Volume II covers topics in finite differences and approximate representation of functions, polynomial interpolation, numerical integration and differentiation, ordinary differential equations and partial differential equations. As in Vol. I the work is illustrated by well chosen examples. The methods presented are standard ones, but the author gives many hints and much practical advice on using the various numerical procedures. The weakest chapter in the book is one on partial differential equations. Only the heat equation and Laplace's equation are actually considered. Even here the author manages to say a lot in a very small space.

These volumes are part of the University Mathematical Text series. The price of each volume is quite modest. In fact, the two volumes can be obtained for considerably less than many single volume treatments of numerical analysis.

RICHARD C. ROBERTS

137[X, Z].—LADIS D. KOVACH, *Computer-Oriented Mathematics*, Holden-Day, Inc., San Francisco, Calif., 1964, vii + 98 pp., 23 cm. Price $3.95.

The purpose of this small volume is to acquaint the interested reader with the ideas behind some of the algorithms which are commonplace in computer programs. It is designed for use in lower-level college programming courses and for advanced high school students. The only concept from elementary calculus which appears is the derivative, and this occurs but once.

Topics included are: characteristics of computers, number bases, initial guesses, interpolation, approximation methods, iteration, relaxation, and Monte Carlo methods.

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138[X, Z].—WILLIAM PRAGER, *Introduction to Basic FORTRAN Programming and Numerical Methods*, Blaisdell Publishing Company, New York, 1965, ix + 203 pp., 28 cm. Price $6.00.

This book, written by one of the most distinguished applied mathematicians of the present time, admirably illustrates that trend in the writing of textbooks on numerical analysis (visible in a number of recent works) in which the author seeks to impart to the student practical experience in the use of a digital computer, to acquaint him with the theory of computation, and to do so within the framework of a balanced and integrated course of study.

With regard to the specific scope and intention of the book it would seem impossible to do better than quote from the publisher's advertisement.

"This text corresponds to a sophomore course, which the author has been teaching for several years. The timing of this course and the choice of its contents was motivated by the desire to introduce students in engineering and the sciences to automatic computation as early as is possible without inviting uncritical use of the new tool.

"An introductory chapter, in which the program for a simple computation (selected partial sums of a series) is presented first in English and then in FOR-

TRAN, is followed by two chapters on FORTRAN Terminology and Ground Rules and Essential FORTRAN Statements. In Chapter IV several programs are discussed, which use only these essential FORTRAN statements. Chapter V is concerned with error analysis and control and Chapter VI with additional FORTRAN statements. Chapters VII–IX are devoted to Computing with Polynomials, Interpolation, and Quadrature. Chapter X reverts to programming and treats the Manipulation of Alphameric Information, the Use of Magnetic Tapes, and Sorting. Chapters on the Solution of Equations and the Integration of Ordinary Differential Equations and an Appendix on the organization of a Monitor conclude the book. Graduated exercises at the end of each chapter enable the reader to practice what he has learned and to check his progress."

This book is not a book for the research student in numerical analysis: vast areas of the subject are left untouched (eigenvalue problems, the numerical solution of partial differential equations, and many other topics are not dealt with). But as a textbook for a one-semester course it is quite outstanding. Given teaching staff of sufficient competence and student material of a suitable calibre it seems probable that this book will serve to introduce to numerous young applied mathematicians, physicists, engineers and many others, the theory, practice, limitations, and possibilities of digital computation.

The book is pleasantly produced: the writing is invested with that degree of formal elegance and clarity in exposition which distinguishes the works of Professor Prager.

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139[Z].—LILIAN TROLLHANN & ALFRED WITTMANN, *Dictionary of Data Processing*, Elsevier Publishing Company, Inc., New York, 1965, 300 pp., 23 cm. Price $17.00.

There are no definitions of words here. The book contains translations of data-processing terms between English/American and German and French. The first section (214 pages long) contains numbered, categorized English/American terms and their translations into German and French. Thus: "**O134 output unit** (dig)" is followed by "Ausgabeeinheit *f*" and "unité *f* de sortie". The category here, (dig), refers to "digital computers". There are 11 other categories: (anal), (math), (tron) = electronics, (datatr) = data transmission, etc. The subsequent German and French lists contain only the term's number, e.g. **O134,** so that one translates between German and French, say, by utilizing the main, English/American listing.

The listings are heavily orientated toward hardware and contain, for example, **relaxation oscillator** (tron) but not **relaxation method** (math). The authors have both worked in the Translation Dept. of Siemens & Halske AG. Although ostensibly English/American is given the central position, it appears likely, from some of the translations, that the authors often began with the German terms. Some of the English/American has a Germanic flavor. On occasion, the definitions do not quite touch bottom. Thus "**F7 factoring** (math) (e.g. an equation containing fractions)" is erroneously translated as "durchmultiplizieren (z.B. eine Bruche enhaltende Gleichung)" while the French translation is given as "_____". Whether the latter implies that the French do not have a word for it, or that it is unprintable in French, is not made clear.

All together there are about 5000 terms. The book is very nicely printed and bound, but quite expensive.

D. S.

140[Z].—W. W. Youden, *Computer Literature Bibliography 1946–1963*, National Bureau of Standards Miscellaneous Publication 266, March 31, 1965, iv + 463 pp., 29 cm. Price $3.75. (Available from the Superintendent of Documents, U. S. Government Printing Office, Washington, D. C. 20402, or from local U. S. Department of Commerce Field Offices.)

There is indexed here the "computer literature" that has appeared during the stated years in *Communications ACM*, *Journal ACM*, *BIT*, *IBM Systems Journal*, *The Computer Journal*, and several other journals; in 21 books; and in over 100 proceedings of computer conferences such as the Joint Computer conferences and IFIP 62. The three indices are by journal (or proceedings), by author, and by every important word in the title. For example, in the last-mentioned index, one finds two pages listing articles containing the word "method" in the title. All together, over 6100 articles are referenced.

This bibliography is, of course, not complete. Articles appearing elsewhere, such as in this journal, are not listed. While all numerical analysis, say, appearing in the aforesaid sources has been indexed, related articles appearing here, in *Numerische Math.*, in the SIAM journals, etc., are not covered.

Nonetheless, the volume is highly useful and instructive, and also has a high browsing-interest quotient. (The latter is the number of pages that catch our attention divided by the total number of pages.) The printing is not always perfect, but usually the invisible information can be restored through redundancy. The price is very reasonable, as is usually the case with this publisher.

D. S.