16[A].—RUDOLPH ONDREJKA, *The First 100 Exact Double Factorials*, ms. of 12 handwritten sheets (undated) deposited in the UMT file.

These unpublished tables consist of two parts: the first consists of the exact values of $(2n - 1)!! = 1 \cdot 3 \cdot 5 \cdots (2n - 1)$ for $n = 1(1)100$; the second consists of the exact values of $(2n)!! = 2 \cdot 4 \cdot 6 \cdots 2n$ for the same range of n. These data were computed by the author on a desk calculator "many years ago" and were subsequently misplaced. Each tabular entry after the first was calculated from its predecessor, and an overall check consisted of forming the product of the last entry in each table and comparing that result with 200!.

The only tables of this kind of comparable size appear to be unpublished ones [1] calculated by J. C. P. Miller on the EDSAC in 1955. His tables cover the same range for the even double factorial and a larger range for the odd double factorial; however, the increment in the argument n is 10 to $n = 100$, and it is 50 beyond that to $n = 250$.

The most extensive published tables of exact values of such numbers are those of Potin [2] and Hayashi [3], which extend to only $n = 25$.

Consequently, the present manuscript tables supply valuable numerical information that has been hitherto unavailable.

J.W.W.

1. A. FLETCHER, J. C. P. MILLER, L. ROSENHEAD & L. J. COMRIE, *An Index of Mathematical Tables*, Vol. I, 2nd ed., Addison-Wesley, Reading, Mass., 1962, p. 53.
2. L. POTIN, *Formules et Tables Numériques*, Gauthier-Villars, Paris, 1925.
3. K. HAYASHI, *Fünfstellige Funktionentafeln*, Springer, Berlin, 1930.

17[A, F].—M. LAL, *Expansion of* $\sqrt{2}$ *to* 19600 *Decimals*, Memorial University of Newfoundland, St. John's, Newfoundland, Canada, ms. of 4 typewritten pp. + 2 tables deposited in the UMT file.

The main table here has an attractively printed value of $\sqrt{2}$ correct to 19600 decimals. This is somewhat more accurate than the recent computation to 14000 decimals [1]. As in [1], we also have here a table of the distribution of the decimal digits and a chi-square analysis of their presumed, and apparent, equidistribution.

This computation required 14.2 hours on an IBM 1620, and the check $(\sqrt{2})^2 = 2$ required 4 hours. Since this computer is a small decimal machine, the author elected to compute $\sqrt{2}$ *one digit at a time*. We may indicate his method as follows:

Let $A_k = [10^k\sqrt{2}]$ and $A_{k+1} = 10A_k + a_{k+1}$, so that a_k is the kth digit. Let $B_0 = 1$ and

$$B_{k+1} = 100B_k - \sum_{n=1}^{a_{k+1}} (20A_k + 2n - 1)$$

where a_{k+1} is the largest value of n which leaves the difference positive. If there is no such n, then $a_{k+1} = 0$. Thus we have

$$\begin{array}{ll} A_0 = 1 & B_0 = 1 \\ A_1 = 14 & B_1 = 4 \\ A_2 = 141 & B_2 = 119, \quad \text{etc.} \end{array}$$

If D decimals are sought, it is clear that the number of operations involved in this method is $O(D^2)$, as is also a binomial series calculation, since the latter converges nearly geometrically. A comparison in speed between these methods would depend on the relative operation-times for subtraction and division. In the second method one would use a large solution of either of the so-called Pell equations

$$x^2 - 2y^2 = \pm 1,$$

and then expand

$$\sqrt{2} = (x/y)(1 \mp x^{-2})^{1/2}.$$

For example, on a binary machine, the evaluation of

$$\sqrt{2} = \frac{941664}{665857}\left[1 + \frac{2^{-9}}{865948329}\right]^{1/2}$$

should be quite fast.

We might note that there is nothing in this data to give encouragement to the stated opinion of J. E. Maxfield to the effect that $\sqrt{2}$ is probably not normal (in the decimal system). On the contrary, the apparent equidistribution mentioned above would suggest that $\sqrt{2}$ is, at least, simply normal. Similarly, the more recent stated opinion of I. J. Good that $\sqrt{2}$ is perhaps not normal in the base 2 has contrary evidence in [1], since it is shown there that $\sqrt{2}$ has apparent equidistribution not only in decimal but also in octal.

D.S.

1. Kōki Takahashi & Masaaki Sibuya, "Statistics of the digits of $\sqrt{n}$," *Jōhō Shori* (*Information Processing*), v. 6, 1965, pp. 221–223. (Japanese) (See also the next review here.)

18[B, K.]—Kōki Takahashi & Masaaki Sibuya, *The Decimal and Octal Digits of $\sqrt{n}$*, The Institute of Statistical Mathematics, Tokyo, August 1966, ms. of iii + 83 pp. deposited in the UMT file.

As stated in the Foreword, the iteration $x_{k+1} = x_k(1.5 - 0.5nx_k^2)$ was used by the authors in the underlying electronic calculations on an HIPAC-103 system.

The numerical output, on standard computer sheets, is arranged in two sections. The first, designated Part I, includes approximations to $\sqrt{2}$ and $\sqrt{3}$ extending to 14000D and to 15360 octal places. The cumulative frequencies of individual digits are given for successive blocks of 100 decimal digits and 128 octal digits, respectively. The frequencies in each of these blocks are separately tabulated for only the first half of the range of digits calculated. The corresponding χ^2 values are given to 3D for the cumulative distributions and to 1D for the others.

In Part II we find similar information for the square roots of the integers 5, 6, 7, 8, and 10. Here, however, the approximations are carried to about one-half the extent of those in Part I. Specifically, the square roots of 5, 6, and 10 are given to 7000D and to 7680 octal digits, whereas those of 7 and 8 appear to 6900D and to 7552 octal digits.

The decimal approximations are conveniently displayed in groups of 10 digits, with 10 such groups in each line, and spaces between successive sets of five lines. A total of 5000D can thereby be shown on each computer sheet. The octal representations are presented in groups of eight digits, with eight groups to a line. Fifty lines are shown on each page, with spaces between successive sets of five lines, as before; thus, a total of 3200 octal digits are accommodated on each sheet.

In a related paper [1] the authors have presented similar statistical information concerning these square roots. While this statistical information is believed correct, the unpublished printed-out values of the roots computed at that time contained several erroneous digits because of a programming error. Discrepancies between those decimal approximations obtained for $\sqrt{2}$ and $\sqrt{3}$ and the values previously published by Uhler [2, 3] were erroneously attributed by the authors to purported errors in Uhler's calculations. The corrected values appearing in the present manuscript are believed by the authors to be free from error. In partial confirmation of this, the reviewer has found complete agreement of the value of $\sqrt{2}$ herein with the unpublished approximation [4] of Lal, which extends to 19600D.

J. W. W.

1. Kōki Takahashi & Masaaki Sibuya, "Statistics of the digits of $\sqrt{n}$," *Jōhō Shori (Information Processing)*, v. 6, 1965, pp. 221–223. (Japanese)
2. H. S. Uhler, "Many-figure approximations to $\sqrt{2}$, and distribution of digits in $\sqrt{2}$ and $1/\sqrt{2}$," *Proc. Nat. Acad. Sci. U. S. A.*, v. 37, 1951, pp. 63–67.
3. H. S. Uhler, "Approximations exceeding 1300 decimals for $\sqrt{3}$, $1/\sqrt{3}$, $\sin(\pi/3)$ and distribution of digits in them," *ibid.*, pp. 443–447.
4. M. Lal, *Expansion of $\sqrt{2}$ to 19600 Decimals*, ms. deposited in the UMT file. (See *Math. Comp.*, v. 21, 1967, pp. 258–259, RMT 17.)

19[F].—L. G. Diehl & J. H. Jordan, *A Table of Gaussian Primes*, Washington State University, Pullman, Washington, a strip of computer paper deposited in the UMT file.

If p, a prime, is of the form $4m + 1$ then $p = A^2 + B^2$. As is known

$$\pm A \pm Bi \quad \text{and} \quad \pm B \pm Ai$$

are then Gaussian primes. This table lists A and B for each $p = 4m + 1 \leqq 90997$.

Whereas such a table is not readily available, a somewhat larger table for $p < 10^5$ was published by A. J. C. Cunningham long ago [1].

The present table was computed in 15 minutes on an IBM 709 at Washington State University. No details are given as to how this was done. It may be of interest to survey briefly known methods that have been used for these and related problems.

Four methods of theoretical interest are reviewed by Davenport [2]. The simplest conceptually is that of Gauss. If $p = 4m + 1$, set

$$A \equiv (2m)!/2(m!)^2, \qquad B \equiv (2m)!\,A \pmod p.$$

It is clear that this is quite inefficient arithmetically speaking. Related to this is Jacobsthal's method. Let

$$S(a) = \sum_{n=1}^{p-1} \left(\frac{n(n^2 - a)}{p}\right)$$

where the quantity summed is the Legendre symbol. Then if R is any quadratic residue, say $R = 1$, and N is any nonresidue, set

$$A = \tfrac{1}{2} \mid S(R) \mid, \qquad B = \tfrac{1}{2} \mid S(N) \mid .$$

Recently Chowla [3] has given an attractive proof of Jacobsthal's method, a consequent simple proof of Gauss's method, and the relation of these Jacobsthal sums to the Riemann Hypothesis.

More practical are the other two methods. That of Legendre is based on the regular continued fraction of $\sqrt{p}$. This would require a number of operations $O(p^{1/2+\epsilon})$, since this is the bound on the period of the continued fraction. This contrasts favorably with the $O(p)$ operations needed for Gauss's method.

But better still is Serret's method. This is based on the finite continued fraction for p/s, where $0 < s < \frac{1}{2}p$ and $p \mid s^2 + 1$. This fraction has only $O(\log p)$ terms. Now, if we were to compute $\sqrt{-1} = s \pmod{p}$ by Wilson's Theorem:

$$s \equiv \pm[(p-1)/2]!$$

we would be back to an $O(p)$ algorithm, but we can do better. We need any non-residue N of p. If $p = 8m + 5$, then 2 will do. If not, but if $p = 24m + 17$, then 3 will do. In $O(\log p)$ attempts we can find an N. Then

$$s \equiv \pm N^{(p-1)/4} \pmod{p}.$$

The power shown can also be computed in $O(\log p)$ operations by expressing $(p - 1)/4$ in binary, and then successively squaring powers of N. For example, if $p = 1429$,

$$(2/1429) = -1,$$

and

$$\pm 2^{357} \equiv s,$$

or

$$-2^{1+4+32+64+256} \equiv 620 = s.$$

Thus, the Serret algorithm, with s found as above is $O(\log p)$. This is perhaps the best one can do for large p. For further discussion of the Legendre and Serret methods see [4, Exercises 145–149, pp. 187–188].

In all of the foregoing the expansion $p = A^2 + B^2$ is being carried out for a single p at a time. There also exists a sieve method based upon the p-adic square roots of -1 which can carry out such computations *en masse*, with the p's and their corresponding s's arising automatically [5].

Still other techniques, less interesting mathematically, but quite feasible programming-wise, have been used. These mostly involved trial-and-error subtractions, or additions combined with a sorting process of some type.

Finally, we might note, for some of the largest known $p = 4m + 1$, such as Brillhart's $p = 2^{457} - 2^{229} + 1$, one has at once, by inspection,

$$p = (2^{228})^2 + (2^{228} - 1)^2.$$

With a bit more effort one can deduce from the third-largest known prime of the form $4m + 1$ (which was found recently by Brillhart and Selfridge):

$$\tfrac{1}{5}(2^{691} - 2^{346} + 1),$$

the *largest* known complex Gaussian prime:

$$\tfrac{1}{5}(3 \cdot 2^{345} - 1) + \tfrac{1}{5}(2^{345} - 2)i.$$

The two largest known primes of the form $4m + 1$:

$$5 \cdot 2^{1947} + 1 \quad \text{and} \quad 7 \cdot 2^{830} + 1,$$

are due to Robinson, but I know of no easy way of finding their Gaussian factors.

Perhaps the best algorithm for the first would again be Serret's, starting from the fact that this prime divides a specific Fermat number.

D. S.

1. A. J. C. Cunningham, *Quadratic Partitions*, Hodgson, London, 1904.
2. H. Davenport, *The Higher Arithmetic*, Harper, New York, 1960, pp. 120–123.
3. S. Chowla, *The Riemann Hypothesis and Hilbert's Tenth Problem*, Gordon and Breach, New York, 1965, Chapters IV, V.
4. D. Shanks, *Solved and Unsolved Problems in Number Theory*, Spartan, Washington, 1962.
5. D. Shanks, "A sieve method for factoring numbers of the form $n^2 + 1$," *MTAC*, v. 13, 1959, pp. 78–86.

20[F].—M. F. Jones, M. Lal & W. J. Blundon, *Table of Primes*, Memorial University of Newfoundland, St. John's, Newfoundland, Canada, June 1966, ms. of 100 computer sheets, 28 cm. Copy deposited in the UMT file.

This table lists all 47273 primes in the eight ranges:

$$10^n(1)10^n + 150{,}000; \qquad n = 8(1)15.$$

It, and its statistics, have been discussed earlier in this journal [1]. As indicated in [1], the primes were computed on an IBM 1620. They are very nicely printed, in an elegant format, 500 to the page. Anyone familiar with programming would note at once the great care that must have been taken here to produce such a format.

The range $10^{12}(1)10^{12} + 10^4$ was checked against Kraitchik's 335 primes in [2], with perfect agreement. (Kraitchik's tables are seldom *that* accurate.) For $n = 9$, a successful spot check was made against Beeger's manuscript table [3].

D. S.

1. M. F. Jones, M. Lal & W. J. Blundon, "Statistics on certain large primes," *Math. Comp.*, v. 21, 1967, pp. 103–107.
2. M. Kraitchik, "Les grand nombres premiers," *Sphinx*, v. 8, 1938, pp. 82–86.
3. N. G. W. H. Beeger, *Tafel van den kleinsten factor de getallen van* 999 999 000–1 000 119 120, *etc.*, deposited in the UMT file and reviewed in UMT 68, *Math. Comp.*, v. 20, 1966, p. 456.

21[F, G].—L. D. Baumert & H. Fredricksen, *The Cyclotomic Numbers of Order Eighteen*, Jet Propulsion Laboratory, California Institute of Technology, 18 computer sheets deposited in the UMT file.

This table presents the cyclotomic numbers of order eighteen. The derivation and computation of these formulas are described adequately in Section 4 of the authors' paper which appears elsewhere in this journal [1].

The identities (2.2) in the paper enable one to group the 324 cyclotomic constants (h,k), $0 \leqq h, k \leqq 17$, into 64 sets. There is a formula for each set, depending on ind 2 (mod 9) and ind 3 (mod 6). Thus there are 54 cases, each with 64 formulas. The table consists of the formulas for sixteen cases; the other formulas can be derived from these formulas. Table 5 of the paper is one of the cases given in the table.

It is interesting to note that not all the formulas in a given case are different.

For example, in Table 5, $(0,3) = (0,6)$, $(1,2) = (1,8) = (2,7) = (2,16)$, $(1,5) = (1,17) = (2,1) = (5,1)$, and $(1,14) = (2,4) = (4,2) = (4,5)$.

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1. L. D. Baumert & H. Fredricksen, "Cyclotomic numbers of order eighteen with applications to difference sets," *Math. Comp.*, v. 21, 1967, pp. 204–219.

22[F, K, X, Z].—BIRGER JANSSON, *Random Number Generators*, Almqvist and Wiksell, Stockholm, 1966, 205 pp., 24 cm. Price Sw. kr. 42.

The most commonly used method of generating pseudo-random numbers with a digital computer is the congruential method. In this method, which was introduced by D. H. Lehmer in 1951, one begins with a number x_1 between 0 and 1, and the next number x_2 is computed as the fractional part of $Nx_1 + \theta$, where N is an integer > 1 and $0 \leqq \theta < 1$. Now x_3 is computed as the fractional part of $Nx_2 + \theta$, etc. The congruential method has been analyzed by many authors. In Chapters 5 and 6 of Birger Jansson's book, he presents a profound study of this method. One wishes to know the serial correlation of such a sequence. If x_1 and θ are rational, as they must be in digital computation, the determination of the serial correlation is a problem in number theory. Beginning with results of Dedekind, Rademacher, and Whiteman, Jansson obtains a practical algorithm for computing the correlation, and he presents extensive tables of exact values of the correlation. This achievement alone will make Jansson's book an indispensable reference in the continuing study of deterministic methods by which we seek to simulate random processes.

The book contains many other topics. There is a survey of statistical tests which have been applied to pseudo-random numbers. There is a collection of special algorithms for computing pseudo-random numbers belonging to distributions other than the uniform distribution. And there is a brief review of the existing theory of what we should *mean* when we call a perfectly well-determined sequence "random."

There is little mention of the *algebraic* theories of random numbers. There is a reference to Zierler's work, but there is no mention of the beautiful and important work of Golomb, Tausworthe, and others on the *P-N* sequences which are used in digital tele-communications. The application of the theory of Galois fields to finite pseudo-random binary sequences has received too little attention by analysts. This theory is particularly challenging because it involves concepts of randomness different from those used in analytical studies.

In summary, Jansson's book is excellent. It is the newest and the most complete guide to the analytical theory of pseudo-random numbers.

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23[G, S].—SHIGETOSHI KATSURA, *Tables of Representations of Permutation Groups for the Many Electron Problem*, Department of Chemistry, University of Oregon, Eugene, Oregon, 1962, unbound report of 326 pp., 28 cm. One copy deposited as Document No. 7567 with the ADI Auxiliary Publications Project, Photoduplication Service, Library of Congress, Washington, D. C. Photoprint $42.00; microfilm copy $14.55 (payable in advance to Chief, Photoduplication Service, Library of Congress).

These tables give to 8D (with a purported maximum error of 5 units in the final decimal places) the elements of those matrices of irreducible unitary representations of the symmetric group on N letters (which are the associates of the representations corresponding to two-element partitions $[(N/2) + S, (N/2) - S]$ of N). These tabulated elements correspond to the elements (pN), for $p = 1(1)N - 1$, of the transposition class of S_N, for $N = 2(1)9$. The dimension of such a representation is the quotient of $(2S + 1)(N!)$ by $[(N/2) + S + 1]![(N/2) - S]!$. When N is as large as 9 this number can be quite large; for example, the dimension of the representation corresponding to $N = 9$, $S = \frac{3}{2}$ is 48, so that the corresponding matrices involve 2304 elements. Since the square of a transposition is the identity permutation, the matrices corresponding to a transposition are symmetric, and it seems uselessly lavish to ignore this fact in printing the tables.

The underlying calculations were performed on an IBM 1620 in the Statistical Laboratory and Computing Center at the University of Oregon and on an IBM 709 in the Pacific Northwest Research Computer Laboratory at the University of Washington.

Following an introductory description of the theory of molecular structure using representation matrices and a discussion of the construction of such matrices, the author appends a list of errata in the smaller tables of Yamanouchi [1], Inui & Yanagawa [2], and Hamermesh [3]. Also included is a list of 11 references.

A brief description of these tables has been published by the author [4].

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1. T. YAMANOUCHI, *Proc. Phys.-Math. Soc. Japan*, v. 18, 1936, p. 623.
2. T. INUI & S. YANAGAWA, *Representation of Groups and Quantum Mechanics of Atoms and Molecules*, 2nd ed., Shohkabo, Tokyo, 1955.
3. M. HAMERMESH, *Group Theory and its Application to Physical Problems*, Addison-Wesley, Reading, Mass., 1962.
4. S. KATSURA, "Tables of representations of permutation groups for molecular integrals," *J. Chem. Phys.*, v. 38, 1963, p. 3033.

24[I].—D. S. MITRINOVIĆ & R. S. MITRINOVIĆ, *Tableaux d'une classe de nombres reliés aux nombres de Stirling*, VI., Belgrade, Mat. Inst., Posebna izdanja, Knjiga 6 (Editions spéciales, 6), 1966, 52pp., 24 cm.

The tables of ${}^{r}S_n^{\,k}$ for $n = 3(1)36$, reviewed in *Math. Comp.*, v. 19, 1965, pp. 151, 690, are here extended, in the same style, to the cases $n = 37$ and 38.

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25[I, L].—HENRY E. FETTIS & JAMES C. CASLIN, *Ten Place Tables of the Jacobian Elliptic Functions*, Report ARL 65–180 Part 1, Aerospace Research Laboratories, Office of Aerospace Research, United States Air Force, Wright-Patterson Air Force Base, Ohio, September 1965, iv + 562 pp., 28 cm. Copies obtainable upon request from the Defense Documentation Center, Cameron Station, Alexandria, Virginia.

This report contains 10D tables of the Jacobi elliptic functions $am(u, k)$, $sn(u, k)$, $cn(u, k)$, and $dn(u, k)$, as well as the elliptic integral $E(am(u), k)$ for $k^2 = 0(0.01)0.99$, $u = 0(0.01)K(k)$ and for $k^2 = 1$, $u = 0(0.01)3.69$. Here, as is conventional, u represents $F(\phi, k)$, the incomplete elliptic integral of the first kind in Legendre's form, and ϕ is the amplitude function, $am(u, k)$.

The integral $E(am(u), k)$ as here tabulated is a by-product of a concurrent calculation of the Jacobi zeta function, defined by the relation

$$Z(u, k) = E(am(u), k) - \frac{E(k)}{K(k)} u,$$

where $K(k)$ and $E(k)$ are the complete elliptic integrals of the first and second kinds, respectively. The integral $K(k)$ and the ratio $E(k)/K(k)$ are also given to 10D for $k^2 = 0(0.01)0.99$.

These tables were calculated on an IBM 7094 computer system, using a subroutine based on the descending Landen transformation (also known as the Gauss transformation).

In addition to this information concerning the calculation of the tables, the authors include the definitions of the tabulated functions and a summary of their various properties.

This set of tables of the Jacobi elliptic functions is the most extensive compiled to date. A relatively inaccessible table prepared by the staff of the Project for Computation of Mathematical Tables [1] gave $sn\, u$, $cn\, u$, $dn\, u$ to 15D for $k^2 = 0(0.01)1$, $u/K = 0.01, 0.1(0.1)1$. The well-known tables of Milne-Thomson [2] give only 5D values of these functions, over a much more restricted set of values of the arguments than the tables under review. It may also be noted here that the 12D tables of Spenceley & Spenceley [3], on the other hand, are arranged with the modular angle and the ratio u/K as parameters.

J.W.W.

1. *Tables of Jacobi Elliptic Functions*, ms. prepared by the Project for Computation of Mathematical Tables, New York City; printed for limited distribution, Washington, D. C., 1942. (See *MTAC*, v. 1, 1943–1945, pp. 125–126, UMT **12**; *ibid.*, p. 425, RMT **207**.)
2. L. M. Milne-Thomson, *Jacobian Elliptic Function Tables*, Dover, New York, 1950. (See *MTAC*, v. 5, 1951, pp. 157–158, RMT **910**.)
3. G. W. Spenceley & R. M. Spenceley, *Smithsonian Elliptic Functions Tables* (*Smithsonian Miscellaneous Collections*, v. 109), Smithsonian Institution, Washington, D. C., 1947. (See *MTAC*, v. 3, 1948–1949, pp. 89–92, RMT **485**.)

26[I, L].—B. I. Korobochkin & Yu. A. Filippov, *Tablitsy modifitsirovannykh funktsiĭ Uittekera*, (*Tables of Modified Whittaker Functions*), Computing Center of the Academy of Sciences of the USSR, Moscow, 1965, xvi + 322 pp., 27 cm. Price 2.66 rubles.

These tables, in the well-known series under the general editorship of V. A. Ditkin, were produced by collaboration between the Computing Center of the Academy of Sciences of the USSR and the Computing Center of the Latvian State University. The functions tabulated are connected with solutions of the hypergeometric equation

$$zy'' + (\gamma - z)y' - \beta y = 0,$$

where we follow the authors in using β, rather than the more usual α, for the first (or numerator) parameter. Much of the introductory text relates to the case in which γ is any positive integer k, but the tables relate entirely to the case $\gamma = k = 2$, to which we shall confine ourselves.

The functions which (with their first derivatives) are actually tabulated are $\Psi_1(\beta, x)$ and $\Psi_2(\beta, x)$. They are connected with confluent hypergeometric functions $Y_{01}(\beta, 2, z)$ and $Y_{02}(\beta, 2, z)$ by the equations

$$\Psi_1(\beta, x) = xe^{-x}[Y_{01}(\beta, 2, 2x)K_1(x) + Y_{02}(\beta, 2, 2x)I_1(x)],$$

$$\Psi_2(\beta, x) = xe^{-x}[-Y_{01}(\beta, 2, 2x)K_0(x) + Y_{02}(\beta, 2, 2x)I_0(x)],$$

where I and K denote the usual Bessel functions, or by the equivalent equations

$$Y_{01}(\beta, 2, 2x) = e^{x}[\Psi_1(\beta, x)I_0(x) - \Psi_2(\beta, x)I_1(x)],$$

$$Y_{02}(\beta, 2, 2x) = e^{x}[\Psi_1(\beta, x)K_0(x) + \Psi_2(\beta, x)K_1(x)].$$

Here $Y_{01}(\beta, 2, z)$ is what is usually denoted by ${}_1F_1(\beta; 2; z)$, $F(\beta, 2, z)$, $M(\beta, 2, z)$ or $\Phi(\beta, 2; z)$, while $Y_{02}(\beta, 2, z)$ is a logarithmic second solution. As the reviewer had considerable difficulty in making quite sure of its meaning, it will be well to state explicitly what $Y_{02}(\beta, 2, z)$ denotes.

In terms of the functional notation used by Miller [1],

$$Y_{02}(\beta, 2, z) = -M(\beta, 2, z)\,[\ln z + C + \psi(\beta) - \psi(2)] - N(\beta, 2, z) - S(\beta, 2, z),$$

where C is Euler's constant $(0.5772 \cdots)$, $\psi(x)$ is the logarithmic derivative of the gamma function (not the factorial function), satisfying

$$\psi(x + 1) - \psi(x) = 1/x,$$

$N(\beta, 2, z)$ is what results on substituting $\gamma = 2$ in

$$N(\beta, \gamma, z) = \left(\frac{1}{\beta} - \frac{1}{\gamma} - 1\right)\frac{\beta}{\gamma}\frac{z}{1!} + \left(\frac{1}{\beta} + \frac{1}{\beta + 1} - \frac{1}{\gamma} - \frac{1}{\gamma + 1} - 1 - \frac{1}{2}\right)\frac{\beta(\beta + 1)}{\gamma(\gamma + 1)}\frac{z^2}{2!} + \cdots,$$

while $S(\beta, \gamma, z)$ reduces for the particular value $\gamma = 2$ to the single term

$$S(\beta, 2, z) = 1/(\beta - 1)z.$$

In the functional notation used by Erdélyi and collaborators [2, p. 261],

$$Y_{02}(\beta, 2, z) = -\Gamma(\beta - 1)\Psi(\beta, 2; z),$$

and in the functional notation used by Miss Slater [3, p. 8]

$$Y_{02}(\beta, 2, z) = -\Gamma(\beta - 1)U(\beta; 2; z).$$

The reviewer's confirmation of the above statement would have been easier if the power series for $\Psi_1(\beta, x)$ and $\Psi_2(\beta, x)$ given by the authors on page x had not been wrong. It would follow from equations (23) and (24) that $a_1(\beta)$ and $b_1(\beta)$ are coefficients of $\frac{1}{2}x$ in the expansions of Ψ_1 and Ψ_2 respectively, but the explicit expressions for $a_1(\beta)$ and $b_1(\beta)$ given in equations (28) and (27) are actually coefficients of x, as may be verified from the tables.

Table I (pp. 2–103) lists 8S values of $\Psi_1(\beta, x)$, $\Psi_2(\beta, x)$ and their first x-derivatives, along with second differences of all four functions with respect to both β and x, for $\beta = 3(0.02)4$, $x = 0(0.05)2.50$.

Table II (pp. 106–269) lists 8S values of the same four functions, again with second differences in both arguments, for $\beta = 3(0.02)4$, $x = 2.5(0.1)10$.

Table III (pp. 272–313) lists 8S values of the products of the same four functions by $A(\beta, x) = (2x)^{3/2-\beta}\,\Gamma(\beta)$, with second differences in both arguments, for $\beta = 3(0.05)4$, $x = 10(0.1)15$. In the sub-title on p. 271, for ρ read β.

Table IV (pp. 316–318) lists 9S values of $A(\beta, x)$, without differences, for $\beta = 3(0.05)4$, $x = 10(0.1)15$.

There is also (pp. 320–321) an 8D table of Everett interpolation coefficients, without differences, at interval 0.001.

Alan Fletcher

1. J. C. P. Miller, "Note on the general solution of the confluent hypergeometric equation," *MTAC.*, v. 11, 1957, pp. 97–99.
2. A. Erdélyi, W. Magnus, F. Oberhettinger & F. G. Tricomi, *Higher Transcendental Functions*, Vol. 1, McGraw-Hill, New York, 1953.
3. Lucy J. Slater, *Confluent Hypergeometric Functions*, Cambridge University Press, Cambridge, 1960.

27[I, M].—J. E. Kilpatrick, Shigetoshi Katsura & Yuji Inoue, *Tables of Integrals of Products of Bessel Functions*, Rice University, Houston, Texas and Tôhoku University, Sendai, Japan, 1966, ms. of 55 typewritten sheets deposited in the UMT file.

This unpublished report tabulates the integral

$$A \int_0^\infty t^\alpha J_{3/2+n_1}(at) J_{3/2+n_2}(bt) J_{3/2+n_3}(ct) f(t)\, dt$$

for the following cases: (1) $A = 4(2\pi)^{1/2}$, $\alpha = -5/2$, $f(t) = 1$, $a = b = c = 1$' and n_i are nonnegative integers $\leqq 20$ such that $n_1 + n_2 + n_3$ is even; (2) $A = 2\pi$, $\alpha = -4$, $f(t) = J_{3/2+n_4}(t)$, $a = b = c = 1$, and n_i are nonnegative integers $\leqq 10$ such that $n_1 + n_2 + n_3 + n_4$ is even; (3) same as the case (1) except that a, b, and c equal 1 or 2, and $n_i \leqq 16$.

Although the tabulated data are given to 16S (in floating-point form), they are generally not that accurate. A short table of the estimated accuracy (6 to 14S), which depends on the maximum value of the integers n_i, is given on p. 3. For some entries the exact value of the integral, as a rational number or as a rational multiple of $\sqrt{2}$, is also given (pp. 10, 27, and 55).

The integrals were evaluated by transforming them into Mellin-Barnes integrals and then applying the calculus of residues. As a by-product of these calculations the authors include a 16S table of $\ln\,[(-1)^s/(-s)!]$ for $s = -25(1)0$ and of $\ln\,\Gamma(s)$ for $s = -24.5(1)0.5(0.5)45$. A spot check revealed that several entries are accurate to only 14S.

Integrals of the type evaluated in this report have also been considered by this reviewer [1].

Y. L. L.

1. Y. L. Luke, *Integrals of Bessel Functions*, McGraw-Hill, New York, 1962, pp. 331–332.

28[K, L].—L. S. Bark, L. N. Bol'shev, P. I. Kuznetsov & A. P. Cherenkov, *Tablitsy raspredeleniia Releia-Raisa* (*Tables of the Rayleigh-Rice Distribution*), Computation Center of the Academy of Sciences of the USSR, Moscow, 1964, xxviii + 246 pp., 27 cm. Price 2.80 rubles.

This volume consists of seven tables. The first five are devoted to data permitting the determination of the function

$$Q(u, v) = \int_u^\infty \rho \exp[-(v^2 + \rho^2)/2] I_0(v\rho)\, d\rho$$

to 6D, by interpolation, for all nonnegative real values of u and v. The remaining two tables represent condensed versions of available tables [1], [2], respectively, of the inverse function $u = u(Q, v)$ and of the function

$$V(K, c) = \frac{1}{c}\int_0^K e^{-B\rho^2/2}\, I_0(A\rho^2/2)\rho\, d\rho = Q(u, v) - Q(v, u),$$

where $A = (c^{-2} - 1)/2$, $B = (c^{-2} + 1)/2$, $u = K(c^{-1} - 1)/2$, $v = K(c^{-1} + 1)/2$. The authors include them for the sake of completeness.

A 28-page introduction contains comments on the various areas in which $Q(u, v)$ occurs, such as statistics, probability, heat conduction, fluid dynamics, chemical and phase decomposition, and information theory. The contents of the tables and the methods used in their computation are described, followed by a discussion of interpolation procedures, accompanied by illustrative examples. Further details are given in another review of these tables (*Math Reviews*, v. 31, 1966, pp. 750–751, #4142).

Table I consists of 6D values of $Q(u, v)$ and its second central difference for $u = 0(0.02)$ max 7.84, $v = 0(0.02)3$. About 80 per cent of the entire volume is taken up by this table.

Table II contains up to 4D values of an auxiliary function $R(q, \epsilon)$ used to evaluate $Q(u, v)$ for $u \geqq v > 3$. It is tabulated for $q = 0(0.0001)0.001(0.001)$-$0.1(0.01)0.57$, $\epsilon = 0(0.005)0.1$, and is defined by the relations $Q(u, v) = q - R(q, \epsilon)$, $q = 1 - \Phi(w)$, $w = u - v - (2v)^{-1}$, $\epsilon = (1 + v^2)^{-1}$. Here $\Phi(w)$ represents the normal distribution function, which is given to 7D in Table IV for argument y over the range $y = 0(0.001)3(0.005)4(0.01)5$.

Table III gives 7D values of $e^{-x} I_0(x)$ for $x = 0(0.001)3(0.01)15(0.1)24.9$; for $x \geqq 25$ the function $L(y) = \exp(-y^2)\, I_0(y^{-2})$ is tabulated to 7D for $y = 0(0.001)0.2$, where $L(x^{-1/2}) = e^{-x} I_0(x)$. The authors advocate use of the relation $Q(u, v) = 1 - Q(v, u) + Q(v - u, 0)\, e^{-uv}\, I_0(u, v)$, where $Q(v, u)$ is found from Table II if $v > u > 3$ or from Table I if $0 \leqq u \leqq 3$ and $v > 3$.

Table V contains 5D values of $\frac{1}{2}\theta(1 - \theta)$ for $\theta = 0(0.001)1$, which facilitates the application of Newton's quadratic interpolation formula to Tables I and II.

This reviewer has checked numerous values selected at random in Table I. No significant errors were found, although many of the values examined erred by about a unit in the final decimal place. The authors' claim that their Table I is the first of its kind is not correct. They are apparently unaware of other tables of $Q(u, v)$, which are not easily accessible. Prior to 1950 the Rand Corporation and the National Bureau of Standards Institute for Numerical Analysis in Los Angeles jointly prepared such a table. It gives 6D values of $Q(u, v)$ for v extending to 24.9; however, the intervals in the arguments are 0.1 and 0.05, as compared to 0.02 in the present table.

All the values in Tables III, VI, and VII were checked by the reviewer. Table III contains two entries that are in error by a unit in the last place: at $x = 0.039$ *read* 0.9621164, and at $x = 9.69$ *read* 0.1299204. Table VII, which is taken directly from [2] (cited as reference (30) in the introduction) contains a single error: at $k = 1.2$, $c = 0.4$ *read* 0.7358558. Table VI is free from error. The remaining tables were not checked.

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1. A. R. Di Donato & M. P. Jarnagin, "A method for computing the circular coverage function," *Math. Comp.*, v. 16, 1963, pp. 347–355.
2. H. L. Harter, "Circular error probabilities," *J. Amer. Statist. Assoc.*, v. 55, 1960, pp. 723–731.

29[K, X].—F. N. David, M. G. Kendall & D. E. Barton, *Symmetric Function and Allied Tables*, Cambridge University Press, Cambridge, England, 1966, x + 278 pp., 29 cm. Price $13.50.

This elaborate, attractively printed set of tables is accompanied by a detailed introduction of 63 pages, which constitutes a self-contained treatment of symmetric functions and their applications in statistics.

The senior author has written a preface giving some of the historical developments in combinatorial algebra and outlining the statistical uses of the tables. He also states there that all the tables were calculated anew and were checked against such similar tables as then existed.

The introduction is divided into nine parts, with the respective headings: Symmetric Functions; Moments, Cumulants and k-Functions; Sampling Cumulants of k-Statistics and Moments of Moments; Partitions; Quantities Based on the First n Natural Numbers; "Runs" Distributions; Randomization Distributions; Tables for the Solution of the Exponential Equations $\exp(-a) + ka = 1$, $\exp a - a/(1 - p) = 1$; and Partition Coefficients for the Inversion of Functions.

The 49 major tables in this collection are arranged according to the relevant parts of the introduction, and cross references thereto are given in the table of contents. At the end of the introduction there appears a list of 48 references; this is augmented at the end of the tables by a supplementary bibliography of 59 publications intended for those table-users who might desire to read more deeply in one or more of the areas covered by these tables.

The entries in most of the tables appear exactly as integers; however, Table 5.4.2, Difference of reciprocals of unity (decimals), gives 10D values of $\Delta^n(1/1^r)$ for $r = 1(1)20$, $n = 1(1)20$, while Tables 8.1 and 8.2 give 7D approximations to the roots of the equations $\exp(-a) + ka = 1$ $(0 < k < 1)$ and $\exp(b) - b/(1 - p) = 1$ $(0 < p < 1)$, respectively, for $k = 0.050\,(0.001)1$ and $p = 0(0.001)\text{-}0.999$. The authors illustrate the use of these roots in obtaining approximations to distributions outside the scope and range of the tables associated with the distribution of runs and the randomization distributions.

The multiplicity of tables represented in this book generally precludes their detailed description here or even their enumeration. It must suffice to state this definitive compilation should be accessible to statisticians and to others working in combinatorial analysis and its applications.

J. W. W.

30[K, X].—PAUL A. MEYER, *Probability and Potentials*, Blaisdell Publishing Company, Waltham, Mass., 1966, xiii + 266 pp., 26 cm. Price $12.50.

Potential theory is one of the more interesting branches of modern mathematics, the main reason being that it has so many useful connections with other branches of mathematics. The application of potential theory to the theory of functions of a complex variable are well known; less well known perhaps are the connections between potential theory and modern probability theory, in particular the theory of Markov processes. In recent years much research has been done in this area and Meyer's book is an attempt to give a systematic account of the probabilistic and analytical techniques that are used in these researches. Before proceeding to discuss the book in more detail I should like to mention some of the more interesting results that have so far been obtained; the reader of this review will then be able to better appreciate the structure and contents of this book.

The first result I should like to mention is due to S. Kakutani (1944). It may rightly be considered as the starting point for all subsequent researches in this area. Kakutani considered the following problem: Let G be a bounded two-dimensional region in the plane with boundary ∂G and let A be a measurable subset of ∂G. Let x be an interior point of G and denote by $\{W(t), t \geqq 0\}$ the two-dimensional Brownian motion process starting at x. Denote by τ_x the first passage time of the Brownian motion process through ∂G (note that τ_x is a random variable, indeed it is what probabilists call a "stopping time"). Kakutani showed that $\Pr\{W(\tau_x) \in A\} = \mu(x, A)$, where $\mu(x, A)$ denotes the harmonic measure of the set A relative to the point x and the region G. More generally, one may solve the Dirichlet problem for the region G with continuous boundary values f in the following "probabilistic way": $u(x) = E\{f(W(\tau_x))\}$, where E denotes the expected value and $u(x)$ is the classical solution to the Dirichlet problem. This probabilistic solution to the Dirichlet problem has been exploited by Doob who, using martingale methods, obtained new results on the boundary behavior of harmonic and superharmonic functions. Hunt, in a series of papers that appeared in the *Illinois J. Math.* (1957–1958) put many of these results in the more general context of Markov processes and their "potential theories".

In all these researches there has been a mutually beneficial interplay between probability and potential theory and what Meyer's book does is to bring together, for the first time, the various techniques that are used in these studies. The book contains 11 chapters divided in the following way: The first three chapters are devoted to probability theory and some of the finer points of measure theory, e.g. Choquet's theory of capacities. The fourth chapter is entitled stochastic processes but as no examples of the concepts discussed are given, its value to an analyst is somewhat doubtful. The next three chapters are devoted to the theory of martingales and includes the author's proof of the existence of a Doob decomposition for continuous parameter martingales satisfying certain conditions. The author does not however discuss the applications of these techniques to Doob's "fine limit theorems". Chapters 9 and 10 discuss those topics in the theory of semigroups which are needed for an exposition of Hunt's approach to potential theory. The book ends with a chapter devoted to Choquet's representation theorem and some applications. Thus, as can be seen from the table of contents, one of the author's main purposes is to give an account of those methods of probability theory which could prove to be of great service to analysts. The reviewer feels that the author should have included for these analysts a section on the potential theory of the Brownian motion process, as this would have illustrated in a concrete and nontrivial way many of the abstract concepts he has defined. For the specialist in this field, on the other hand, this well written book, with its careful and complete discussion of new and important results, some of them due to the author himself, is highly recommended.

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31[K, X].—ARISTARKH KONSTANTINOVICH MITROPOL'SKIĬ, *Correlation Equations for Statistical Computations*, Authorized translation from the Russian by Edwin S. Spiegelthal, Consultants Bureau Enterprises, Inc., New York, 1966, viii + 103 pp., 28 cm. Price $9.50.

In this book, the author describes various methods for performing calculations associated with correlational analysis. Most of these methods have little value when the calculations are performed on an automatic computer. Matrix notation is not used, and the notation of the author is quite awkward.

One can only be more overwhelmed by the price of this book than by the fact that it was translated at all.

G. H. G.

32[K, X].—E. S. PEARSON & H. O. HARTLEY, Editors, *Biometrika Tables for Statisticians*, Vol. I, Third Edition, Cambridge University Press, New York, 1966, xvi + 264 pp., 29 cm. Price $6.50.

This new edition of a standard source of statistical tables is welcomed. The review by Milton Abramowitz (this journal, Volume 9, (1955), 205–211) remains a valid assessment. We now add details to his review to reflect the changes of the new editions.

For both the second and third editions the basic changes were made in the tables, although corresponding changes have been made in the Introduction. In these editions there is an Index of the tables. The following is a list of the changed or new tables. Tables with a number followed by a lower case letter are new in the third edition.

8 Percentage points of the χ^2-distribution.
Removal of cut-off errors in the last figure tabled.

11 Test for comparisons involving two variances which must be separately estimated.
Addition of 2.5 % and 0.5 % critical levels.

16 Percentage points of the B-distribution.
Removal of cut-off errors in the last figure tabled. Addition of 0.25% and 0.1% points.

20 Moment constants of the mean deviation and of the range.
Removal of cut-off errors in the last figure tabled.

22 Percentage points of the distribution of the range.
Removal of cut-off errors in the last figure tabled.

23 Probability integral of the range, W, in normal samples of size n.
Removal of cut-off errors in the last figure tabled.

26 Percentage points of the extreme studentized deviate from sample mean, $(x_n - \bar{x})/s_\nu$ or $(\bar{x} - x_1)/s_\nu$.

In the second edition this table was extended to $n = 12$ and errors were corrected. Formerly lower percentage points were given in this table but now page 50 gives these values for $\nu = 10$ and suggested methods of interpolation. The range of the table is $n = 3(1)10, 12$; $\nu = 10(1)20, 24, 30, 40, 60, 120, \infty$ for the 10%, 2.5%, 0.5% and 0.1% points and $\nu = 5(1)20, 24, 30, 40, 60, 120, \infty$ for the 5% and 1% points. 3D for all except the 0.1% points which are 2D.

26a Percentage points of $(x_n - \bar{x})/S$ or $(\bar{x} - x_1)/S$ (where $S^2 = \sum_{i=1}^{n} (x_i - \bar{x})^2 + \nu s_\nu^2$).

Note: $S^2 = \sum_1^n (x_i - \bar{x})^2 + \sum_1^{\nu+1} (y_i - \bar{y})^2$, with y_i independent of x_i.

The range of the table is $n = 3(1)10, 12, 15, 20$; $\nu = 0(1)10, 12, 15, 20, 24, 30, 40, 50$ and the 5% and 1% points. The 5% points are 3D (except $n = 3$, $\nu = 0$ is 4D) and the 1% points are 4D.

26b Percentage points of max $| x_i - \bar{x} | /S$ (where $S^2 = \sum_{i=1}^{n} (x_i - \bar{x})^2 + \nu s_\nu^2$)

Note: $S^2 = \sum_1^n (x_i - \bar{x})^2 + \sum_1^{\nu+1} (y_i - \bar{y})^2$, with y_i independent of x_i.

The range is as in 26a; all values are 3D except those for $n = 3$, $\nu = 0$ are 4D.

29 Percentage points of the studentized range, $q = (x_n - x_1)/s_\nu$.

The lower percent points are no longer included. The 10% points have been included and the accuracy of the 5% and 1% points has been improved. All values are either 2D or 4S.

29a Two-sample analogue of Student's test. Values of $u = | \bar{x}_1 - \bar{x}_2 | / (w_1 + w_2)$ exceeded with probability α.

Note: $\bar{x}_1, \bar{x}_2$ are the means and w_1, w_2 the ranges in two independent samples containing n_1, n_2 observations, respectively.

The range of the table is $n_1 = 2(1)20$, $n_2 = n_1(1)20$ for 10%, 5%, 2% and 1% points. 3D accuracy.

29b Upper percentage points of the ratio of two independent ranges, $F' = w_1/w_2$.

Note: w_1, w_2 are the ranges in two independent samples containing n_1, n_2 observations, respectively.

The range of the table is n_1 and $n_2 = 2(1)15$ for 50%, 25%, 10%, 5%, 2.5%, 1%, 0.5% and 0.1% points. 4S throughout.

29c Percentage points of the ratio of range to standard deviation, w/s, where w and s are derived from the same sample of n observations.

The range of the table is $n = 3(1)20(5)100(50)200$, 500, 1000 for upper and lower 0.0 %, 0.5 %, 1.0 %, 2.5 %, 5.0 % and 10.0 % points. At least 3S.

31a Percentage points of the ratio $s_{\max}^2/\sum_{t=1}^k s_t^2$.

Note: $s_{\max}^2$ is the largest in a set of k independent mean squares, s_t^2, each based on ν degrees of freedom.

The range of the table is $k = 2(1)10, 12, 15, 20$; $\nu = 1(1)10, 16, 36, 144, \infty$ for the 5 % and 1 % points. Values to 4D.

31b Percentage points of the ratio $w_{\max}/\sum_{t=1}^k w_t$. Upper 5 % points.

Note: $w_{\max}$ is the largest in a set of k independent ranges, w_t, each derived from a sample of n observations.

The range of the table is $k = 2(1)10, 12, 15, 20$; $n = 2(1)10$ and the accuracy is 3D.

34c Tests for departure from normality. Percentage points of the distribution of $b_2 = m_4/m_2^2$.

The values of $n = 50(25)\ 150(50)\ 700(100)\ 1000(200)\ 2000(500)\ 5000$.

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33[L].—Henry E. Fettis & James C. Caslin, *Ten Place Tables of the Jacobian Elliptic Functions* II. *Theta Functions and Conversion Factors*, Report ARL 66-0069, Aerospace Research Laboratories, Office of Aerospace Research, United States Air Force, Wright-Patterson Air Force Base, Ohio, April 1966, iv + 109 pp., 28 cm. Copies obtainable upon request from the Defense Documentation Center, Cameron Station, Alexandria, Virginia.

This report concludes a compilation in two parts of new tables by the authors relating to the Jacobian elliptic functions. In this second part we find a 10D table, without differences, of the ratio $\Theta(u, k)/\Theta(0, k)$, in the original notation of Jacobi, for $k^2 = 0.01(0.01)1$ and $u = 0(0.01)N$, where N ranges from 1.60 to 4.00 with increasing values of k^2 in a manner too complicated for simple description. This table was calculated on an IBM 1620 system by use of Gauss's transformation, as described in the introduction. A supplementary two-page table gives 10D conversion factors (involving the theta functions of zero) that permit the calculation of the remaining three theta functions (expressed in Jacobi's earlier notation) from the present tabular data in conjunction with the values of the Jacobi elliptic functions tabulated in the first report [1]. The basic formulas are given for such calculations.

The abbreviated introduction to the present tables contains no discussion of the problem of interpolation therein nor of the precautions, if any, that were taken to insure accuracy in the printed data.

In the introductory text this reviewer detected five typographical errors in addition to three noted by the authors on an insert sheet. Furthermore, the exponent was advertently omitted from k^2 in the tabular headings on pages 18, 19, and 107. It is regrettable that such careless errors should have been allowed to mar this unique table.

J. W. W.

1. HENRY E. FETTIS & JAMES C. CASLIN, *Ten Place Tables of the Jacobi Elliptic Functions*, Report ARL 65–180 Part 1, Aerospace Research Laboratories, Wright-Patterson Air Force Base, Ohio, September 1965.

34[L, M].—HENRY E. FETTIS & JAMES C. CASLIN, *Elliptic Integral of the Third Kind*, Applied Mathematics Research Laboratory, Aerospace Research Laboratories, Wright-Patterson Air Force Base, Ohio, two manuscript volumes, each of 180 computer sheets, deposited in the UMT file.

The authors have herein tabulated to 10D the elliptic integral of the third kind in Legendre's form for $\theta = 0°(1°)90°$, and arcsin $k = 0°(1°)90°$, $\alpha^2 = 0.1(0.1)1$, except that when $\alpha^2 = 1$, the upper limit of the argument θ is 89°. This voluminous table is intended as a companion to the authors' manuscript 10D tables of the elliptic integrals of the first and second kinds [1].

The published 10D tables [2] of elliptic integrals by the same authors employ the modulus k or its square rather than the modular angle as one argument. However, it is possible to compare a few of the entries in those tables with corresponding entries in the tables under review; in particular, those entries corresponding to the erroneous entries [3] in the published tables are thus found to be given correctly.

The authors have informed this reviewer that the present tables were calculated on an IBM 7094 system by means of a program adapted from that used on an IBM 1620 in preparing their previous tables of elliptic integrals.

The impressive series of 10D tables of elliptic integrals and elliptic functions by the present authors reflect the continually increasing capabilities of electric digital computer systems used in such calculations.

J.W.W.

1. HENRY E. FETTIS & JAMES C. CASLIN, *Elliptic Integral of the First Kind and Elliptic Integral of the Second Kind*, ms. tables deposited in the UMT file. (See *Math. Comp.*, v. 20, 1966, pp. 626, RMT **99.**)
2. HENRY E. FETTIS & JAMES C. CASLIN, *Tables of Elliptic Integrals of the First, Second and Third Kind*, Applied Mathematics Research Laboratory Report ARL 64-232, Aerospace Research Laboratories, Wright-Patterson Air Force Base, Ohio, 1964. (See *Math. Comp.*, v. 19, 1965, p. 509, RMT **81.**)
3. *Math. Comp.*, v. 20, 1966, p. 639, MTE **398.**

35[L, X].—V. S. AIZENSHTADT, V. I. KRYLOV & A. S. METEL'SKIĬ, *Tables of Laguerre Polynomials and Functions*, translated by Prasenjit Basu, Pergamon Press, Inc., Long Island, New York, 1966, xv + 150 pp., 24 cm. Price $8.00.

This is an English translation of some work originally published by the Academy of Sciences of the BSSR, Minsk, in 1963. It is closely related to some work done by the same authors in 1962 and previously reviewed in these annals, see *Math. Comp.*, v. 17, 1963, p. 93.

Let $L_n^s(x)$ denote the generalized Laguerre polynomial which we express in hypergeometric form as $L_n^s(x) = (s + 1)_n\, {}_1F_1(-n; s + 1; x)$. Note that the polynomials are usually normalized by the factor $(s + 1)_n/n!$ The related function

$\psi_n^s(x) = e^{-x/2}x^{s/2} L_n^s(x)$ is called a Laguerre function. In some circles use of the nomenclature Laguerre function may be misleading as this usually refers to $L_n^s(x)$ where n is an arbitrary parameter.

The following are tabulated:

$$L_n^{(s)}(x), \psi_n^{(s)}(x) \quad \text{for} \quad n = 2(1)7, \qquad s = 0(0.1)1.0,$$

$$x = 0(0.1)10.0(0.2)30.0, \text{ 6S.}$$

Coefficients of the polynomials $L_n^{(s)}(x)$ for $n = 2(1)10$, $s = 0(0.05)1.0$, to 8S. (Note that these coefficients are not always exact.) Zeroes of the polynomials for $n = 2(1)10$, $s = 0(0.05)1.0$, 8S.

See the references [1]–[5] below and the sources they quote for tables relating to the abscissae and weights for numerical evaluation of $\int_0^\infty x^s e^{-x} f(x)dx$.

Y. L. L.

1. M. Abramowitz & I. Stegun, *Handbook of Mathematical Functions with Formulas, Graphs and Mathematical Tables*, Applied Mathematics Series No. 55, U. S. Government Printing Office, 1964. (See *Math. Comp.*, v. 19, 1965, pp. 147–149.)
2. P. Concus, D. Cossat, G. Jaehnig & E. Melby, "Tables for the evaluation of $\int_0^\infty x^\beta e^{-x} f(x)dx$ by Gauss-Laguerre quadrature, *Math. Comp.*, v. 17, 1963, pp. 245–256.
3. P. Concus, "Additional tables for the evaluation of $\int_0^\infty x^\beta e^{-x} f(x)dx$ by Gauss-Laguerre quadrature," *Math. Comp.*, v. 18, 1964, p. 523.
4. T. S. Shao, T. C. Chen & R. M. Frank, "Tables of zeros and Gaussian weights of certain associated Laguerre polynomials and the related generalized Hermite polynomials," *Math. Comp.*, v. 18, 1964, pp. 598–616.
5. A. H. Stroud & D. Secrest, *Gaussian Quadrature Formulas*, Prentice-Hall, Englewood Cliffs, N. J., 1966. (See *Math. Comp*, v. 21, 1967, pp. 125-126, RMT **14.**)

36[L, X].—V. A. Ditkin, Editor, *Tables of Incomplete Cylindrical Functions*, Computing Center of the Academy of Sciences of the USSR, Moscow, 1966, xxix + 320 pp., 27 cm.

Consider the functions

$$\text{(1)} \qquad \frac{1}{2} J_\nu(\alpha, \rho) = \frac{\rho^\nu}{A_\nu} \int_0^\alpha \cos(\rho \cos u) \sin^{2\nu} u \, du,$$

$$\text{(2)} \qquad \frac{1}{2} H_\nu(\alpha, \rho) = \frac{\rho^\nu}{A_\nu} \int_0^\alpha \sin(\rho \cos u) \sin^{2\nu} u \, du,$$

$$\text{(3)} \qquad F_\nu^+(\alpha, \rho) = \frac{\rho^\nu}{A_\nu} \int_0^\alpha e^{\rho \cos u} \sin^{2\nu} u \, du,$$

$$\text{(4)} \qquad F_\nu^-(\alpha, \rho) = \frac{\rho^\nu}{A_\nu} \int_0^\alpha e^{-\rho \cos u} \sin^{2\nu} u \, du,$$
$$A_\nu = 2^\nu \Gamma(\nu + \tfrac{1}{2}) \Gamma(\tfrac{1}{2}).$$

Note that

$$J_\nu(\pi/2, \rho) = J_\nu(\rho), \qquad H_\nu(\pi/2, \rho) = H_\nu(\rho),$$
$$F_n^\pm(\pi/2, \rho) = \tfrac{1}{2}[I_n(\rho) \pm L_n(\rho)],$$

where $J_\nu(\rho)$ and $I_\nu(\rho)$ are the Bessel and modified Bessel functions of the first kind, respectively, and $H_\nu(\rho)$ and $L_\nu(\rho)$ are the Struve and modified Struve functions respectively.

The volume gives tables of (1) and (2) for

$$\nu = 0, 1, \qquad \rho = 0.1(0.1)5.0, \qquad \alpha = 0.01(0.01)1.57, \qquad \pi/2, \text{ 6D}.$$

Items (3) and (4) are also tabulated to 6S for the same ν, ρ and α as above. Reliefs of the functions are presented.

The introduction describes the method of computation and delineates numerous properties of the functions including series expansions, recurrence formulas and relations to other functions. Alternate methods of computation are not fully referenced. For example, the introduction shows how the tables may be used to evaluate the integrals

$$\int_0^\rho e^{-\alpha t} I_0(t)\, dt, \qquad \int_0^\rho e^{-\alpha t} K_0(t)\, dt,$$

$$\int_0^\rho e^{-i\alpha t} J_0(t)\, dt, \qquad \int_0^\rho e^{-i\alpha t} Y_0(t)\, dt,$$

but no mention is made to alternate schemes of computation which appear in the literature, e.g., the reviewer's *Integrals of Bessel Functions*, McGraw-Hill, New York, 1962. (See *Math. Comp.*, v. 17, 1962, 318–320.)

As the computations are for integer values of ν, we would have preferred tables of the functions

$$i_n(\alpha, \rho) = \int_0^\alpha e^{\rho \cos u} \cos nu\, du. \tag{5}$$

Thus, for example,

$$F_0^+(\alpha, \rho) = \frac{1}{\pi} i_0(\alpha, \rho), \qquad F_1^+(\alpha, \rho) = \frac{\rho}{2\pi} [i_2(\alpha, \rho) - i_0(\alpha, \rho)], \quad \text{etc.}$$

An efficient scheme to evaluate (5) would be to expand exp $(\rho \cos u)$ in series of Bessel functions and termwise integrate to get

$$\begin{aligned} i_0(\alpha, \rho) &= \alpha I_0(\rho) + 2 \sum_{m=1}^{\infty} \frac{\sin m\alpha}{m} I_m(\rho), \\ i_n(\alpha, \rho) &= \frac{\sin n\alpha}{n} I_0(\rho) + 2\left(\frac{\alpha}{2} + \frac{\sin 2n\alpha}{4n}\right) I_n(\rho) \\ &\quad + 2 \sum_{m=1; m\neq n}^{\infty} (m^2 - n^2)^{-1}(m \sin m\alpha \cos n\alpha - n \sin n\alpha \cos m\alpha) I_m(\rho), \\ &\qquad n > 0. \end{aligned} \tag{6}$$

The evaluation of (6) is then quite easy on an electronic calculator, since the Bessel functions are readily computed by use of the backward recurrence formula.

Y. L. L.

37[L, X].—H. E. Hunter, D. B. Kirk, T. B. A. Senior & E. R. Wittenberg, *Tables of Prolate Spheroidal Functions for m* = 0, Vols. I & II, College of Engineering, The University of Michigan, under contract with Air Force Cambridge Research Laboratories, Bedford, Mass., April 1965, Report No. AFCRL-65-283(I), Vol. I, 69 + 218 unnumbered pp., Report No. AFCRL-65-283(II), Vol. II, 4 + 348 unnumbered pp., 28 cm. Government agencies or their contractors may obtain copies from Defense Documentation Center (DDC), Cameron Station, Alexandria, Va. All others should apply to Clearinghouse for Federal Scientific and Technical Information (CFSTI), Sills Building, 5285 Port Royal Road, Springfield, Va.

Spheroidal wave functions result when the scalar Helmholtz equation is separated in spheroidal coordinates, either prolate or oblate. The angular prolate spheroidal wave functions, for example, satisfy a differential equation of the form

$$\frac{d}{dz}\left[(1 - z^2)\frac{du}{dz}\right] + \left(\lambda_{mn} - c^2z^2 - \frac{m^2}{1 - z^2}\right)u = 0.$$

The solutions of this equation are much more complicated than either Bessel or Legendre functions, in which, in fact, series solutions of the spheroidal functions are most often expanded. The complexity arises from the fact that the spheroidal differential equation has an irregular singular point at ∞ and two regular ones at $z = \pm 1$, in contrast to the three regular ones of the Legendre equation and to the one regular and one irregular singularity of the Bessel equation.

The construction of tables of spheroidal wave functions involves the calculation of the eigenvalues λ_{mn} of the differential equation, that is, those values of λ for which there are solutions that are finite at $z = \pm 1$, and the calculation of the coefficients in expansions in terms of either Legendre or spherical Bessel functions. In the past, such calculations have been, for the most part, sporadic and in many cases not very accurate.

In Volume I the eigenvalues $\lambda_{on}(c)$ and normalization constants $N_{on}(c)$ are tabulated for $c = 0.1(0.1)10.0$ and n ranging from 0 up to a maximum of 20. The $m = 0$ radial functions of the first and second kinds, and their first derivatives, are given for the same c and n for $\xi = 1{\cdot}0000500$, $1{\cdot}0050378$, $1{\cdot}0206207$ and $1{\cdot}1547005$, corresponding to prolate spheroids of length-to-width ratios 100:1, 10:1, 5:1 and 2:1 respectively. Values of the $m = 0$ angular functions and their first derivatives are presented in Volume II for $c = 0.1(0.1)10.0$ with n ranging from 0 up to a maximum of 20, and $\eta = 0(0.05)1.0$.

All computations were carried out on The University of Michigan IBM 7090 computer. The program is described in some detail.

These tables should be very useful for the calculation of various acoustical problems that involve prolate spheroids. In electromagnetic theory, however, only rather simple problems can be treated by means of the functions for $m = 0$. In most electromagnetic problems, the functions with $m = 1$ and often higher values of m are required.

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38[L, X].—L. N. Nosova & S. A. Tumarkin, *Tables of Generalized Airy Functions for the Asymptotic Solution of the Differential Equation* $\epsilon(py')' + (q + \epsilon r)\, y = f$, translated by D. E. Brown, Pergamon Press, Inc., Long Island, New York, 1965, xxxiv + 89 pp., 25 cm. Price $12.00.

This is a translation from the Russian originally published in 1961. The original version does not seem to have been previously reviewed in these annals although some of the tables have appeared in another work of the authors [1]. Tumarkin [2] studied asymptotic expressions for the solution of the inhomogeneous differential equation

$$(1) \qquad \epsilon \frac{d}{dx}\left[p(x)\frac{dy}{dx}\right] + [q(x) + \epsilon r(x)]y = f(x)$$

for small ϵ where $q(x)$ has a simple zero at the origin. It is also assumed that $p(x)$ is positive. By a change of variables, namely

$$(2) \qquad y = (p(du/dx))^{1/2}\eta,$$

(1) may be put in the form

$$(3) \qquad \epsilon(d^2\eta/du^2) + u\eta = g,$$

where u is expressed by the asymptotic series

$$(4) \qquad u \sim \sum_{k=0}^{\infty} \epsilon^k u_k(x).$$

To generate the desired solution of (1) it is shown in [2] and in the present volume that it is sufficient to have tables for the solution of

$$(5) \qquad y'' + ty = 1.$$

To describe the functions tabulated, we use the standard notation of [3] and [4]. Solutions of (5) may be taken in the form

$$(6) \qquad e_0(t) = (2\pi/3)\, B_i(-t) + \pi\, T_i(-t) = \pi\, H_i(-t),$$

$$(7) \qquad \tilde{e}_0(t) = (-\pi/3)\, B_i(-t) + \pi\, T_i(-t) = -\pi\, G_i(-t)$$

so that the Airy integral

$$(8) \qquad B_i(-t) = e_0(t) - \tilde{e}_0(t).$$

In the above

$$\pi\, T_i(-t) = \tfrac{1}{2}t^2\, {}_1F_2(1; 4/3, 5/3; -\xi^2/4) = \tfrac{2}{3}t^{1/2} s_{0,1/3}(\xi), \qquad \xi = \tfrac{2}{3}t^{3/2},$$

where $s_{0,1/3}(\xi)$ is a Lommel function, see [4], and the notation $H_i(t)$ and $G_i(t)$ is that introduced by Scorer [5] and used further by Rothman [6, 7]. We also have need for the notation

$$(9) \qquad h_n(t) = \xi^{1/3} H_{1/3}^{(n)}(\xi), \qquad n = 1, 2,$$

where $H_{1/3}^{(n)}(z)$ is the usual notation for the Hankel functions. Also

$$(10) \qquad e_1(t) = 1 - te_0(t), \qquad e_2(t) = -te_1(t)/2.$$

It is useful to record the integral representations

$$(11) \quad e_0(t) = \int_0^\infty \exp(-tx - x^3/3)\, dx, \qquad e_1(t) = \int_0^\infty x^2 \exp(-tx - x^3/3)\, dx.$$

Finally, following [4], we can show that

$$(12) \qquad \int_0^t B_i(-x)\,dx = e_0(t)B_i'(-t) - e_0'(t)B_i(-t),$$

$$(13) \qquad \int_0^t A_i(-x)\,dx = \frac{2}{3} + e_0(t)A_i'(-t) - e_0'(t)A_i(-t).$$

A description of the Tables follows.

TABLE 1.

$$t = is, \qquad e_n(is) = R(e_n) + iI(e_n)$$

$$e_n' = de_n/ds = R(e_n') + iI(e_n')$$

$$n = 0:\ 0 \leqq s \leqq 8,\ 7d; \qquad 8 \leqq s \leqq 9,\ 9D.$$

$$n = 1:\ 0 \leqq s \leqq 6,\ 7d; \qquad 6 \leqq s \leqq 9,\ 7D.$$

$$n = 2:\ 0 \leqq s \leqq 3,\ 7d; \qquad 3 \leqq s \leqq 7,\ 6D, \qquad 7 \leqq s \leqq 9,\ 5D.$$

Note the convention $e_n' = de_n/ds$ so that when $t = is$, $de_n/dt = -i\,de_n/ds$. This practice is followed throughout.

TABLE 2.

$$e_n(t),\ e_n'(t)$$

$$n = 0, 1, 2, \qquad 0 \leqq t \leqq 10, \qquad -1 \leqq t \leqq 0,\ 7D.$$

TABLE 3.

$$\tilde{e}_n(t),\ \tilde{e}_n'(t)$$

$$n = 0:\ -10 \leqq t \leqq 0, \qquad 0 \leqq t \leqq 1,\ 7D.$$

$$n = 1:\ -6 \leqq t \leqq 0, \qquad 0 \leqq t \leqq 1,\ 7D; \qquad -10 \leqq t \leqq -6,\ 6D.$$

$$n = 2:\ -3 \leqq t \leqq 0, \qquad 0 \leqq t \leqq 1,\ 7D;$$

$$-8 \leqq t \leqq -3,\ 6D; \qquad -10 \leqq t \leqq -8,\ 5D.$$

Note on p. 63 for $e_n(t)$ read $\tilde{e}_n(t)$.

TABLE 4.

$$t = is, \qquad h_n = R(h_n) + iI(h_n),$$

$$h_n' = dh_n/ds = R(h_n') + iI(h_n'),$$

$$n = 1, 2, \qquad s = 0(0.1)6,\ 6D.$$

These are rounded from the 8D Harvard tables [8]. In the latter $h_n' = dh_n/dt$.

Comparison of the present tables with those in [5]–[8] shows that essentially only Table 1 is new. See [9] for tables of (12–13).

The introduction describes the method of calculation, checks used, and ranges for which the tables may be linearly interpolated.

Y. L. L.

1. L. N. Osipova & S. A. Tumarkin, *Tables for the Calculation of Toroidal Shells*, Akad. Nauk SSSR, Moscow, 1963. (See *Math. Comp.*, v. 18, 1964, pp. 677–678.)
2. S. A. Tumarkin, "Asymptotic solution of a linear nonhomogeneous second order differential equation with a transition point and its application to the computations of toroidal shells and propeller blades," *Prikl. Mat. Meh.*, v. 23, 1959, pp. 1083–1094; English transl., *J. Appl. Math. Mech.*, v. 23, 1959, pp. 1549–1565.
3. Milton Abramowitz & Irene A. Stegun, Editors, *Handbook of Mathematical Functions with Formulas, Graphs and Mathematical Tables*, Applied Mathematics Series No. 55, U. S. Government Printing Office, Washington, D. C., 1964. (See *Math. Comp.*, v. 19, 1965, pp. 147–149.)
4. Y. L. Luke, *Integrals of Bessel Functions*, McGraw-Hill, New York, 1962. (See *Math. Comp.*, v. 17, 1963, p. 318–320.)
5. R. S. Scorer, "Numerical evaluation of integrals in the form $I = \int_{x_1}^{x_2} f(x)e^{i\varphi(x)}dx$ and the tabulation of the function $G_i(z) = 1/\pi \int_0^\infty \sin(uz + u^3/3)\, du$", *Quart. J. Mech. Appl. Math.*, v. 3, 1950, pp. 107–112. (See *MTAC*, v. 4, 1950, p. 215.)
6. M. Rothman, "The problem of an infinite plate under an inclined loading with tables of the integrals of $A_i(\pm x)$ and $B_i(\pm x)$," *Quart. J. Mech. Appl. Math.*, v. 7, 1954, pp. 1–7. (See *MTAC*, v. 8, 1954, p. 162.)
7. M. Rothman, "Tables of the integrals and differential coefficients of $G_i(x)$ and $H_i(-x)$," *Quart. J. Mech. Appl. Math.*, v. 7, 1954, p. 379–384. (See *MTAC*, v. 9, 1955, pp. 77–78. On the latter pages are descriptions of further tables related to Airy functions and their integrals. See also [3].)
8. Harvard University Computation Laboratory, Annals, Vol. 2, *Tables of the Modified Hankel Functions of Order One-Third and Their Derivatives*, Harvard Univ. Press, Cambridge, Mass., 1945. (See *MTAC*, v. 2, 1946, pp. 176–177.)
9. K. Singh, J. F. Lumley & R. Betchov, *Modified Hankel Functions and their Integrals to Argument* 10, Engineering Research Bulletin B-87, The Pennsylvania State University, University Park, Penn., 1963. (See *Math. Comp.*, v. 18, 1964, p. 522.)

39[L, X].—F. Tölke, *Praktische Funktionenlehre*, Band II: *Theta-Funktionen und spezielle Weierstrasssche Funktionen*, Springer-Verlag, Berlin, 1966, vii + 248 pp., 28 cm. Price DM 84.

This is the first part of a monumental work on theta functions and elliptic functions. It contains an incredible wealth of formulas and theorems involving the elliptic theta functions and those of Weierstrass' elliptic functions which have periods 1 and $i\kappa$ or 1 and $\frac{1}{2} + i\kappa/2$. Four future volumes will treat the Jacobi elliptic functions, special Weierstrass Zeta and Sigma functions, elliptic integrals and Jacobi elliptic functions in the complex domain, general Weierstrass elliptic functions and derivatives with respect to the parameter, integrals of Theta functions and bilinear expansions. The final volume will contain numerical tables.

The present volume consists of four chapters and altogether 107 sections. Everything that can be expected to be helpful to the applied mathematician is derived briefly and stated in full detail, including approximation formulas for the parameter functions which are correct up to the fifth decimal. Partial differential equations, derivatives, values for specialized arguments and addition theorems for the Theta functions are given in great detail. Many expansions are given with a large number of numerical coefficients. The Weierstrass elliptic integrals in normal form (of the 2nd and 3rd kinds) are expressed in terms of the logarithms of the Theta functions. There are 765 numbered formulas, but many of them are groups of formulas. To the best of the knowledge of the reviewer, nothing comparable in completeness and abundance of details exists in the literature.

There is no index, but the table of contents provides a very good orientation. The list of references consists of 147 entries; however, the *Handbook of Elliptic Integrals for Engineers and Physicists* by P. F. Byrd and M. D. Friedman, Springer, Berlin, 1954, is missing, probably because it has little in common with the present work.

This is certainly no textbook, but it is a very valuable source of information for anyone who needs detailed knowledge of these functions for problems in physics and technology. (The algebraic aspects, e.g. the transcendental solution of the generic equation of fifth degree in terms of Theta functions seem to have been excluded entirely.) However, there is one more aspect of the book which should be mentioned since it may appeal to a large number of mathematicians of all professional denominations. The behavior of several functions, in particular of Weierstrass' $\wp$-function in the complex domain is illustrated by numerous magnificent drawings. Of these, the figures on pp. 168, 169, 175, 177 and 199 deserve special mention.

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40[P, T, V].—JERRY DEAN PEARSON & ROBERT C. FELLINGER, *Thermodynamic Properties of Combustion Gases*, The Iowa State University Press, Ames, Iowa, 1965, xv + 213 pp., 24 cm. Price $7.50.

The book is a collection of tables of the equilibrium thermodynamic properties of the products of combustion of a hydrocarbon fuel at high temperatures. The fuel must be of the type $C_K H_{2K}$ where K is any integer.

The tables give the enthalpy, entropy, molecular weight, specific heat ratio, and sonic velocity of the combusion products as a function of the total pressure and temperature of the gas mixture in various percentages for stoichiometric oxygen. The pressures range from .01 to 25 atmospheres and temperatures from 1500°K to 3500°K.

The book should be of use primarily to mechanical or aeronautical engineers interested in combustion problems.

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41[P, W, X, Z].—LÁSZLÓ KALMÁR, Editor, *Colloque Sur les Fondements des Mathématiques, les Machines Mathématiques, et Leurs Applications*, Gauthier-Villars, Paris, 1965, 317 pp., 24 cm. Price $8.00.

This volume contains a selection from the lectures presented at the Colloquium "Foundations of Mathematics, Mathematical Machines and Their Applications", held at Tihany, Hungary, September 11–15, 1962. The papers presented are grouped into seven categories:

1. Foundations of Mathematics and Mathematical Logic. This section contains papers by the following authors: J. Bečvář, A. Church, H. B. Curry, Gy. Graetzer, K. Haertig, H. Hermes, M. Makkai, G. H. Mueller, H. Rasiowa, E.-J. Thiele, V. Vučkovič, S. Watanabe.
2. Abstract Theory of Automata and Computers. Papers by: T. Frey, M. A. Gavrilov, L. Kalmár, R. Péter.
3. Circuit Theory. Papers by: A. Ádám, Ya. Barat, M. Coroi-Nedelcou, I. Fenyoe, H. Rohleder.

4. Mathematical Linguistics and Machine Translation. Papers by: S. Abraham & G. Salapina, M. Bierwisch, L. Deshoe, Gy. Hell, F. Kiefer, R. B. Lees, S. Marcus, P. Sgall, Gy. Sipoeczy.

5. Digital Computers and Programming. Papers by: B. Doemoelki, I. Friš & P. Liebl, M. A. Kartsev & E. B. Glivenko, Z. Pawlak, Gy. Révész.

6. Application of Computers in Economics. Papers by: C. A. Aleskerov, J. L. Destouches, S. Ganczer, I. Kiss, B. Krekó, B. Martos, J. Piehler.

7. Artificial Intelligence, Machine Learning. Papers by: H. L. Gelernter, V. M. Glushkov & A. A. Ctogniĭ.

E. I.

42[P, X].—S. O. Asplund, *Structural Mechanics: Classical and Matrix Methods*, Prentice-Hall, Inc., Englewood Cliffs, N. J., 1966, xiii + 474 pp., 24 cm. Price $14.00.

The purpose of this book is to serve as a text for advanced undergraduates and graduate students in structural mechanics. It treats both determinant and indeterminant structures in great detail. However, the emphasis in this book is somewhat different from the one normally found in a structural mechanics text. The author is very cognizant of the impact of modern high-speed computers on this field and has written his text accordingly. Thus, while classical methods are discussed, matrix methods are emphasized as being the more useful to the practicing engineer.

In many ways this is a remarkable book. It is contemporary and every page reflects the author's familiarity with his subject. Each topic is given a consistently polished development and logic is never sacrificed to intuition. The style of the book may tend to be overly succinct and the notation may cause some difficulty to those familiar with more common notations. However, with these reservations, this is a highly recommended book.

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43[P, X].—A. Halanay, *Differential Equations: Stability, Oscillations, Time Lags*, Academic Press, New York, 1966, xii + 528 pp., 24 cm. Price $19.50.

This monograph is essentially a translation of the original Rumanian edition of 1963. It presents a unified treatment, for ordinary differential equations and functional-differential equations, of those topics in stability theory and the theory of oscillations which have been at the center of interest during the past decade.

Chapter I concerns the various types of stability, including total and integral stability, that may be defined for the solutions of ordinary differential equations. The setting is that of Lyapunov's Second Method, and the stability criteria given, except for Perron's condition, are either deduced from, or stated directly in terms of appropriate Lyapunov functions. In Chapter II sufficient conditions for the absolute stability of regulator systems are derived, using both Lurie's approach and the method of Popov. Chapter III deals with the existence of periodic and almost-periodic solutions of ordinary differential equations. It contains concise expositions of the method of averaging, the method of successive approximations, and a treatment of singular perturbations. Chapter IV, by far the longest, takes up the questions of the previous chapters in the setting of functional-differential equations. Stability theory is developed in terms of Lyapunov functionals, the stability of a regulator system with time lag is discussed by the method of Popov, criteria for the existence of periodic solutions are given, and an extension of the method of averaging is introduced in detail.

Each chapter is provided at the end with some brief notes containing references to the origin of some of the theorems and to further work. The book closes with a fairly complete bibliography up to 1964.

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44[P, X, Z].—B. A. VOLYNSKIĬ & V. YE. BUKHMAN, *Analogues for the Solution of Boundary-Value Problems*, International Tracts in Computer Science and Technology, and their Application, Volume 13, Pergamon Press, New York, 1965, xi + 460 pp., 24 cm. Price $15.00.

The primary aim of the authors is to describe the development of network analyzers in the Soviet Union for the approximate solution of boundary-value problems in partial differential equations. To this end, the book is divided essentially into two parts.

First, representative scientific and engineering problems which lead to boundary-value problems are introduced, and finite-difference, integral and Monte Carlo methods for their solution are investigated. The construction of electrical circuits corresponding to the approximate expressions for the solutions of the mathematical equations are presented for numerous cases. The Dirichlet, Neumann and mixed-boundary value problems are covered. The treatment is well-organized and lucid, with particular emphasis placed on the nature of errors in the solutions and on methods for improving accuracy. This should prove of great value to the engineer who seeks a practical, clearly written approach to the subject.

The second portion of the book is concerned with general-purpose and special-purpose network analyzers. The former are applied to equations of the Laplace, Poisson and Fourier type; the latter are invoked when there are more stringent requirements, e.g., greater number of nodes, improved accuracy, greater speed of solution. There is a detailed discussion of the construction of these analyzers and of techniques for measuring the physical quantities which yield the solutions.

One subject which receives special attention is the use of a "star" configuration of resistors to represent the integral form of solution of boundary-value problems for unbounded domains. This English edition contains a supplementary chapter which reviews the research, carried out after the book appeared in the Soviet Union in 1960, on the simulation of integral methods and of more complicated boundary conditions. A few hybrid applications are also included.

The material in this second part of the book is highly specialized; much of it is the result of extensive analyses carried out by the authors and others. Devoted, as it is, primarily to special-purpose network analyzers, its audience is a somewhat limited one. Nonetheless, the authors are to be commended for their contributions to a complex subject, and for their constant attention to the theoretical as well as to the practical aspects of their analogues.

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45[P, Z].—HELLMUT GOLDE, *Fortran* II *and* IV *for Engineers and Scientists*, The Macmillan Company, New York, 1966, xvi + 224 pp., 28 cm. Price $4.50.

"Primarily a text for beginning students in engineering and science on the college level," this book represents, with a few exceptions, a complete treatment of the subject matter in a clear and lucid style. After the usual introductory material, the elements of the FORTRAN language are presented in Chapters 3–9. The dialect is that of level one of the proposed American Standard for FORTRAN. The FORTRAN IV extension is discussed in Chapter 10. A presentation of the elements of the language is motivated by means of one or more coding problem(s) in each chapter. Examples of correct and incorrect coding are given throughout the text. Basic numerical problems associated with fixed precision floating point quantities and the necessary programming to avoid these difficulties are also discussed. Complete statistics are available concerned with the characteristics of the compilers for various computers (appendices A and B).

Although the overall treatment of the subject matter is good, there are a few weaknesses. Scanty material is presented on the generation and review of binary information stored peripherally, an important aspect of many large scientific problems. A discussion of the computed GØ TØ statement and EQUIVALENCE statement is left for the concluding Chapter 11. Thus, the frequent use of these statements, which occurs in everyday situations, is not reflected in the coding problems of the text. A glossary of terms is not included.

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46[S].—B. L. MOISEIWITSCH, *Variational Principles*, John Wiley & Sons, New York, 1966, x + 310 pp., 24 cm. Price $14.00.

Variational principles have long played two major roles in mathematical physics; one as great unifying principles through which the different equations can be expressed in elegantly simple form, and the other as remarkably useful computational tools for the accurate determination of discrete eigenvalues such as the vibration frequencies of classical systems and the bound state energies of quantum mechanical systems. In the latter role, variational principles represent a small triumph of man over nature. The fractional error in the quantity to be determined, the "output," is proportional to the square of the fractional error in the "input" information, almost giving one the eerie feeling that some law of thermodynamics is being violated; the "input" information is represented, for example, by a guess at the shape of a vibrating string. Furthermore, the sign of the "output" error is known.

A number of exciting developments in the field of variational principles have taken place in the past 20 years, particularly with regard to the analysis of scattering problems. (In the quantum mechanical case, the concern is then with the continuous portion of the energy spectrum and the problem is to determine not the energy but quantities such as phase shifts which determine the scattering.) The present time is therefore appropriate for a good review of the subject. *Variational Principles* provides just that.

The introductory material, on the problem of expressing the equations of motion of classical mechanics in the form of variational principles, as, for example, the principle of least action, is, almost of necessity, primarily a standard treatment. The author then turns to the basic variational principle of optics, Fermat's principle of least time, and to the establishment of an analogue of Hamilton's principle which provides the quantum dynamical equations of motion in a Lagrangian form. The latter material, the work of Schwinger, is not yet standard textbook fare; the treatment is good but too terse.

The variational principle formulation of the field equations of physics is enriched by a variety of applications from electromagnetism, sound waves, etc., that should be very helpful to the student of mathematical physics. One might hope that in any later edition Moiseiwitsch will also include examples from magnetohydrodynamics, nonequilibrium thermodynamics, and other topics not often studied by graduate students of physics or mathematics. It is my impression that developments in variational principles in one field have taken unconscionably long to spread to other fields even when they were essentially immediately applicable. The rate of flow of information from one area to another can be increased by the proper selection of examples from the different areas.

Many detailed examples of the determination of discrete eigenvalues are covered, a number from atomic physics, where the "input" is a trial function. A recent development of potentially great significance, a variational principle for an arbitrary operator, or rather for matrix elements of the operator, is discussed.

The last third of the book is devoted to the use of variational principles in scattering theory. This much space is fully justified by the developments in the field, and is desirable since the author has been among the most active workers in the field.

Variational principles are established for scattering phase shifts and scattering amplitudes, and a number of examples, largely from atomic physics, are considered in sufficient detail to be highly educational. The original formulation of scattering theory provided a variational principle which was weaker than its discrete energy eigenvalue analogue; it provided a second order error but not the sign of the error and therefore did not provide a bound. (In the space of the parameters used in the trial function, there might be a saddle point rather than an absolute extremum.) More recently the full analogue of the Rayleigh-Ritz variational bound on discrete eigenvalues was developed. The treatment in the book is limited to that incident energy, zero, for which the variational bound formulation of scattering theory assumes its simplest form. Variational bounds are studied in a number of important and by no means trivial examples, including that of the zero energy scattering of electrons by hydrogen atoms.

The book concludes with a treatment of the basic work by Lippmann and Schwinger on formal time-dependent scattering theory, but, for only the second instance in the volume, the treatment is probably too terse to be really useful. In general, the treatment of material throughout the text is sufficiently thorough to enable second year graduate students of physics not only to follow but to profit considerably; with the possible exception of some of the formal material on quantum mechanics and the treatment of the Dirac equation, the same should be true for students of mathematics.

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47[S, X].—HARRY H. DENMAN, WILFRIED HELLER & WILLIAM J. PANGONIS, *Angular Scattering Functions for Spheres*, Wayne State University Press, Detroit, Michigan, 1966, xix + 294 pp., 24 cm. Price $7.50.

Let

$$i_\perp = \left| \sum_{n=1}^{\infty} \{A_n \pi_n (\cos \alpha) + B_n \tau_n (\cos \alpha)\} \right|^2,$$

$$i_\parallel = \left| \sum_{n=1}^{\infty} \{A_n \tau_n (\cos \alpha) + B_n \pi_n (\cos \alpha)\} \right|^2.$$

$\pi_n(x) = dP_n(x)/dx$, $\tau_n(x) = x\pi_n(x) - (1 - x^2)d\pi_n(x)/dx$, where $P_n(x)$ is the Legendre polynomial of degree n. Also the coefficients A_n and B_n depend on the Riccati-Bessel functions.

$S_n(x) = (\pi x/2)^{1/2}J_{n+1/2}(x)$, $C_n(x) = (-1)^n(\pi x/2)^{1/2}J_{-n-1/2}(x)$. A_n and B_n are functions of $S_n(\alpha)$ and $S_n(\beta)$ where $\beta = m\alpha$. This volume tabulates $i_\perp/\alpha^3$ and $i_\parallel/\alpha^3$ to 5S for $\alpha = 0.2(0.2)25$, $\alpha = 0°(5°)180°$, $m = 1.05(0.05)1.30, 1.333$.

The method of computation and the checks used are explained in detail, and the authors conclude that "the fifth figure in these tables is *correct* in most cases, and is *significant* almost always." (The italics are theirs.) The present tables are the most complete on the subject. For a description of the physical aspects of the problem to which the tables relate and previous tables, see *MTAC*, v. 3, 1949, p. 483–484 and *MTAC*, v. 6, 1952, p. 95–97.

Y. L. L.

48[W, X].—V. S. NEMCHINOV, Editor, *The Use of Mathematics in Economics*, The M. I. T. Press, Cambridge, Massachusetts, 1965, xxi + 377 pp., 26 cm. Price $12.50.

The present work constitutes a sample of recent East European (principally Soviet) work on mathematical economics.

The first article, by V. S. Nemchinov, gives an introductory discussion of industrial input-output matrices and their applications, with emphasis on planning of uniform growth. A gross national balance sheet for the Soviet economy (years

1923–1924!) is also given. The second article, by V. V. Novozhilov, is considerably longer, and essentially constitutes a pedagogical monograph, written to appeal to the Russian economic manager (presumably as nervous about these new-fangled inventions of the ivory-tower types as are U. S. managers) of elementary linear programming. Profitability (in percent per year) is emphasized as an investment criterion, this criterion is shown to be equivalent to various criteria of desirability: maximum growth rate, maximum return on total investment, etc. A certain amount of effort is devoted to establishing the compatibility (in a suitably 'higher' sense) of the new rational methods with the obligatory Marxist ideological base.

Two additional articles, by L. V. Kantorovich, develop the linear programming method in more detail, bringing out the mathematical bases of the method in terms of the dual problem of linear programming—called by Kantorovich the method of resolving multipliers. A variety of elegant small applications to machine shop scheduling, plywood cutting, etc. are discussed in detail, and some indication of numerical methods given. The first of these two papers (published 1939) is in fact one of the pioneering works on linear programming.

A good bibliography by A. A. Korbut surveys the development of linear programming, both in the USSR and abroad.

A short article by Oscar Lange discusses models of economic growth in the context of input-output analysis; these models are regarded as multi-sector developments of primitive two- and three-sector ones which may be construed out of Marx.

An article by A. L. Lure derives useful practical algorithms for the solution of rail-transport optimization problems by elementary linear programming and graph theoretical means.

Apparent in the whole volume is the convergence of Soviet planning economics with American single-firm efficiency economics. As Novozhilov puts it, quoting a Russian proverb: "Every vegetable has its season."

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49[X].—F. Ceschino & J. Kuntzmann, *Numerical Solution of Initial Value Problems*, translated by D. Boyanovitch, Prentice-Hall, Inc., Englewood Cliffs, N. J., 1966, xvi + 318 pp., 24 cm. Price $10.50.

In Chapter 1 the authors introduce notational conventions, terminology (canonical forms, resolved forms, equilibrated resolved forms, etc.), brief statements of existence theorems (assuming functions are differentiable), and brief comments on numerical procedures.

Chapters 2–5 cover approximately 100 pages and are devoted mainly to deriving various single-step formulas, from Euler's method, through Runge-Kutta methods, to variants on these methods, such as one due to Blaess, and the implicit Runge-Kutta methods. Some Runge-Kutta formulas of order 5 and 6 are given, as well as a proof that formulas of order 5 cannot be obtained with only five function evaluations.

In Chapter 6 (Adams Method and Analogues) some special multistep formulas are derived, including explicit and implicit Adams formulas, and those due to Cowell, Nystrom, and Milne.

Chapter 7 (Different Multistep Formulas) is devoted mainly to a discussion of stability, without attempting to prove any of the basic theorems due to Dahlquist, even though both authors have made interesting contributions to this area in at least one of their earlier papers. Brief mention is also made of special formulas, such as those "which appeal to higher order derivatives," and also some "of the Obrechkoff type."

Chapter 8 (Application of the Runge-Kutta Principle to the Multistep Methods) considers very briefly the idea of combining Runge-Kutta and multistep ideas into composite formulas.

Chapter 9 (Theoretical Considerations) consists mainly of remarks about the characteristic roots of linear homogeneous systems with constant coefficients, the "propagation matrix" for the variational equations, and the use of something called a "coaxial" in investigating the errors associated with various methods.

A final chapter (Practical Considerations) is concerned with a variety of topics, such as different ways of estimating local truncation errors, choosing the step-size, and changing the step-size.

Numerical results for relatively simple problems are used frequently, in almost every chapter, to illustrate the methods being discussed.

There is a bibliography of nearly 600 items, including a few for 1962. (The text from which the translation has been made was copyrighted in 1963.)

This book provides a large number of formulas for the numerical integration of ordinary differential equations. Unfortunately, despite the claim in the preface that the authors have tried to group the methods around a central idea, the result is a hodge-podge of formulas, facts, near-facts, and opinions. The treatment is sometimes incomplete, and often superficial.

The presentation is frequently rather vague or misleading. For example, when "methods of approximate solution" are first introduced on p. 11, it is stated that "These methods do not give the general integral but only a well-determined integral. These methods can furnish, instead of the exact solution which we do not know or do not wish to write down, an approximate solution in the sense of numerical calculus. We thus understand that this solution is defined in a finite interval by a procedure actually executable and that we possess certain information on the error by which it is affected." Does this help the reader to understand the nature of numerical methods?

To illustrate some of the carelessness with which this book is written, consider the way in which the method of Euler-Cauchy is introduced, after the tangent method has been described, and subsequently improved. On p. 40, it is stated that "Another manner of improving the tangent method consists of noting that, on a small arc, the slope of the chord is obviously the arithmetic mean of the slopes of the tangents at the end points."

The English is not good, and the fact that the book is a translation is frequently obvious. But the main difficulties with the presentation must have existed in the original version.

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50[X].—I. G. PETROVSKI, *Ordinary Differential Equations*, translated by Richard A. Silverman, Prentice-Hall, Inc., Englewood Cliffs, N. J., 1966, x + 232 pp., 24 cm. Price $7.95.

The work under review covers a variety of topics in the field of ordinary differential equations, and also contains a brief supplement on first order partial differential equations. The scope and level of the material is such that, in terms of an American university curriculum, it fits a senior level or first year graduate level one semester course for mathematics majors.

In principle no prior acquaintance with the field of differential equations is required. In the first two chapters the basic ideas are introduced. The discussion of the special solvable cases is unusually careful and detailed. The chapter devoted to existence and uniqueness theorems is among the best in the book. Arzéla's theorem is proved and used to prove Peano's existence theorem. Uniqueness questions are then discussed via Osgood's uniqueness theorem. The method of successive approximations is then discussed as an application of the fixed point theorem for contracting mappings. The classical Cauchy theorem regarding analytic cases is also discussed. The same chapter covers in detail the continuous dependence of solutions on initial data and parameters.

The chapters covering linear systems are adequate. The sections devoted to the canonical form of linear systems with constant coefficients are unnecessarily cumbersome. A simple statement of the Jordan form for square matrices and its application to linear systems would have been adequate. Lyapunov's second method is introduced to discuss some stability questions. The proof of the theorem on p. 151 regarding asymptotic stability is faulty. The function V must also be assumed to have an "infinitesimal upper bound" to guarantee asymptotic stability (see Massera, *Ann. of Math.* **50** (1949), 118-126.)

The last chapter is devoted to a number of topological questions; limit cycles are discussed briefly. A proof of the Brouwer fixed point theorem is given and some nice applications of that theorem are provided. The supplement is devoted to those aspects of first order partial differential equations that can be discussed in terms of ordinary differential equations. A brief and good introduction to generalized solutions is provided.

As has been indicated, those topics covered in the book are done well. Unfortunately there are many other topics that are completely omitted. For example, there is no mention of classical stability theory, linear systems with periodic coefficients, perturbation theory, boundary value problems (Sturm-Liouville problems), Green's functions, and equations with singularities, especially Fuchsian singularities (Legendre polynomials, Bessel functions etc.). There are many aspects of nonlinear differential equations which have come to prominence in recent years. These are, by and large, ignored. From this reviewer's point of view these omissions are serious and will severely limit the utility of this book. Nevertheless there will be some who will find the exceptional features of this book adequate compensation for its shortcomings.

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51[X].—JAMES B. SCARBOROUGH, *Numerical Mathematical Analysis*, Sixth Edition, The Johns Hopkins Press, Baltimore, Md., 1966, xxi + 600 pp., 24 cm. Price $8.50.

This veteran textbook, originally copyrighted in 1930, is well known to numerical analysts and has been reviewed time and again. In this, the sixth edition, nothing has been added which alters the basic strengths and weaknesses of the previous editions. The alterations are in fact quite minor. They consist of a belated introduction of the trapezoidal rule for quadrature, of a slight modification of the regula-falsi method for root-finding, and of some formulas of Runge-Kutta type (due to Kooy and Uytenbogaart) for solving systems of second order differential equations. It is unfortunate that while introducing the trapezoidal rule the author did not deem it advisable to introduce the related (but more useful methods) of Romberg integration and the trapezoidal rule with end corrections. For those interested in solving problems on desk calculators this book should continue to be appealing. For those interested in solving problems using computers there are a number of books now on the market which should be preferred.

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52[X].—BURTON WENDROFF, *Theoretical Numerical Analysis*, Academic Press, New York, 1966, xi + 239 pp., 24 cm. Price $10.95.

An accurate and brief description of this book is given by the author,

"My purpose in writing this book is to present numerical analysis as a legitimate branch of mathematics, deserving attention from mature mathematicians and students alike. In keeping with this theme the reader will find himself traveling a narrow, often deep path through five basic fields of numerical analysis: interpolation, approximation, numerical solution of ordinary and partial differential equations, and numerical solution of systems of equations. The direction and depth of the path, while largely a matter of my own taste, are constrained when feasible so as to lead to a consideration of good computing technique (large scale digital)."

He succeeds admirably! This book should prove to be an excellent reference work for practical numerical analysts, for advanced graduate students and for others interested in the elegant and unified presentation of tastefully selected topics.

E. I.

53[X, Z].—IVAN FLORES, *Computer Software, Programming Systems for Digital Computers*, Prentice-Hall, Inc., Englewood Cliffs, N. J., 1965, x + 493 pp., 24 cm. Price $12.00.

One of the problems of writing in a rapidly developing field such as computer science is obsolescence. The problem is aggravated in this case by the recent introduction of a 'third generation' of computers by the large manufacturers. The blurb in the Preface of this book begins "here is an advanced programming book which builds software before your very eyes. . .," but no consideration is given to compilers, multiprogramming, multiprocessing, or timesharing. The book is actually devoted to a detailed description of an assembler and a batch-type operating system for a hypothetical machine of the 7090 class.

The choice of a hypothetical machine and system rather than an existing one is questionable, and is only justifiable if it allows the author to improve the treatment in some way. Unfortunately, there is little of novelty in this book apart from what can only be described as the author's idiosyncrasies.

For example, the word 'transfer' is used for transfer of information instead of control, and 'list-processing' is used in connection with instruction loops, almost as though the author knew it was an 'in' word, but was unaware of its meaning. But when the author fails to make clear the difference between macros and subroutines, and confuses compilation with assembly-time calculation, the situation becomes more serious.

Certain parts of the book, mainly those describing the I/O control structure, could be of interest, but the reader's time would be better spent in a study of FAP and IBSYS.

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54[X, Z].—F. H. GEORGE, *An Introduction to Digital Computing*, Pergamon Press, New York, 1966, xiii + 276 pp., 20 cm. Price $4.50.

This is an unusual text on digital computing in that it is written for "programmed instruction." The reader is asked to read some text and questions are asked about the contents of the material covered. Depending upon the answer given the reader is directed to another page. This is fine in principle but, unfortunately, much time may be spent in flipping pages.

If this were the only fault of the book it would not be of great consequence, but actually, even the reader who correctly answers each question never really gets anywhere apart from learning that a computer is composed of a memory, a register, instructions and numbers. He is presented with material which can better be covered in a very elementary "straight" manner and he would probably be better off saving his thumb work for programming. At no point in the text is the reader introduced to programming in the more meaningful sense of the word.

The text is more suitable to the junior grades of an elementary school rather than for 18–19 year old high school and college students who served as the author's validation group.

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55[X, Z].—FREDERIC STUART, *Introductory Computer Programming*, John Wiley and Sons, Inc., New York, 1966, xi + 155 pp., 24 cm. Price $5.95.

If it were not for the fact that this book is devoted entirely to the description of two uncommon brands of FORTRAN programming, NCE* and AFIT,† it would serve as a useful guide to learning FORTRAN. There are a great number of useful examples geared towards statistics and the questions listed in the exercises are well posed. However, the differences between these two forms of IBM 1620 FORTRAN and the commonly used FORTRAN I, II and IV are sufficiently great to keep the prospective programmer from learning these versions which would be unacceptable on the majority of the scientific computers in general use today.

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* NCE = Newark College of Engineering.
† AFIT = Air Force Institute of Technology.