

REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS

35[2.05].—LESLIE F. BAILEY, *Tables of Folded-sin x/x Interpolation Coefficients*, U. S. Government Printing Office, Washington, D. C., 1966, xv + 161 pp., 30 cm. Price \$2.75. (Obtainable from the Superintendent of Documents, U. S. Government Printing Office, Washington, D. C. 20402.)

Although not so identified explicitly by the author, these interpolation tables consist of 11D values (without differences) of

$$A(k) = \frac{\sin \pi(p-k)}{n \sin \pi(p-k)/n} \text{ for } n = 3(2)11, \quad k = -\frac{n-1}{2} (1) \frac{n-1}{2},$$

and of

$$A(k) = \frac{\sin \pi(p-k)}{n \tan \pi(p-k)/n} \text{ for } n = 4(2)12, \quad k = -\frac{n-2}{2} (1) \frac{n}{2},$$

where in both cases $p = 0(0.001)1$.

The underlying calculations were performed to 14D on an IBM 7030 system. The pertinent Fortran program (which includes three redundant lines of coding) is appended to the introductory text. Also included is a bibliography of nine publications.

The author states in the Preface that these tables originated in the course of developing a method for designing digital filters of specified characteristics for processing data from seismometers of geophones in a seismic array. Other applications are referred to in a special section of the text. Two numerical examples of the use of the tables are also given.

The arrangement of the tables has been patterned after the WPA tables [1] for Lagrangian interpolation.

A mathematical motivation and explanation of these tables has not been supplied by the author. One procedure for deriving these coefficients is as follows. Suppose a function $f(t)$ is given at n points:

$$t = -\frac{n-1}{2} (1) \frac{n-1}{2} \text{ when } n \text{ is odd,}$$

and

$$t = -\frac{n-2}{2} (1) \frac{n}{2} \text{ when } n \text{ is even.}$$

Assume further that $f(t+n) = f(t)$ for all t . Then the interpolation formula

$$f(t) = \sum_{m=-\infty}^{\infty} \frac{\sin \pi(t-m)}{\pi(t-m)} f(m)$$

reduces to

$$f(t) = \sum_{k=-(n-1)/2}^{(n-1)/2} \sum_{m=-\infty}^{\infty} \frac{\sin \pi(t-k+mn)}{\pi(t-k-mn)} f(k)$$

when n is odd. Thus, in this case

$$f(t) = \sum_{k=-(n-1)/2}^{(n-1)/2} A(k) f(k),$$

where

$$A(k) = \frac{\sin \pi(t-k)}{\pi} \left[\frac{1}{t-k} + \sum_{m=1}^{\infty} (-1)^m \frac{2(t-k)}{(t-k)^2 - (mn)^2} \right] = \frac{\sin \pi(t-k)}{n \sin \pi(t-k)/n}.$$

When n is even, a similar manipulation leads to the result

$$\begin{aligned} A(k) &= \frac{\sin \pi(t-k)}{\pi} \left[\frac{1}{t-k} + \sum_{m=1}^{\infty} \frac{2(t-k)}{(t-k)^2 - (mn)^2} \right] \\ &= \frac{\sin \pi(t-k) \cos \pi(t-k)/n}{n \sin \pi(t-k)/n}. \end{aligned}$$

From this derivation of the formulas for the interpolation coefficients $A(k)$ it is evident that the author's descriptive phrase "folding, accordion style" is inaccurate in characterizing this type of interpolation.

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1. MATHEMATICAL TABLES PROJECT, WORK PROJECTS ADMINISTRATION, *Tables of Lagrangian Interpolation Coefficients*, Columbia Univ. Press, New York, 1944.

36[2.05, 2.10, 2.20, 2.25, 2.30, 2.35, 3, 5, 7, 8, 12].—ANTHONY RALSTON & HERBERT S. WILF, Editors, *Mathematical Methods for Digital Computers*, Vol. 2, John Wiley & Sons, Inc., New York, 1967, x + 287 pp., 27 cm. Price \$11.95.

In a format similar to that of the first volume, but considerably improved, the editors have collected the contributions of experts in special areas of numerical analysis, calculation and programming languages. The first chapter gives a critical analysis of FORTRAN versus ALGOL and then makes comments about their limitations. Each subsequent writer presents a theoretical treatment of a method or class of methods for accomplishing a particular numerical calculation. In addition, flow charts, FORTRAN programs, estimates of running time and other practical hints are provided.

The depth of the analyses is variable, but appropriate to the complete understanding of the different methods. The list of the chapter headings and authors attests to the usefulness and the excellence of the exposition.

Part I. Programming Languages

1. An Introduction to FORTRAN and ALGOL Programming—Niklaus Wirth

Part II. The Quotient-Difference Algorithm

2. Quotient-Difference Algorithms—Peter Henrici

Part III. Numerical Linear Algebra

3. The Solution of Ill-Conditioned Linear Equations—J. H. Wilkinson
4. The Givens-Householder Method for Symmetric Matrices—James Ortega
5. The LU and QR Algorithms—B. N. Parlett

Part IV. Numerical Quadrature and Related Topics

6. Advances in Numerical Quadrature—Herbert S. Wilf
7. Approximate Multiple Integration—A. H. Stroud
8. Spline Functions, Interpolation, and Numerical Quadrature—T. N. E. Greville

Part V. Numerical Solution of Equations

9. The Solution of Transcendental Equations—J. F. Traub
10. The Numerical Solution of Polynomial Equations and the Resultant Procedures—Erwin H. Bareiss
11. Alternating Direction Methods Applied to Heat Conduction Problems—Jerome Spanier

Part VI. Miscellaneous Methods

12. Random Number Generation—Jack Moshman
13. Rational Chebyshev Approximation—Anthony Ralston

E. I.

37[2.05, 2.10, 2.20, 2.35, 3, 4, 5].—ROYCE BECKETT & JAMES HURT, *Numerical Calculations and Algorithms*, McGraw-Hill Book Co., New York, 1967, xi + 298 pp., 24 cm. Price \$9.95.

The purpose of this text is to train advanced undergraduate and beginning graduate engineers in numerical methods. The scope of the book may be indicated by listing the chapter headings: Introduction to Computers, The Flow Chart, Non-linear Algebraic Equations, Simultaneous Linear Equations, Determinants and Matrices, Interpolation and Numerical Integration, Initial-Value Problems, Finite Differences and Boundary-Value Problems, and Data Approximation. Each chapter contains flow charts of suitable algorithms for implementing the numerical procedures and a large selection of problems.

The material in the book is carefully selected and appears to be well written.

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38[2.05, 3, 131].—S. I. ZUKHOVITSKIY & L. I. AVDEYEVA, *Linear and Convex Programming*, W. B. Saunders Co., Philadelphia, Pa., 1966, viii + 286 pp., 25 cm. Price \$8.00.

What sets this book aside from most linear programming books are the two last chapters on Chebyshev approximation and convex programming. The simplex method is used for the solution of inconsistent linear systems (with or without constraints) and an algorithm is presented for Chebyshev approximation by rational functions. The chapter on convex programming gives a description, a convergence proof and a numerical example, worked out in detail, of an algorithm of feasible directions for solving convex programs. The case of quadratic programming is also discussed in detail including a finite algorithm. The first four chapters cover the basic material of linear programming and applications to production planning, optimal trimming problems, agricultural problems, allocation problems, military problems, game theory and transportation problems. There is a brief section on integer programming.

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39[2.05, 3, 4, 5, 9, 13.05, 13.15].—J. WALSH, Editor, *Numerical Analysis: An Introduction*, Thompson Book Co., Washington, D. C., 1967, xiv + 212 pp., 24 cm. Price \$12.00.

This informative book is based on a series of lectures delivered at a symposium organized by the Institute of Mathematics and its Applications at Birmingham, England in July 1965. It is addressed principally to nonspecialists in numerical analysis engaged in applying numerical techniques to the solution of problems in both pure and applied mathematics.

The lectures have been arranged in twelve chapters, which are largely self-contained. Especially helpful to the reader are the up-to-date references appended to the successive chapters, which direct his further study up to the frontiers of current research.

Following a condensed general survey by L. Fox, the topics considered in the succeeding chapters include linear algebraic equations, eigensystems of matrices, the numerical solution of both ordinary and partial differential equations, polynomial and rational approximation to functions of one variable, minimization of functions of several variables, application of computers to pure mathematics, mathematical techniques in operations research, industrial applications of numerical analysis, and computation in British school and university teaching.

In addition to Professor Fox, the contributors are D. W. Martin, J. H. Wilkinson, J. C. P. Miller, Joan Walsh, H. H. M. Pike, A. R. Curtis, M. J. D. Powell, H. P. F. Swinnerton-Dyer, S. Vajda, H. P. Y. Hitch, H. H. Robertson, M. E. Silvester, R. Hetherington, A. J. Moakes, and J. Crank.

This series of lectures constitutes a valuable, timely supplement to the similar series of lectures offered by the National Bureau of Standards in 1957 and 1959, which are also available in book form [1].

J. W. W.

1. JOHN TODD, Editor, *A Survey of Numerical Analysis*, McGraw-Hill Book Co., 1962. (See *Math. Comp.*, v. 17, 1963, p. 89, RMT 3.)

40[2.10].—BRUCE S. BERGER & ROBERT DANSON, *Tables of Zeros and Weights for Gauss-Laguerre Quadrature*, ms. of 4 typewritten pp. + 15 computer sheets deposited in the UMT file.

These manuscript tables consist of 24S approximations in floating-point form to the abscissas, x_{kN} ; the weights, a_{kN} ; and the weights multiplied by $\exp x_{kN}$ associated with the Gauss-Laguerre quadrature formula for $N = 100, 150, 200$, and 300.

According to the introductory text, the underlying calculations were performed on a CDC 6600 system, using double-precision floating-point operations accurate to about 30S. The zeros of the corresponding Laguerre polynomials were calculated by Newton's method and were checked by the relations $\sum_{k=1}^N x_{kN} = N^2$ and $\prod_{k=1}^N x_{kN} = N!$ to 26S and 24S, respectively. Similar summation tests were applied to a_{kN} and to $a_{kN} \cdot x_{kN}^n$, for $n = 1, 7$, and 15. These check relations were adapted from those employed by Rabinowitz and Weiss [1].

The present tables constitute a useful supplement to the extensive 30S tables of Stroud and Secrest [2].

J. W. W.

1. PHILIP RABINOWITZ & GEORGE WEISS, "Tables of abscissas and weights for numerical evaluation of integrals of the form $\int_0^\infty e^{-x} x^j f(x) dx$," *MTAC*, v. 13, 1959, pp. 285-294.
2. A. H. STROUD & DON SECREST, *Gaussian Quadrature Formulas*, Prentice-Hall, Englewood Cliffs, N. J., 1966, pp. 254-275.

41[2.10].—H. TOMPA, *Abscissae and Weight Factors for Gaussian Integration with $N = 192$* , one typewritten p. + two computer sheets deposited in UMT file.

In a recent note [1] the author has described an algorithm for performing Gaussian quadrature with $N = 2^j$ and $3 \cdot 2^{j-1}$. For such values of N , 30S approximations to the abscissas and weight factors are available [2] up to $j = 9$ and 8, respectively, except for $N = 192$. This omitted case is covered by the present manuscript table, which gives the pertinent data to 21D.

The underlying calculations were performed on an IBM 1620 using floating-point arithmetic with a 25-digit mantissa and a greatly simplified version of the FORTRAN program given on pp. 29 and 30 of [2].

J. W. W.

1. H. TOMPA, "Gaussian numerical integration of a function depending on a parameter," *Computer J.*, v. 10, 1967, pp. 204-205.
2. A. H. STROUD & DON SECREST, *Gaussian Quadrature Formulas*, Prentice-Hall, Englewood Cliffs, N. J., 1966.

42[2.10, 2.15, 2.25, 2.35, 3, 8, 12, 13].—WILLIAM S. DORN & HERBERT J. GREENBERG, *Mathematics and Computing: with Fortran Programming*, John Wiley & Sons, Inc., New York, 1967, xvi + 595 pp., 24 cm. Price \$8.95.

This text is neither fish nor fowl, but a tasty mixture of both: the mathematics is eminently practical in its orientation, and the computing avoids a "cook-book" approach. The authors manage to weave together a number of subjects in a way that leads the student in a variety of interesting directions—differential and integral calculus, infinite series, iterative and finite methods for linear and nonlinear systems, logic, and probability. The level of the text is appropriate for perhaps freshmen or sophomores, or for bright high school students.

It might be appropriate to characterize the book as a beginner's introduction to numerical analysis, since there is emphasis on how to go about finding solutions to real-life problems and how to handle the difficulties that typically arise. Since it is written in an easy-going style with extensive discussion of each topic, the book should give the lecturer freedom to emphasize particular aspects in greater detail with the assurance that the student will be able to cover others on his own.

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43[2.10, 2.35].—PHILIP J. DAVIS & PHILIP RABINOWITZ, *Numerical Integration*, Blaisdell Publishing Co., Waltham, Mass., 1967, ix + 230 pp., 23 cm. Price \$7.50.

"In writing this book, we have tried to keep our feet on the ground and our head in the clouds: By ground we imply utility in day-to-day computation and

know-how of the computer laboratory; by clouds, theoretical topics that underlie numerical integration. The prerequisite for this book is a course in advanced calculus. It would also be helpful, though not strictly necessary, if the reader had an introductory course in numerical analysis so that he will be familiar with the motivation and the goals of computing. There are several places where some mathematics beyond calculus is used, but they are relatively few. The book is not wholly self-contained; nor has it been possible to include proofs for all statements made. Where these gaps occur, references to other texts or to original articles have been given."

The authors succeed by presenting the subject from a mature point of view and in delightfully readable prose. The mathematics is as sophisticated as the subject matter demands (and on rare occasions, complex variable theory and functional analysis are appealed to). The modest size of this comprehensive monograph reflects the care with which the authors have selected their material. This book will prove to be invaluable to anyone interested in using and/or studying numerical integration and sets a standard for the Blaisdell series which it is going to be hard to maintain. A listing of the chapter headings and appendices will serve to outline the wide scope of the book.

1, Introduction; 2, Approximate Integration over a Finite Interval; 3, Approximate Integration over Infinite Intervals; 4, Error Analysis; 5, Approximate Integration in Two or More Dimensions; 6, Automatic Integration;

Appendix 1, "On the Practical Evaluation of Integrals" by Milton Abramowitz;

Appendix 2, Some FORTRAN Programs;

Appendix 3, Bibliography of ALGOL Procedures;

Appendix 4, Bibliography of Tables;

Appendix 5, Bibliography of Books and Articles.

E. I.

44[2.10, 7].—E. N. DEKANOSIDZE, *Tabitsy kornei i vesovykh mnozhitel' obobshchennykh polinomov Lagerra* (Tables of the zeros and weight factors of the generalized Laguerre polynomials), Computing Center, Acad. Sci. USSR, Moscow, 1966, xx + 306 pp., 27 cm. Price 2.33 rubles.

The main table (Table I, pp. 1–301) consists of 15S approximations (in floating-point form) to the zeros $x_{i^{(n); \alpha}}$ of the generalized Laguerre polynomials, defined by the Rodrigues formula

$$L_n^{(\alpha)}(x) = \frac{x^\alpha e^x}{n!} \frac{d^n}{dx^n} (x^{\alpha+n} e^{-x}),$$

and to the associated weight factors $A_{i^{(n); \alpha}}$ and $B_{i^{(n); \alpha}} (= A_{i^{(n); \alpha}} \exp [x_{i^{(n); \alpha}}])$ occurring in the Gaussian quadrature formulas

$$\int_0^\infty e^{-x} x^\alpha f(x) dx \approx \sum_{i=1}^n A_{i^{(n); \alpha}} f(x_{i^{(n); \alpha}}),$$

$$\int_0^\infty x^\alpha f(x) dx \approx \sum_{i=1}^n B_{i^{(n); \alpha}} f(x_{i^{(n); \alpha}}).$$

The tabular ranges of the parameters are $i = 1(1)n$, $n = 2(1)16$, $\alpha = 0(-0.01) - 0.99$. The values for $n = 1$ are omitted; however, in the introduction the author

gives the formulas $x_1^{(1; \alpha)} = \alpha + 1$, $A_1^{(1; \alpha)} = \Gamma(1 + \alpha)$, $B_1^{(1; \alpha)} = \Gamma(1 + \alpha)e^{1+\alpha}$, with the remark that values of the weight factors in this case can be obtained by use of well-known tables.

Table I constitutes an excellent supplement to the closely related tables of Rabinowitz & Weiss [1], Aizenshtat, Krylov & Metleskiĭ [2], Concus, Cassatt, Jaehnic & Melby [3], Concus [4], and Shao, Chen & Frank [5].

Table II (pp. 303–306) includes 15S approximations to the zeros $x_{\pm i}^{(2n)}$, $x_{\pm i}^{(2n+1)}$ of the Hermite polynomials, which, as the author notes in the introduction, are related to the zeros of the generalized Laguerre polynomials by means of the equations

$$x_{\pm i}^{(2n)} = \pm [x_i^{(n; -0.5)}]^{1/2}, \quad x_{\pm i}^{(2n+1)} = \pm [x_i^{(n; 0.5)}]^{1/2}, \quad (1 \leq i \leq n).$$

Also included in this table are 15S values of the weight factors associated with the Hermite quadrature formulas

$$\int_{-\infty}^{\infty} e^{-x^2} f(x) dx \approx C_0^{(2n+1)} f(0) + \sum_{i=1}^n C_i^{(2n+1)} [f(-x_i^{(2n+1)}) + f(x_i^{(2n+1)})],$$

$$\int_{-\infty}^{\infty} f(x) dx \approx D_0^{(2n+1)} f(0) + \sum_{i=1}^n D_i^{(2n+1)} [f(-x_i^{(2n+1)}) + f(x_i^{(2n+1)})],$$

for $i = 0(1)n$, $n = 2(1)16$. Exact values of the zeros and weight factors corresponding to $n = 0, 1$ are given in the introduction (p. viii). Equations relating the tabulated Hermite integration coefficients to the weight factors $A_i^{(n; 0.5)}$ and $B_i^{(n; 0.5)}$ appear on p. vii.

The 19-page introduction also includes mathematical details (including omnibus checks) of the calculation of these tables by double-precision arithmetic on the BESM2, as well as illustrative numerical examples of Newton interpolation in Table I and of two applications of that table. The remainder term for generalized Gauss-Laguerre quadrature is given explicitly and was evaluated to 2S when $\alpha = 0$ and -0.995 for the case $f(x) = x^{2n+j}$, $n = 2(1)16$, $j = 0(1)18$. However, only those computed values of this remainder are tabulated (p. xv) which are less than 0.33 when $n < 4$, and which are less than 0.2 when $n \geq 4$.

Appended to the introduction is a list of 12 references, which includes all the published tables cited in this review.

This reviewer has repeated the author's comparison of relevant sections of his tables with corresponding entries in the tables appearing in [1], [3], and [5]. None of the observed discrepancies exceeds a unit in the least significant tabulated figure; however, a study of these discrepancies suggests that most of the entries in the present tables have been left unrounded.

J. W. W.

1. PHILIP RABINOWITZ & GEORGE WEISS, "Tables of abscissas and weights for numerical evaluation of integrals of the form $\int_0^\infty e^{-x} x^n f(x) dx$," *MTAC*, v. 13, 1959, pp. 285–294.

2. V. S. AIZENSHTAT, V. I. KRYLOV & A. S. METLESKIĬ, *Tablitsy dlia chislennogo preobrazovaniia Laplasa i vychisleniia integralov vida $\int_0^\infty x^n e^{-x} f(x) dx$* (Tables for Calculating Laplace Transforms and Integrals of the form $\int_0^\infty x^n e^{-x} f(x) dx$), Izdat. Akad. Nauk SSSR, Minsk, 1962. (See *Math. Comp.*, v. 17, 1963, p. 93, RMT 9.)

3. P. CONCUS, D. CASSATT, G. JAEHNIG & E. MELBY, "Tables for the evaluation of $\int_0^\infty x^j e^{-x} f(x) dx$ by Gauss-Laguerre quadrature," *Math. Comp.*, v. 17, 1963, pp. 245–256.

4. P. CONCUS, *Additional Tables for the Evaluation of $\int_0^\infty x^j e^{-x} f(x) dx$ by Gauss-Laguerre Quadrature*, ms. in UMT file. (See *Math. Comp.*, v. 18, 1964, p. 523, RMT 81.)

5. T. S. SHAO, T. C. CHEN & R. M. FRANK, "Tables of zeros and Gaussian weights of certain associated Laguerre polynomials and the related generalized Hermite polynomials," *Math. Comp.*, v. 18, 1964, pp. 598–616.

- 45[2.20].—PAUL CONCUS, *Table of the Solutions of $a \tan(\pi x) = -b \tan(a\pi x)$* , Report UCRL-17609, Ernest O. Lawrence Radiation Laboratory, University of California, Berkeley, California, June 2, 1967, iii + 6 pp., 28 cm. Obtainable from the Clearinghouse for Federal Scientific and Technical Information, Springfield, Virginia 22151. Price \$3.00 (printed copy), \$0.65 (microfiche).

The first ten positive solutions of the trigonometric equation in the title are tabulated to 4D for $a = 0.001, 0.005, 0.01(0.01)0.1(0.05)1, b = 0.001, 0.01, 0.03, 0.1, 0.2, 0.5, 1, 2, 5, 10, 30, 100, 1000$.

According to the introductory explanatory text these roots were calculated to 8D by Newton's method on a CDC 6600 system and then were rounded to 4D for publication.

We are also informed that this unique table arose from the need for the numerical values of certain eigenvalues required in the series solution of a certain diffusion problem [1].

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1. PAUL CONCUS & DONALD R. OLANDER, *Transient Diffusion in a Composite Slab*, Lawrence Radiation Laboratory Report UCRL-17606, June 1967.

- 46[2.50].—D. S. MITRINOVIĆ, *Nejednakosti (Inequalities)*, Izdavačko Preduzeće, Građevinska Knjiga, Beograd, 1965, 240 pp. (In Serbo-Croatian.)

The author has collected elementary inequalities from elementary analysis and geometry and presents a selection, some with solutions and some with references to their sources.

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- 47[3].—D. B. YUDIN & E. G. GOL'SHTEIN, *Linear Programming*, translated from Russian, Daniel Davey & Co., Inc., 1966, x + 509 pp., 25 cm. Price \$25.25.

The outrageous price of \$25.25 demanded for this book will make it a rare volume in most linear programming libraries. This however is no misfortune. For the material covered in this book is covered just as adequately if not better in many of the available texts on linear programming (for example see the review in this issue, RMT 48, of M. Simonard, *Linear Programming*). This fact can be deduced from the following chapter headings:

1. Fundamental concepts of linear programming
2. Convex polyhedral sets and linear programming
3. Duality
4. Theoretical principles of the simplex method
5. The simplex computational procedure
6. The dual simplex method
7. The Hungarian method [primal-dual method]
8. Finite methods of linear programming

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48[3, 9, 10, 13].—MICHEL SIMONNARD, *Linear Programming*, translated by William S. Jewell, Prentice-Hall, Inc., Englewood Cliffs, N. J., 1966, xxiv + 430 pp., 24 cm. Price \$9.00.

This is an exceptionally well-written and well-translated book on linear programming and related subjects which will serve as an excellent text for a one- or two-semester course or for independent study. The book contains the usual linear programming material (primal-simplex method, dual-simplex method, duality, primal-dual method, parametric programming), two chapters on linear programming in integers and mixed integer programming (there is only a brief description by the translator of branch-and-bound methods), one chapter on bounded variables and the Dantzig-Wolfe decomposition principle, and five chapters on transportation problems. Two appendices present a self-contained review of linear algebra and convex polyhedra, and a third appendix gives some basic definitions of graph theory. There is also a four-page English-French glossary and a 169-item bibliography. (The addition of numerical exercises at the end of each chapter of this book would probably make it an unbeatable textbook on the subject.)

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49[4, 5, 13.15, 13.35].—JACK K. HALE & JOSEPH P. LASALLE, Editors, *Differential Equations and Dynamical Systems*, Academic Press, New York, 1967, xvii + 544 pp., 24 cm. Price \$18.00.

We copy the dedication of this book:

This volume, as was the Symposium itself, is dedicated to Solomon Lefschetz for the many contributions he has made over the past 20 years to the development of this subject. He has influenced most of us either directly or indirectly, and to many of us he has been a mentor and a father mathematician through his inspiration, encouragement, and guidance. In life as well as in mathematics he has been a true friend and a true companion.

An excerpt from the preface follows:

This volume is the proceedings of the third of a series of international symposia on differential equations and related topics. Speakers were invited to discuss advances in the theory of ordinary differential equations including both qualitative and geometric theory, the theory of stability and control, hereditary phenomena (functional differential equations), and special topics in the theory of partial differential equations. In addition, papers containing significant new results were accepted for presentation. Invited speakers whose reports are included in this volume are: H. A. Antosiewicz, R. F. Arenstorf, L. Cesari, K. L. Cooke, A. Friedman, U. Grenander, A. Halanay, J. K. Hale, P. Hartman, J. Kurzweil, J. P. LaSalle, Solomon Lefschetz, N. Levinson, J. Moser, N. Onuchic, M. Peixoto, and S. Smale.

The International Symposium on Differential Equations and Dynamical Systems was held at the University of Puerto Rico, Mayaguez, Puerto Rico, from December 27 to 30, 1965, and was sponsored by the United States Air Force Office of Scientific Research, the University of Puerto Rico, and Brown University.

The complete list of papers appears below:

Geometric Differential Equations: Recent Past and Proximate Future—Solomon Lefschetz, On the Theory of Quasiperiodic Solutions of Differential Equations—J. Moser, Existence of Quasiperiodic Solutions to the Three-Body Problem—Burton B. Lieberman, Applications of the Cunningham Integral to the Oblate Earth Problem—W. T. Kyner, A Perturbation Theorem for Invariant Riemannian Manifolds—Robert J. Sacker, New Periodic Solutions of the Plane Three-Body Problem—Richard F. Arenstorf, Periodic Solutions of a Nonlinear Wave Equation—P. H. Rabinowitz, A Minimax Proof of the Existence of an Optimal Control Making a Finite Number of Jumps in the Linear Bang-Bang Theory—N. Levinson, The Existence of Optimal Controls for a Performance Index with a Positive Integrand—R. Datko and M. Anvari, Adaptive Stochastic Control—Ulf Grenander, Control of Linear Time Delay Systems with Essentially Linear Cost Functionals—D. H. Chyung and E. B. Lee, Existence Theorems for Multidimensional Problems of Optimal Control—Lamberto Cesari, Algebraic Aspects of the Theory of Dynamical Systems—R. E. Kalman, An Extension Theorem for Asymptotic Stability and Its Application to a Control Problem—Nam P. Bhatia and George P. Szegő, Discontinuous Vector Fields and Feedback Control—H. Hermes, Functional-Differential Equations: Some Models and Perturbation Problems—Kenneth L. Cooke, A Note on Periodic Solutions of Nonlinear Differential Equations with Time Lags—Carlos Perello, Hereditary Dependence in Dynamics—G. Stephen Jones, Invariant Manifolds for Systems with Time Lag—A. Halanay, On the Bifurcation of Periodic Solutions in Non-linear Difference-Differential Equations of Neutral Type—R. K. Brayton, The Asymptotic Distribution of Characteristic Exponents for Periodic Differential-Difference Equations—J. C. Lillo, On the Asymptotic Behavior of the Solutions of Functional-Differential Equations—Nelson Onuchic, On “In the Large” Application of the Contraction Principle—S. C. Chu and J. B. Diaz, On the Existence of Solutions to Linear Differential-Difference Equations—K. R. Meyer, Geometric Theory of Functional-Differential Equations—Jack K. Hale, Asymptotic Behavior of Solutions near Integral Manifolds—Taro Yoshizawa and Junji Kato, An Invariance Principle in the Theory of Stability—Joseph P. LaSalle, A Stability Criterion for a Panel Flutter Problem via the Second Method of Liapunov—P. C. Parks, Variational Methods for Stability of Periodic Equations—R. W. Brockett, Stability Criteria for n th Order, Homogeneous Linear Differential Equations—E. F. Infante, Domains of Attraction of Some Quasiperiodic Solutions—P. R. Sethna, Inverse Liapunov Problems—Ruey-Wen Liu and R. Jeffrey Leake, Finite Time Stability under Perturbing Forces and on Product Spaces—Leonard Weiss and E. F. Infante, Stability of Periodic Linear Lagrangian Systems—Michel Cotsaftis, Asymptotically Self-Invariant Sets and Conditional Stability—V. Lakshmikantham and S. Leela, On Global Weak Attractors in Dynamical Systems—Nam P. Bhatia, Alan C. Lazer, and George P. Szegő, On General Dynamical Systems and Finite Stability—T. G. Windeknecht and M. D. Mesarovic, On Quasilinear Elliptic Functional-Differential Equations—Philip Hartman, Asymptotic Behavior of Solutions of Parabolic Differential Equations and of Integral Equations—Avner Friedman, Nonlinear Boundary Value Problems—H. A. Antosiewicz, Invariant Manifolds for Flows—Jaroslav Kurzweil, Qualitative Theory of Differential Equations and Structural Stability—M. M. Peixoto, Structural Stability on M^2 —Charles C. Pugh, Dynamical Systems on n -Dimensional Manifolds—S. Smale, Dynamical

Systems on Group Manifolds—Lawrence Markus, A Conjecture on Local Trajectory Equivalence—Courtney Coleman, Forced Oscillations in Second Order Systems with Bounded Nonlinearities—O. Benediktsson and B. A. Fleishman, Uniqueness Theorems for Initial Value Problems—Robert D. Moyer, Existence and Uniqueness of Solutions to the Second Order Boundary Value Problem—Paul B. Bailey, Lawrence F. Shampine, and Paul E. Waltman, An Application of Popov's Method in Network Theory—Vaclav Dolezal, Topologies for Linear Spaces of Nonlinear Differential Equations—Hubert Halkin, On Nonelementary Hyperbolic Fixed Points of Diffeomorphisms—Kuo-Tsai Chen, Nonautonomous Differential Equations as Dynamical Systems—George R. Sell.

E. I.

50[5].—FRANÇOISE MICHAUD, "Sur le calcul numérique des valeurs propres de l'équation de Hill," *Comptes Rendues de l'Académie des Sciences de Paris*, Series A, v. 264, 1967, pp. 867–870.

This note presents concise details of an application of the method of "intermediate problems," developed by Weinstein [1], Aronszajn [2], and Bazley [3], to the calculation of the eigenvalues, θ_0 , of Hill's equation

$$u'' + \left(\theta_0 + 2 \sum_{j=1}^p \theta_j \cos 2jz \right) u = 0$$

for which there exist even periodic solutions of period π corresponding to given values of θ_j ($j \geq 1$).

The first N eigenvalues are tabulated, correct to from 11S to 3S (as N and p increase), for the four cases: $\theta_1 = 1, \theta_2 = 0.2, p = 2, N = 6$; $\theta_1 = 3, \theta_2 = -0.4, p = 2, N = 6$; $\theta_1 = 1, \theta_2 = 0.2, \theta_3 = 1, p = 3, N = 11$; and $\theta_1 = 1, \theta_2 = 0.2, \theta_3 = 1, \theta_4 = 0.5, \theta_5 = 2.1, \theta_6 = 0.2, \theta_7 = 1, p = 7, N = 15$. In the first two cases good agreement is shown to exist between the decimal approximations found here for the first two eigenvalues and the corresponding 4D values found by Klotter and Kotowski [4] by the method of continued fractions.

J. W. W.

1. A. WEINSTEIN, *Etude des spectres des équations aux dérivées partielles*, Mémorial des Sciences Mathématiques, No. 88, Paris, 1937.

2. N. ARONSZAJN, "Approximation methods for eigenvalues of completely continuous symmetric operators," *Proc. Symposium on Spectral Theory and Differential Problems*, Stillwater, Oklahoma, 1951.

3. N. W. BAZLEY, "Lower bounds for eigenvalues," *J. Math. Mech.*, v. 10, 1961, pp. 289–307.

4. K. KLOTTER & G. KOTOWSKI, "Über die Stabilität der Lösungen Hillscher Differentialgleichungen mit drei unabhängigen Parametern," *Z. Angew. Math. Mech.*, v. 23, 1943, pp. 149–155.

51[5, 6].—R. D. RICHTMYER & K. W. MORTON, *Difference Methods for Initial-Value Problems*, Interscience Publishers, New York, 1967, xiv + 405 pp., 24 cm. Price \$14.95.

The brilliantly conceived and executed, the most often quoted monograph on difference methods for solving partial differential equations, has been improved upon in this second, completely revised, edition! Here the blending of theoretical analysis and intuitive formulation of methods is ideal and au courant with the current state of the art and science of computing.

"The principal theoretical advances are (1) the rounding-out or completion of

the theory for pure initial-value problems with constant coefficients by the general sufficient conditions for stability obtained by Buchanan and Kreiss, and (2) the rigorous stability theory for certain classes of problems with variable coefficients, of mixed initial-boundary-value problems, and of quasi-linear problems. Among the ideas that we believe should be of value to people engaged in the solution of practical problems are (1) the notion of dissipative difference schemes, (2) the Lax-Wendroff method for systems of conservation laws, (3) the alternating-direction methods for multidimensional parabolic problems, (4) practical stability criteria for cases in which stability as normally defined is inadequate, and the use of energy methods in the analysis of stability."

The authors have produced a remarkably clear and careful treatment under the chapter headings:

Part I—General Considerations

1. Introduction, 2. Linear Operators, 3. Linear Difference Equations, 4. Pure Initial-Value Problems with Constant Coefficients, 5. Linear Problems with Variable Coefficients; Non-Linear Problems, 6. Mixed Initial-Boundary-Value Problems, 7. Multi-Level Difference Equations;

Part II—Applications

8. Diffusion and Heat Flow, 9. The Transport Equation, 10. Sound Waves, 11. Elastic Vibrations, 12. Fluid Dynamics in One Space Variable, 13. Multi-Dimensional Fluid Dynamics; References.

The bibliography (in References) should prove most useful.

E. I.

52[7].—NBS COMPUTATION LABORATORY, *Tables Relating to Mathieu Functions: Characteristic Values, Coefficients, and Joining Factors*, NBS Applied Mathematics Series, Vol. 59, National Bureau of Standards, Washington, D. C., 1967, xlvii + 311 pp., 27 cm. Price \$3.25. (Obtainable from the Superintendent of Documents, U. S. Government Printing Office, Washington, D. C. 20402.)

The first edition [1] of these definitive tables, published by Columbia University Press in 1951, has been out of print since early in 1965. To remedy this situation, the National Bureau of Standards has reissued this book, with additions, in August 1967 as Volume 59 in its Applied Mathematics Series.

The original tables, together with the elaborate introduction by Gertrude Blanch, have been reprinted, with the correction of a few misprints. On the other hand, the bibliography has now been increased to 35 items through the addition of the six most significant publications on Mathieu functions between 1951 and 1964.

An extensive article [2] by Dr. Blanch and Ida Rhodes, containing supplementary tables of characteristic values, is reproduced in its entirety and is appended to the main tables in this new edition.

A valuable service has been rendered researchers in applied mathematics through the publication of this updated, enlarged edition of these fundamental tables.

J. W. W.

1. NBSCL, *Tables Relating to Mathieu Functions*, Columbia University Press, New York, 1951. (See *MTAC*, v. 6, 1952, pp. 29–30, RMT 952.)

2. GERTRUDE BLANCH & IDA RHODES, "Tables of characteristic values of Mathieu's equation for large values of the parameter," *J. Washington Acad. Sci.*, v. 45, 1955, pp. 166–196.

53[7].—R. W. WYNDRUM, JR. & E. S. MITCHELL, JR., *Values of the Complete Elliptic Integral $K(k)$ when the Ratio K'/K is Large*, ms. of three typewritten pp. + two computer sheets deposited in UMT file.

In this manuscript table the authors have tabulated 16S approximations (in floating-point form) to the complete integrals $K(k)$, $K'(k)$ and the moduli k , k' for argument $K'/K = 2(0.2)20$. Double-precision floating-point arithmetic was employed in duplicate calculations performed on GE 635 and IBM 7094 systems. Because of rounding errors, the accuracy of the tabular results is guaranteed to only 14S.

The basic formulas used in the calculation, involving the Jacobi nome, q , and the well-known q -series for the Jacobi theta functions of zero, are given in the introduction.

The present unusual table constitutes a considerable extension of the closely related table of Eagle [1], to which the authors refer.

J. W. W.

1. A. EAGLE, *The Elliptic Functions as They Should Be*, Galloway & Porter, Cambridge, 1958.

54[7].—M. I. ŽHURINA & L. N. KARMAZINA, *Tablitsy modifitsirovannykh funktsii Besseliā mnimogo indeksa $K_{i\tau}(x)$* (*Tables of the modified Bessel function of imaginary order $K_{i\tau}(x)$*), Computing Center, Acad. Sci. USSR, Moscow, 1967, xii + 341 pp., 27 cm. Price 3.18 rubles.

The modified Bessel function of the second kind of argument x and pure imaginary order $i\tau$ has the integral representation

$$K_{i\tau}(x) = \int_0^\infty \exp(-x \cosh u) \cos \tau u du, \quad R(x) > 0,$$

which shows that it is real for $x > 0$ and τ real.

This function arises in many contexts. For example, values of it are required in the numerical evaluation of the integrals in the Kantorovich-Lebedev transform pairs

$$F(\tau) = \int_0^\infty G(x) K_{i\tau}(x) dx, \quad 0 \leq \tau < \infty,$$

$$G(x) = \frac{2x}{\pi^2} \int_0^\infty \tau F(\tau) \sinh \pi \tau K_{i\tau}(x) d\tau, \quad 0 < x < \infty.$$

Tables of $K_{i\tau}(x)$ are given in this volume to 7S for $x = 0.1(0.1)10.2$, $\tau = 0.01(0.01)10$.

The introduction contains a description of the methods of computing these tables and a detailed analysis for interpolation of the entries in both directions.

Tables of $K_{i\tau}(v)$, $v = e^x$, have been previously prepared by Morgan [1] and by Luke & Weissman [2].

Y. L. L.

1. S. P. MORGAN, JR., *Tables of Bessel Functions of Imaginary Order and Imaginary Argument*, California Institute of Technology Book Store, Pasadena, 1947. (See *MTAC*, v. 3, 1948–1949, pp. 105–107, RMT 504.)

2. S. LUKE & S. WEISSMAN, *Bessel Functions of Imaginary Order and Imaginary Argument*, University of Maryland, Institute for Molecular Physics, Report DA-ARO(D)-31-124-G466 No. 1, College Park, Md., 1964. (See *Math. Comp.*, v. 19, 1965, pp. 343–344, RMT 50.)

55[8].—DOROTHY KERR, *Run Test for Randomness*, Applications Analysis Division, Department of Computer Services, Army Map Service, Washington, D. C. 20315, ms. in two volumes, 155 and 169 computer sheets, respectively (photographically reduced and unnumbered), deposited in the UMT file.

If u represents the number of runs in a random linear arrangement of two different kinds of objects (m of one kind, n of the other), then the probability that u does not exceed a given number u' can be found from the formula

$$P\{u \leq u'\} = \sum_{u=2}^{u'} f_u \div \binom{m+n}{m}$$

where

$$f_u = 2 \binom{m-1}{k-1} \binom{n-1}{k-1}, \quad k = \frac{u}{2}, \text{ when } u \text{ is even};$$

and

$$f_u = \binom{m-1}{k-1} \binom{n-1}{k-2} + \binom{m-1}{k-2} \binom{n-1}{k-1},$$

$$k = \frac{u+1}{2}, \text{ when } u \text{ is odd [1].}$$

The tables under review consist of 7D approximations to the value of P for $15 \leq m \leq n$, $m+n \leq 100$. When $m = n$, the maximum theoretical value of u' is $2m$; when $m < n$, this maximum is $2m+1$. However, the printed tabular values do not generally extend to these limits in u because of suppression as soon as they first equal unity when rounded to 7D. Likewise, all the entries equal to zero to 7D are omitted.

These tables constitute a direct continuation of a similar table by Swed & Eisenhart [2], to which reference is made in the brief introduction.

The underlying calculations were performed on a Honeywell 800 system, and several entries were checked against related data in a report of Argentiero & Tolson [3].

These new tables in conjunction with those in [2] should be particularly useful in connection with testing a large range of samples of data for randomness of grouping when the asymptotic formulas of Wald & Wolfowitz [4] do not yield the desired precision.

J. W. W.

1. WILLIAM FELLER, *An Introduction to Probability Theory and its Applications*, Vol. I, John Wiley & Sons, New York, 1950, pp. 56–58.

2. FRIEDA S. SWED & C. EISENHART, "Tables for testing randomness of grouping in a sequence of alternatives," *Ann. Math. Statist.*, v. 14, 1943, pp. 66–87.

3. P. D. ARGENTIERO & R. H. TOLSON, *Some Nonparametric Tests for Randomness in Sequences*, NASA Technical Note D-3766, December 1966.

4. A. WALD & J. WOLFOWITZ, "On a test whether two samples are from the same population," *Ann. Math. Statist.*, v. 11, 1940, pp. 147–162.

56[8].—FRANCIS J. WALL, *Tables of the Generalized Variance Ratio or U-Statistic*, ms. of 32 computer sheets deposited in the UMT file.

Univariate analysis of variance possesses a direct generalization for vector

variables, leading to an analysis of vector sums of squares. A multivariate analogue of the variance σ^2 of a univariate distribution is the determinant of the covariance matrix Σ , called the "generalized variance" [1].

The generalized variance ratio or U -statistic, here tabulated, is the ratio of the likelihood estimate of the generalized residual variance assuming that the hypothesis is false to the likelihood estimate assuming that the hypothesis is true. The parameters for the U -statistic are the dimension p of the covariance matrix Σ and the degrees of freedom, q and n , for the hypothesis and error, respectively.

These unpublished tables, computed on an IBM 360 Model 40 system, consist of 6D values of $U(p, q, n)$ for $p = 1(1)8$, $q = 1(1)15(3)30(10)40(20)120$, $n = 1(1)30(10)40(20)140(30)200, 240, 320, 440, 600, 800, 1000$, at confidence levels $\alpha = 0.05$ and $\alpha = 0.01$, respectively.

As a partial check, recent 3D tables of Schatzoff [2] have been used by the author to recompute the U -statistic for $p = 4(2)10$, $q = 4$; $p = 5(7)9$, $q = 6$; $p = 3, 7$, $q = 8, 10$. These results were found to agree to at least 3D with the corresponding data in the more extended tables under review, which are the most elaborate of this type thus far calculated.

J. W. W.

1. T. W. ANDERSON, *An Introduction to Multivariate Statistical Analysis*, John Wiley & Sons, New York, 1958.

2. MARTIN SCHATZOFF, "Exact distributions of Wilks's likelihood ratio criterion," *Biometrika*, v. 53, 1966, pp. 347-358.

57[12].—J. M. FOSTER, *List Processing*, American Elsevier Publishing Co., Inc., New York, 1967, 54 pp., 23 cm. Price \$4.50.

This is an excellent little book. It introduces the concepts of list-processing within a programming language which is an extension of ALGOL. This has the advantage that many of the techniques which are used in languages like LISP or IPL-V can be illustrated rather simply, and in a way which makes them easily accessible to the programmer who only knows ALGOL, or even FORTRAN. It has the slight disadvantage that the more innovative features of list-processing languages are lost, such as the lack of distinction between program and data in LISP, or the form of the replacement rule in SNOBOL.

The description of list-processing facilities is based mainly on LISP, both from the point-of-view of the user and of the implementer. Other established list-processing languages are also discussed.

The book reads very easily, but is by no means superficial, and would be very useful in an introductory course on programming or machine intelligence.

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58[12].—JAMES T. GOLDEN & RICHARD M. LEICHUS, *IBM 360 Programming and Computing*, Prentice-Hall, Inc., Englewood Cliffs, N. J., 1967, ix + 342 pp., 26 cm. Price \$5.50.

This recent entry to the steadily increasing ranks of IBM 360 programming texts

is probably the most detailed and complete to date. Its twelve chapters deal with decimal and binary programming, indexing, logical bit and byte manipulations, floating-point operations, macros, subroutines and linkages, 360 I/O operations, I/O software, operating systems in general, and the 360 Disk Operating System in particular.

Although intended as an introductory text to programming and operating systems, with reference to the IBM 360, this book turns out to be much too sophisticated to be termed "Introductory" and much too involved with the inner workings of the 360 to be considered general (e.g., hexadecimal).

However, the authors have treated their subject remarkably well, leaving very little to be desired. The text is adequately prepared with flow charts, diagrams, and programming examples. Most examples are discussed in great detail and are easy to follow. The result is an excellent book in 360 machine language programming for both reference and self-instruction. While seasoned programmers might find the book well suited to their tastes, the inexperienced novice could have a fairly rough time with the material. But that might be a problem more inherent in the machine involved than in the book itself.

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59[13].—LESLIE C. EDIE, ROBERT HERMAN & RICHARD ROTHERY, Editors, *Vehicular Traffic Science*, American Elsevier Publishing Co., Inc., New York, 1967, x + 373 pp., 24 cm. Price \$16.00.

We quote from the preface:

This volume contains the Proceedings of The Third International Symposium on the Theory of Traffic Flow held under the auspices of the Transportation Science Section of the Operations Research Society of America.

The Symposium was held in New York City during June, 1965. Forty-five technical papers were presented, all of which are published in these Proceedings in full or in the form of summaries. They cover a variety of traffic phenomena relating to single-lane, two-lane, and multi-lane traffic flow; general theory and experiment; networks and intersections; pedestrian and vehicle gap acceptances; simulation; and economics and scheduling. The program of this meeting reflected a continuation and expansion of the fields of research which were covered in the first two symposia as well as the development of new ideas. The aim of the work is to develop an understanding of vehicular traffic which will contribute to the solution of the pressing problems of traffic congestion, delays, and accidents with their rising economic costs to society as a whole, and their rising personal costs to individual members of society in terms of human frustration and suffering.

E. I.

60[13.05].—M. CHRETIEN & S. DESER, Editors, *Axiomatic Field Theory*, Vol. I, Brandeis University Summer Institute in Theoretical Physics, 1965, Gordon & Breach, Science Publishers, Inc., New York, 1966, xi + 516 pp., 24 cm. Price \$32.50.

[13.05].—M. CHRETIEN & S. DESER, Editors, *Particle Symmetries*, Vol. II, Brandeis University Summer Institute in Theoretical Physics, 1965, Gordon & Breach, Science Publishers, Inc., New York, 1966, xi + 691 pp., 24 cm. Price \$35.00.

This publication of lecture notes fills a gap between current textbooks and the advanced literature in two areas of theoretical physics. The first volume, devoted to axiomatic field theory, will appeal to the more mathematically minded reader. It shows what mathematical methods and concepts are especially important in this area, how they are used by physicists, and what the specific problems are for physicists to solve. The analytic and the algebraic approach to field theory as well as a fundamental discussion on group theory are contained in this volume.

The second volume deals with elementary particle physics, mainly stressing the symmetric aspect. It gives a survey of the present experimental situations and covers some major attempts to understand it on a phenomenological basis. Group theoretical as well as dynamical models are discussed.

Naturally, completeness is not in the scope of publications of lecture notes. Instead, the volumes under review emphasize the presentation of very advanced material on subjects which, due to intensive research, are developing extremely rapidly.

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