

A Note on Sums of Four Cubes

By M. Lal, W. Russell and W. J. Blundon

Abstract. A search for the integral solutions of the Diophantine equation $x^3 + y^3 + 2z^3 = k$, for $|x|$, $|y|$ and $|z| < 10^5$ was made on an I.B.M. 1620 Model 1. These results showed that there are now just 19 values of k in the range $1 \leq k \leq 999$ for which no solution is known. ■

Sierpiński [4] asks the question, "Can every natural number k be put in the form $x^3 + y^3 + 2z^3$, where x, y, z are integers?". Partial answers have been given by Ko [1], Makowski [2] as well as by Schinzel and Sierpiński [3]. Ko found a representation for every natural number up to 100 with the exception of the num-

TABLE 1
Integral Solutions of $x^3 + y^3 + 2z^3 = k$.

X	Y	Z	K
1118	2723	-2210	99
-2212	2549	-1421	99
-2276	-2627	2464	229
4636	-5699	3496	229
1099	-1199	583	274
5185	-5201	865	274
1523	-1525	191	284
-8413	8507	-2161	284
778	-2072	1615	454
-4991	-9710	8041	571
1168	-1169	127	589
3088	7168	-5837	598
2926	4405	-3809	643
7210	7939	-7592	643
-2415	2507	-942	692
-2631	2653	-613	692
2647	-3003	1622	692
3385	3955	-3692	724
-1867	2318	-1438	725
4931	5894	-5455	725
-944	-1155	1060	741
-1007	8148	-6463	755
680	1113	-946	825
-2218	2531	-1384	851
2972	3371	-3184	851
-1274	1849	-1286	913
-8561	8626	-1931	913
-9142	9587	-3883	941
-4117	6521	-4699	950
6220	9268	-8033	958

bers 76 and 99. Makowski investigated the range $101 \leq k \leq 220$ and gave representations for all but the cases $k = 113, 148, 183, 190, 195$.

In the investigations [1], [2] the parameters were small; in fact, the absolute value of each is less than 260. Since it is possible that some of the missing solutions might be just outside that range, we investigated the problem of finding further solutions of the Diophantine equation

$$(1) \quad x^3 + y^3 + 2z^3 = k,$$

where the parameters $|x|, |y|, |z|, k$ were allowed to vary up to 999.

This search gave representations for all but 39 values of k . We note the following three solutions missing in [2].

$$(-133)^3 + (-46)^3 + 2 \times (107)^3 = 113,$$

$$(-602)^3 + 450^3 + 2 \times (399)^3 = 190,$$

$$(-79)^3 + 126^3 + 2 \times (-91)^3 = 195.$$

The representation for $k = 113$ is reported in [4] to have been found by K. Moszynski and J. Swianiewicz.

On an I.B.M. 1620 Model 1, a Fortran program for this problem took about 15 hours to run. Later on, we rewrote the program in the assembler language which ran about 15 times faster and decided to extend the search to 10^4 . This required a computing time of 1000 hours and the run was made over a period of one year on a low priority basis. This extended search gave us solutions for 20 more values of k and they are given in Table 1.

As a result, we now have just 19 values of k in the range $1 \leq k \leq 999$ for which no solution of (1) is known. These values are recorded in the Table 2 below.

TABLE 2
*Values of k in the Range $1 \leq k \leq 999$ for which
no Solution of $x^3 + y^3 + 2z^3 = k$ is known.*

76	356	491	788
148	418	519	923
183	428	580	931
230	445	671	967
253	482	734	

The detailed results of our investigations are being prepared for depositing in the Unpublished Mathematical Tables file of this journal.

Department of Mathematics
Memorial University of Newfoundland
St. John's, Newfoundland, Canada.

1. C. Ko, "Decompositions into four cubes," *J. London Math. Soc.*, v. 11, 1936, pp. 218-219.
2. A. MAKOWSKI, "Sur quelques problèmes concernant les sommes de quatre cubes," *Acta Arith.*, v. 5, 1959, pp. 121-123. MR 21 #5609.
3. A. SCHINZEL & W. SIERPIŃSKI, "Sur les sommes de quatre cubes," *Acta Arith.*, v. 4, 1958, pp. 20-30.
4. W. SIERPIŃSKI, *A Selection of Problems in the Theory of Numbers*, Macmillan, New York, 1964, p. 115. MR 30 #1078.