REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS

The numbers in brackets are assigned according to the indexing system printed in Volume 22, Number 101, January 1968, page 212.

1[2.05, 2.35].—J. Kowalik & M. R. Osborne, *Methods for Unconstrained Optimization Problems*, American Elsevier Publishing Co., Inc., New York, 1969, xii + 148 pp., 24 cm. Price $9.50.

Within the last ten years, there has been increasing interest in the problem of minimizing a function of n variables numerically. During that time several new methods have appeared and numerical experiments as well as mathematical analyses have shed much light on some of these newer as well as the older methods. The authors of this new book, the first dealing solely with this subject, address themselves to giving a survey of this material.

The first, very short, chapter contains some preliminaries and a second short chapter treats search methods such as that of Rosenbrock and of Hook and Jeeves. The third chapter considers the classical method of steepest descent as well as the more recent variation of Davidon and methods employing conjugate directions. In Chapter 4 the authors discuss the special case of least squares problems, and after a discussion of the linear problem as well as the Newton and secant methods for solving nonlinear systems of equations, the Gauss-(Newton) method together with the Levenberg modification is treated. The chapter ends with a description of a method of Powell. Chapter 5 considers various ways in which constrained problems may be handled by methods for unconstrained problems. Chapter 6 records the results of various numerical experiments. The book ends with short appendices on matrices and convexity as well as some "Notes on Recent Developments."

In contradiction to the publisher's claim, it is difficult to classify this work as a textbook. There are no exercises and the style is mostly descriptive with only a few convergence theorems proved. However, it should be useful as supplemental reading for courses in both numerical analysis and nonlinear programming as well as providing a readable introduction to the subject for practicing scientists and engineers.

J. M. O.

2[2.10].—Bruce S. Berger, Robert Danson & Robert Carpenter, *Tables of Zeros and Weights for Gauss-Hermite Quadrature for* $N = 200, 400, 600, 800, 1000,$ *and* 2000, ms. of 3 typewritten pp. + 50 computer sheets deposited in the UMT file. (Copies also obtainable from Professor Berger, Department of Mechanical Engineering, The University of Maryland, College Park, Md. 20742.)

The authors continue herein their tabulation of the abscissas and weights associated with certain Gauss quadrature formulas [1], [2].

As stated in the title, the present tables relate to the Gauss-Hermite formula when the number, N, of abscissas is equal to 200(200)1000 and 2000. The abscissas x_{kN} (the zeros of the Hermite polynomial $H_N(x)$), the corresponding weights, and the products of the weights multiplied by exp (x_{kN}^2) are all tabulated in floating-point form to 27S, except for $N = 2000$, where the precision is limited to 26S. (Because of symmetry, only the positive zeros and corresponding weights are given.)

The zeros and weights were checked to the precision of the printed entries by the respective relations

$$\prod_{k=1}^{N/2} x_{kN}^2 = N!/[2^N(N/2)!] \quad \text{and} \quad 2\sum_{k=1}^{N/2} a_{kN} = \sqrt{\pi}\,.$$

Further details of the calculations are presented in the introduction to these tables, which constitute a unique supplement to the valuable tables of Stroud & Secrest [3].

J. W. W.

1. Bruce S. Berger & Robert Danson, *Tables of Zeros and Weights for Gauss-Laguerre Quadrature*, ms. deposited in UMT file. (See *Math. Comp.*, v. 22, 1968, pp. 458–459, RMT **40.**)
2. Bruce S. Berger, Robert Danson & Robert Carpenter, *Tables of Zeros and Weights for Gauss-Laguerre Quadrature to* 24S *for* $N = 400$, 500, *and* 600, and *Tables of Zeros and Weights for Gauss-Laguerre Quadrature to* 23S *for* $N = 700$, 800, *and* 900, mss. deposited in the UMT file. (See *Math. Comp.*, v. 23, 1969, p. 882, RMT **60.**)
3. A. H. Stroud & Don Secrest, *Gaussian Quadrature Formulas*, Prentice-Hall, Englewood Cliffs, N. J., 1966. (See *Math. Comp.*, v. 21, 1967, pp. 125–126, RMT **14.**)

3[2.20, 13.00].—R. J. Herbold & P. N. Ross, *The Roots of Certain Transcendental Equations*, Professional Services Department, The Procter and Gamble Co., Winton Hill Technical Center, Cincinnati, Ohio, ms. of 32 typewritten pp. deposited in the UMT file.

Herein are tabulated 15S values (in floating-point form) of the first 50 positive roots of the equations $x \tan x - k = 0$ and $x \cot x + k = 0$, for $k = 0(0.001)0.002(0.002)0.01$, $0.02(0.02)0.1(0.1)1(0.5)2(1)10(5)20(10)60(20)100$. These decimal approximations to the roots were calculated on an IBM 360/65 system, using Newton-Raphson iteration programmed in double-precision Fortran.

As the authors note in their introduction, these tables can be considered to be an extension of the tables of Carslaw & Jaeger [1] for the same range of the parameter k.

A specific example of the numerical solution of a problem in the field of chemical engineering is adduced to show the practical need for these more elaborate tables.

J. W. W.

1. H. S. Carslaw & J. C. Jaeger, *Conduction of Heat in Solids*, 2nd ed., Oxford University Press, Oxford, 1959.

4[2.35, 6].—Louis B. Rall, *Computational Solution of Nonlinear Operator Equations*, John Wiley & Sons, Inc., New York, 1969, viii + 224 pp., 24 cm. Price $14.95.

The topic of this book has received an increasing amount of research in the last several years, and many books devoted at least in part to this subject have already appeared (for example, the books by Collatz, Goldstein, Kantorovich and Akilov, Keller, Kowalik and Osborne, and Ostrowski). In comparison with these existing works, the present book places somewhat more emphasis on computational aspects and practical error analysis.

Chapter 1, comprising about a quarter of the book, is an introduction to Hilbert and Banach spaces and the solution of linear operator equations, while a somewhat shorter Chapter 3 contains additional background material on differentiation of nonlinear operators. In between is a short (29 pages) chapter on the contraction mapping principle, and some of its variants, with an application to an integral equation.

The last and longest chapter (85 pages) is devoted to Newton's method and a few of its modifications. The author presents the now famous Kantorovich analysis and then discusses in some detail the implications of these results for practical programming. This discussion, together with the point of view put forth, is perhaps the strongest and most novel part of the book and concerns primarily the author's own research using interval arithmetic and similar ideas in order to obtain error bounds. Additional results relating to error estimation, as well as applications to various differential and integral equations, are also given.

Although the book makes a valuable contribution in those areas that it covers, its scope is rather limited. There is little or no mention of the important class of minimization methods nor of a large number of variants of Newton's method including, in particular, the secant methods and more recent "quasi-Newton" methods. Even within the confines of Newton's method, the author has restricted his analysis to the setting of normed linear spaces which precludes mention of the powerful results available in partially ordered linear spaces.

Nevertheless, the book will make useful supplementary reading in various graduate courses in both computer science and mathematics. Unfortunately, however, it seems to be overpriced, for its size, by a factor of almost two.

J. M. O.

5[2.45, 12].—Julius T. Tou, Editor, *Advances in Information Systems Science*, Vol. 1, Plenum Press, New York, 1969, xv + 303 pp., 23 cm. Price $14.00.

This volume is part of a proposed series which attempts "(1) to provide authoritative review articles on important topics which chart the field with some regularity and completeness, and (2) to organize the multidisciplinary core of knowledge needed to build a unified foundation." The articles in this volume do indicate some of the most prominent directions in the field of computer or information systems science. The series is aimed at "a wide audience, from graduate students to practicing engineers and active research workers." However, in order to avoid the learning of appropriate responses to the terminology of the field without learning the meanings of the terms, the prospective reader should have some familiarity with the field, especially the history and justification of the current lines of investigation.

The first article, "Theory of Algorithms and Discrete Processors," by V. M. Glushkov and A. A. Letichevskii (translated by Edwin S. Spiegelthal), is slightly out of place in this collection. In a modified form (even including its brief and incomplete excursion into the history of the development of automata theory) it might have been published as original research. Still, it is representative enough to serve as a description, by example, of one of the main directions of automata theory.

Having discussed "discrete processors" in an automata theory context, the authors develop a theory of "algorithmic algebras" aimed finally at studying various equivalence problems of automata theory. The references give the researcher a copious supply of entry points to the Russian literature on automata. The student should definitely be familiar with what the authors call "classical automata theory" before beginning this chapter.

The second article, "Programming Languages," by Alfonso Caracciolo de Forino, is to be commended for its attempt to integrate the theory of programming languages into a general philosophical framework. The philosophy is Carnap's Semeiotics, "the general theory of signs and of their significance." Unfortunately, the philosophical discussion cannot be taken literally. It requires an insight into what the author is trying to say rather than what he says when, for example, he refers to "the computation of a quantity not finitely representable, such as the number π." Except for trouble caused by Carnap's terminology and the continually strained insistence that the metaphorical references to the concepts of "language," "meaning," "knowledge," "understanding," etc. are not metaphorical, the article is easily accessible to anyone familiar with a higher-level programming language such as ALGOL. Aside from the philosophy, the article is a genuine survey of the field of programming languages emphasizing work on their formal definition. The references represent the current literature of the field well. Many problem areas are mentioned and the author's suggestions might be useful to those interested in entering the field.

The most accessible article of the volume is "Formula Manipulation—The User's Point of View," by M. E. Engeli. This article reviews its area by introducing a programming language SYMBAL as an example of the direction in which formula manipulation oriented programming languages should be moving. The language is a modification of ALGOL. The syntax is presented in Backus Normal Form; but the author does not get carried away by formalism. The study of SYMBAL in some depth complements the survey of the previous chapter with very little overlap. Again, this chapter provides areas of interest to those who are entering the field, at a level accessible to anyone who has been briefly acquainted with ALGOL. The references are so current as to include D. E. Knuth's *The Art of Computer Programming*, Vol. II.

The fourth article, "Engineering Principles of Pattern Recognition," by Julius T. Tou (the editor of the series) is another survey of pattern recognition methods: "Distance Functions," "Potential Functions," "Likelihood Functions," and "Entropy Functions." The chapter emphasizes adaptive methods and "training algorithms."

There is significant overlap between the fourth article and the fifth, "Learning Control Systems," by K. S. Fu. Fortunately, the notations are sufficiently similar that the reader can adjust to this situation. Both of these articles rely on the reader's familiarity with "statistical decision theory" and refer to information theory for many of the concepts discussed.

The volume, as a whole, is good as a descriptive work but short on justification. The interested reader should be warned against accepting certain problem areas as worthy of study merely because they are, as the book reports, being studied. If this warning is heeded, the proposed series should serve as a good introduction to current research in "information systems science."

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6[3, 13.15].—RALPH A. WILLOUGHBY, Editor, *Proceedings of the Symposium on Sparse Matrices and Their Applications*, IBM Corporation, Thomas J. Watson Research Center, Yorktown Heights, New York 10598.

The symposium was held at the Thomas J. Watson Research Center on September 9 and 10, 1968, with 124 registered participants representing many fields of application. Included in this volume are summaries of the talks, usually of about eight or ten pages, together with an "edited version" of a panel discussion forming the closing session.

The eigenvalue problem came up only during the panel discussions and the contributions were meager. Otherwise, only inversion and the solution of linear systems were discussed. The treatment of large sparse systems is not yet to be found in the textbooks, and only occasionally in the periodicals concerned with numerical analysis. But special techniques have been devised for linear programming problems and for the analysis of power networks, in particular, and these are described in the literature dealing with these areas. This seems to be the first effort to bring together mathematicians and programmers, and specialists in their diverse areas, in order to coordinate and systematize their work. It is claimed that in some cases 100-fold reductions are achieved. This, and the range of applications, provide impressive evidence of the worthwhileness of the project.

The novice will not find in this an easy introduction to the subject in general or to any one technique in particular. But he can find indications of the various methods of approach and sometimes extensive lists of publications for further study. And the expert may well learn of other approaches he had not previously come across.

A. S. H.

7[3].—J. A. WILKINSON, *Rundungsfehler*, translated from English into German by G. Goos, Springer-Verlag, New York, 1969, x + 208 pp., 21 cm. Price $3.70 (paperbound).

This translation contains minor corrections of the earlier English version: *Rounding Errors in Algebraic Processes*. See review RMT **90**, vol. 18, no. 88, p. 675.

E. I.

8[3].—H. R. SCHWARZ, H. RUTISHAUSER & E. STIEFEL, *Numerik Symmetrischer Matrizen*, B. G. Teubner Verlag, Stuttgart, 1968, 243 pp., 22 cm. Price DM 34 —.

The names of the three authors should be sufficient to recommend this book to anyone interested in the material covered. Hence the reviewer's job is simplified. It is sufficient to indicate what material is covered. The title as it appears on the cover is "Matrizen-numerik" which is a bit misleading, but the proper title, given above, is sufficiently explicit. Only real symmetric matrices are considered.

There are five chapters. The first is elementary, introducing linear vector spaces, and norms and condition numbers, then passing to conditions for definiteness, and finally developing the method of Cholesky. The next chapter is on "relaxation methods" including, of course, the conjugate gradient method. Next comes the least-squares problem with the Schmidt orthogonalization. The eigenvalue problem, Chapter 4, takes up the most space, 90 pages, and the book concludes with something over forty pages on boundary value problems. There is a bibliography of 80 items, and a five-page index. Several ALGOL programs are included, and a number of numerical illustrations, but familiarity with ALGOL is no prerequisite. On the whole, this is a clear, careful, and authoritative exposition requiring very little for background.

A. S. H.

9[3].—Donald D. Spencer, *Game Playing with Computers*, Spartan Books, New York, 1968, 441 pp., 24 cm. Price $12.95.

Game Playing with Computers is written by an amateur games player who is a novice at computing and an even greater amateur at book writing. By extending the notion of a "game" to include any sort of recreational aspect of computing, the author has assembled a strange mishmash of information. The book will delight freshman students of computing, since it includes complete programs (some in Fortran, some in Basic) for playing Blackjack, the 15 Puzzle, constructing magic squares, and sifting small primes. Flowcharts are given for playing Tic-Tac-Toe, Roulette, and the construction of knight's tours. In almost all cases, the packaged solutions represent bad computing in the sense of using brute force rather than intelligence.

Many games are described without relating them in any way to the book's title. In one case (Checkers and Kings), a flowchart and program are given for which the point seems to be the logic of counting the elements in an array. In several instances, the author throws in photographs of the punched cards he used (very badly reproduced by the publisher).

It is difficult to deduce the point of this book, or its possible audience, or just where a computer enters the game. It might be that it will reduce the number of student programs to play Tic-Tac-Toe (or then it might increase the number), but it will hardly foster any good computing. Perhaps the greatest value of the book is in its descriptions of many games of chance, most of which are presented with no suggestion of how to apply a computer to their analysis.

Brief appendices provide a reference list of books and articles on game playing and descriptions of the Fortran and Basic languages.

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10[7].—Rudolph Ondrejka, 1273 *Exact Factorials*, one ms. volume of 671 unnumbered computer sheets deposited in the UMT file.

This large volume gives the exact values of $n!$ for $n = 1(1)1273$, with no explanatory text. However, private correspondence with this reviewer revealed that the calculation of the table was performed on a Honeywell 200 computer system and the first 10S of each entry were checked with the corresponding entry in the table of Reid & Montpetit [1].

Furthermore, the value of 1000! has been found by the reviewer to agree with the final entry in the table of Lal & Russell [2], which was not available to the author.

J. W. W.

1. J. B. Reid & G. Montpetit, *Table of Factorials* 0! *to* 9999!, Publication 1039, National Academy of Sciences—National Research Council, Washington, D. C., 1962. (See *Math. Comp.*, v. 17, 1963, p. 459, RMT **67.**)
2. M. Lal & W. Russell, *Exact Values of Factorials* 500! *to* 1000!, Department of Mathematics, Memorial University of Newfoundland, St. John's, Newfoundland, Canada, undated. (See *Math. Comp.*, v. 22, 1968, pp. 686–687, RMT **68**, and the references cited therein.)

11[7].—Rudolph Ondrejka, *Tables of Double Factorials*, two ms. volumes, each of 618 unnumbered computer sheets deposited in the UMT file.

The first of these impressive volumes gives the exact values of $(2n - 1)!!$ for $n = 1(1)1162$; the second gives the companion values of $(2n)!!$ for $n = 1(1)1161$, all computed on a Honeywell 200 computer system.

The author collated these tables with the corresponding entries in his earlier table [1] of double factorials and found complete agreement.

The present elaborate tables should prove more than adequate to satisfy the requirements of most users of such data.

J. W. W.

1. Rudolph Ondrejka, *The First* 100 *Exact Double Factorials*, ms. in the UMT file. (See *Math. Comp.*, v. 21, 1967, p. 258, RMT **16.**)

12[7].—James D. Talman, *Special Functions, A Group Theoretic Approach*, W. A. Benjamin, Inc., New York, 1969, xii + 260 pp., 24 cm. Price \$13.50 cloth, \$5.95 paper.

An intriguing aspect of mathematical thinking is the variegated approaches and analyses used to extend well-established theories and construct new ones. The properties of the special functions of mathematical physics are usually studied on the basis of their analytic character, the principal tools being the theory of analytic functions. Thus special functions can be studied as the solution of differential and difference equations. They can be characterized by definite integrals, power series, addition theorems and so on.

The volume under review is based on lectures by E. P. Wigner and approaches the subject of special functions from the group theoretic standpoint. The general introduction is written by E. P. Wigner. He points out that the lectures began with the observation that the results of the analytic theory are more general than those derived from the group-theoretic analyses. Since the time of the lectures, this drawback has been considerably reduced by subsequent developments. "This in no way diminishes the beauty and elegance of the analytic theory, or the inventiveness that was necessary to its development. Rather, the claim of the present volume is to point to a role of the 'special functions' which is common to all, and which leads to a point of view which permits the classification of their properties in a uniform fashion."

Wigner continues as follows:

"The role which is common to all the special functions is to be matrix elements of representations of the simplest Lie groups, such as the group of rotations in three-space, or the Euclidean group of the plane. The arguments of the functions are suitably chosen group parameters. The addition theorems of the functions then just express the multiplication laws of the group elements. The differential equations which they obey can be obtained either as limiting cases of the addition theorems or as expressions of the fact that multiplication of a group element with an element in the close neighborhood of the unit element furnishes a group element whose parameters are in close proximity to the parameters of the element multiplied. The integral relationships derive from Frobenius' orthogonality relations for matrix elements of irreducible representations as generalized for Lie groups by means of Hurwitz's invariant integral. The completeness relations have a similar origin. Further relations derive from the possibility of giving different equivalent forms to the same representation by postulating that the representatives of one or another subgroup be in the reduced form. Finally, some of the Lie groups can be considered as limiting cases of others; this furnishes further relations between them. Thus, the Euclidean group of the plane can be obtained as a limit of the group of rotations in three-space. Hence, the elements of the representations of the former group (Bessel functions) are limits of the representations of the latter group (Jacobi functions)."

The author has attempted to make the book self-contained. Thus, about the first third of the volume is a preliminary discussion of abstract groups, Lie groups and algebras, group representations and related topics (Chapters 1–7). The remaining chapters consider various groups, and the special functions are studied in connection with the group with which they are related. Chapters 8 and 9 take up rotation in 2-space and 3-space, respectively. Some associated special functions are the so-called $3 - j$ coefficients, harmonic polynomials and spherical harmonics. Chapter 10 is called Rotation in Four Dimensions, and properties of Gegenbauer polynomials are considered. Chapter 11, called Euclidean Group in the Plane, studies properties of Bessel functions of integer order and Chapter 12, called The Euclidean Group in Space, studies properties of spherical Bessel functions. Finally, the title of Chapter 13 is The Quantum-Mechanical Group, and here the Laguerre polynomials enter.

The acid test of any scientific theory is that it should include existing knowledge and produce results not forthcoming from extant developments. To date I know of no results on the special functions which follow from the group approach that cannot be deduced from the conventional analytic approaches. On the other hand, there are numerous results on the special functions which do not arise from the group-theoretic analyses. Thus there is a challenge—can future developments alter this situation? The present volume is very readable and should prove valuable to workers in this area.

Y. L. L.

13[7].—HENRY E. FETTIS & JAMES C. CASLIN, *Table of Modified Bessel Functions*, Report ARL-69-0032, Aerospace Research Laboratories, Wright-Patterson Air Force Base, Ohio, February 1969, iv + 232 pp., 27 cm. Price $3.00. (Obtainable from Clearinghouse, U. S. Department of Commerce, Springfield, Virginia 22151.)

The main table in this report is a photographic reproduction of a manuscript table [1] compiled by the authors in 1967, using an IBM 7094 system. It consists of 15S values of $I_0(x)$ and $I_1(x)$ and their respective products with e^{-x} for $x = 0(0.001)10$.

To this is now appended a 16S table of $I_n(x)$ and $e^{-x}I_n(x)$, for $x = 1(1)10$, $n = x(1)x + 25$. All entries in both tables are given in floating-point form.

The report concludes with a list of terminal-digit errors in Table 9.8 in the NBS *Handbook*, which the authors have previously announced [2], except for one new round-off error; namely, the final digit in the NBS value of $e^{-x}I_1(x)$ for $x = 1$ should read 3 instead of 4.

J. W. W.

1. HENRY E. FETTIS & JAMES C. CASLIN, *Tables of the Modified Bessel Functions* $I_0(x)$, $I_1(x)$, $e^{-x}I_0(x)$, *and* $e^{-x}I_1(x)$, Aerospace Research Laboratories, Wright-Patterson Air Force Base, Ohio, March 1967, deposited in the UMT file. (See *Math. Comp.*, v. 21, 1967, pp. 736–737, RMT **91.**)
2. *Math. Comp.*, v. 22, 1968, p. 244, MTE **418.** (This errata notice is incorrectly cited on p. 4 of the report.)

14[8].—V. I. PAGUROVA, *A Comparison Test for the Mean Values of Two Normal Samples*, Reports in Computational Mathematics, No. 5, Computing Center of the Academy of Sciences of the USSR, Moscow, 1968, 59 pp. (In Russian.)

Suppose we are given samples of size n_1, n_2 respectively from two (scalar) normal populations. How do we best test whether or not the mean values of the two populations differ, using a criterion conservative enough so that if the means are the same, we decide otherwise at most $\alpha\%$ of the time? If the variances σ_1^2, σ_2^2 of the two populations are the same, this is a standard problem; the best procedure is to calculate a certain statistic ν_1 and then see whether or not $|\nu_1| \geqq f$, where f is the $(1 - \alpha/2)$-percentile of the Student t-distribution with $n_1 + n_2 - 2$ degrees of freedom. The general problem (if the ratio of the variances is unknown) is known as the Behrens-Fisher problem and is much more difficult (there is apparently no best test for all values of the ratio of the variances).

In tackling this problem, the author extends an approach due to the statistician Wald for $n_1 = n_2$. A statistic ν_2 similar to (but not identical with) the statistic ν_1 is found, and also a rejection level $f = f[\eta(c), n_1, n_2, \alpha]$, where n_1, n_2 are the sample sizes, α is a "nominal" level of significance, and c is an estimate (from the data) of $c' = (\sigma_1^2/n_1)/(\sigma_1^2/n_1 + \sigma_2^2/n_2)$. The test $|\nu_2| \geqq f$ then has level of significance of order α, with better approximation for large n_1 and n_2. Indeed, the author tabulates min α and max α, which are the minimum and maximum possible values of the true level of significance of the test over all c', $0 \leqq c' \leqq 1$, for fixed n_1, n_2, α.

For the theoretical arguments behind his approach, see the author's article in *Theor. Probability Appl.*, v. 13, 1968, No. 3 (English translation).

The author gives tables for $f[\eta(c), n_1, n_2, \alpha]$ and min α, max α, for "nominal" $\alpha = 10\%, 5\%, 2\%, 1\%$, and $\frac{1}{2}\%$, $c = 0(.1)1$, $n_1 = 1$, $n_2 - 1 = 3(1)10, 12, 15, 20,$

24, 30, 40, 60, 120, ∞, $n_1 \leqq n_2$. An ALGOL code (in English) is given for the calculation of f, min α and max α, as well as for the $\alpha/2$-percentiles of the Student t-distribution.

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15[9].—JOSEPH B. MUSKAT & ALBERT L. WHITEMAN, *The Cyclotomic Numbers of Order Twenty*, University of Pittsburgh, Pittsburgh, Pennsylvania, and University of Southern California, Los Angeles, California, 40 computer sheets deposited in the UMT file.

This table presents formulas for the cyclotomic numbers of order 20. The derivation and computation of these formulas are described in [1].

The 400 cyclotomic numbers (h, k), $0 \leqq h, k \leqq 19$, can be grouped into 77 sets. There is a formula for each set, a linear combination of the prime p, a constant, and sixteen variables associated with Jacobi sums. The formulas depend, however, on ind 2 (mod 10) et al., so that there are forty different cases. All forty cases are given, one per sheet. Considerably fewer are necessary, for some cases can be derived from others merely by changing the primitive root used in generating the cyclotomic numbers.

AUTHORS' SUMMARY

1. JOSEPH B. MUSKAT & ALBERT L. WHITEMAN, "The cyclotomic numbers of order twenty," *Acta Arithmetica*, v. 17, no. 2, (to appear).

16[12].—R. E. GRISWOLD, J. F. POAGE & I. P. POLONSKY, *The SNOBOL 4 Programming Language*, Prentice-Hall, Inc., Englewood Cliffs, N. J., x + 221 pp., 28 cm. Price $6.50 (paperbound).

SNOBOL 4 is a general-purpose string manipulation language and includes many novel features. Wider use has been hampered by the low availability of information about SNOBOL 4, except for photocopied journal extracts. This book clearly and cleanly fills this gap. It includes descriptions and examples of all currently implemented facilities. Many common problems of SNOBOL 4 users are resolved. Also included are seven complete working programs, although none seem to be real solutions of real problems. The book is aimed at advanced students and those with some programming experience and problems which may be solved by SNOBOL 4, and should hit this target well. It should be read by all with any possible interest in SNOBOL 4.

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17[12].—JOHN A. N. LEE, *The Anatomy of a Compiler*, Reinhold Publishing Corp., New York, 1967, xi + 275 pp., 24 cm. Price $13.75.

This discursive book, far more readible than anything previously available in its subject field, surveys compiler writing, touching in an introductory way on many principal compiler issues. It is quite suitable for classroom use in an introductory course, and also as a guide for the experienced programmer wishing to learn something of the inner workings of compilers. Its point of view is principally shaped by experience with FORTRAN compilers.

The layout of topics in this book is as follows. A first chapter discusses, in general terms, such basic terms as symbolic language, interpreter, compiler, bootstrapping, syntax-oriented translation. Chapter 2 introduces BNF as a mechanism for the definition of languages, and describes various possible additions to the basic BNF apparatus, that is, additions potentially useful in shortening syntactic descriptions. Chapter 3 is a broad introduction to the parsing problem, outlining top-down, bottom-up, and catch-as-catch-can approaches to compiling. After three additional chapters devoted to semantic issues, this discussion of parsing is continued in Chapter 7, which describes precedence parsing in its application to algebraic expressions and the use of precedence methods for translation from ordinary algebraic infix notation to Polish strings.

The remaining chapters of the book are concerned with the semantic portions of compilers, i.e., with the symbol table manipulating and code generating routines which compilers contain. Chapter 4 describes symbol tables in general, outlines the hash schemes by which they may be addressed, and surveys the lexical scan processes used to enter items into such tables. The same chapter goes on to discuss some of the basic object-code issues arising in the assignment of addresses to symbol table items: layout of arrays, analysis of equivalence declarations, treatment of COMMON blocks. Chapter 5 describes target code styles for the treatment of control statements, emphasising techniques available for use in "single-pass" compilers. Chapter 6 discusses some of the special issues arising in connection with FORMAT-controlled I/O statements, describing the structure of a FORMAT interpreter, and the way in which links between a program, its I/O subroutines, and an operating system may be constructed. Chapter 9 gives a general discussion of target code questions connected with subroutine linkages, indicating the manner in which these linkages may be compiled, describing the treatment of arrays when they occur as subroutine parameters, and discussing the special issues which arise when subroutine names are to be transmitted as parameters. Chapter 8 describes the code generation process in additional detail, indicating the manner in which straightforward code may be generated either from pre-compiled Polish strings or directly from algebraic formulae during a precedence parse and the manner in which simple local optimization may be included in this process. The same chapter also details code generation both for the addressing of indexed variables and for the invocation of separately compiled functions.

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18[12, 13.35].—F. Genuys, Editor, *Programming Languages*, Academic Press, New York, 1968, x + 295 pp., 24 cm. Price $15.00.

This book contains a collection of five articles based upon a series of lectures given at the NATO Advanced Study Institute in 1966. The articles are all on a high technical level, though there is considerable variation in clarity of writing.

The first article is by C. C. Elgot, and is entitled "Abstract Algorithms and Diagram Closure." It is a difficult and highly mathematical paper concerned with the theory of computation. An abstract algorithm is recursively defined, starting with a class of elementary imperative sentences. These imperative sentences assume the existence of an abstract machine with cells capable of holding contents. An elementary imperative sentence causes the contents of certain cells to be examined, the contents of certain other cells to be replaced, and control to be transferred to some other imperative sentence. Each such sentence is defined by a certain mapping function, known as a *direction*. A network of these sentences is considered to be a diagram, and every diagram may itself be treated as a direction. Thus the closure $\overline{D}$ of a set of directions D consists of those directions that can be computed from diagrams composed of directions in D. The paper consists of an exploration of the concept of diagram closure and related computability results. My own, somewhat subjective appraisal is that this work is highly overspecialized and is primarily a mathematical exercise. However, one's reaction depends upon one's general attitude towards current work in theory of computation; and Elgot's article is in any case representative of the best work in the field.

The second article is by E. W. Dijkstra, and is concerned with cooperating sequential processes. This article was my favorite, and I consider the book worth buying for Dijkstra's article alone. It is a brilliant and clear exposition of the subtle difficulties inherent when sequential computing processes, operating in parallel with unknown relative speeds, must cooperate and share information. Dijkstra's basic synchronizing device is the *semaphore*, which can be operated on by two operations: the P-operation and the V-operation. The V-operation adds one to the value of the semaphore; the P-operation subtracts one from the value of the semaphore as soon as the resulting value would be nonnegative. Thus the V-operation is, roughly speaking, a go-ahead signal issued by one process to others; the P-operation is a wait-for-go-ahead. Dijkstra shows how semaphores can be applied to the management of input-output buffering, to communications systems, and to storage management. Although the article does not even assume that the reader knows ALGOL, it is nevertheless a highly sophisticated treatment of the problems of parallel processing.

The third article, "Compiler Writing Techniques" by L. Bolliet, occupies nearly half of the entire book. Although the article is not well written, it contains a great deal of useful information about compiling algorithms, particularly those concerned with syntactic analysis. Much of the article consists of listings of actual ALGOL programs (sparsely commented). The basic orientation is towards the construction of an interpreter (rather than a compiler) for ALGOL, though the difference is relevant only in the post-syntactic phases of a compiler. The last section of the article treats compilation for multi-access systems and the problems of incremental compilation.

The fourth article, "Record Handling," is by C. A. R. Hoare. A *record* is a computational entity used to represent an object that has several distinguished subparts. Objects with the same subparts may be lumped into a *record class*: the subparts are known as *fields*. Since fields may themselves refer to other records, complex structures can thus be developed. Many ramifications of the record concept are explored, and the application of records in different programming languages is discussed. An appendix specifies the necessary additions to ALGOL 60 in order to include records. Although by this time Hoare's concepts are quite well known, the article is still well worth reading and helps to provide a unifying framework for a number of related approaches to record handling.

The final article, by Ole-Johan Dahl, is a survey of discrete event simulation languages. Considering that Dahl is one of the authors of SIMULA, a leading simulation language, this is a remarkably even-handed and impartial discussion of the field. The author considers five well-known simulation languages: GPSS, SIMSCRIPT, CSL, SOL, and SIMULA. Examples are drawn from all of them. Dahl discusses the peculiar requirements of simulation languages, and takes pains to warn the reader of the hazards of making predictions from computer simulations. This article is in much the same spirit as the Hoare article, and develops a unifying framework in which the different languages can be viewed. There are interesting interrelationships between simulation languages and Hoare's record concept, since simulated entities usually have just the kind of structure that Hoare is concerned with. However, since the articles are separately written, this connection unfortunately is not made explicit.

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19[13.35].—Robert E. Larson, *State Increment Dynamic Programming*, American Elsevier Publishing Co., Inc., New York, New York, 1968, xvi + 256 pp., 24 cm. Price $14.50.

Of all the primary ideas in optimization theory, dynamic programming has perhaps the most immediate and intuitive appeal. So fecund of application is "the principle of optimality" that one can readily forgive its ignorance of the subjunctive for the sake of the constant challenge it presents to the ingenuity in adapting it to a vast range of situations. Its basic notion can be seen from the simplest of examples, yet it reaches to the most recondite problems. It is in fact—as I once heard Bellman remark—"just mathematics; not difficult; all it requires is intelligence."

The two main drawbacks of dynamic programming were its lack of precision and what Bellman colorfully called "the curse of dimensionality." The work of Berkovitz and Dreyfus in 1964 laid a rigorous foundation and permitted the proper conditions of continuity to be imposed on optimal solutions. At about the same time Larson was beginning to overcome the practical difficulties that arise with more than one or two state variables. It is this work that he has now most usefully presented in book form. State increment dynamic programming starts from the standard functional equation of dynamic programming, but uses two ingenious modifications to reduce the computing time and storage requirements. The first is to choose the time-like increment so as to keep each successive state within a prescribed hypercube centered on the previous one. The second is to carry out the computations by blocks rather than in the sequence of time-like increments. The first saves storage and time in the high speed memory; the second saves time in transferring between low speed and high speed storage.

After an outline of the conventional procedure, the book gives a very clear description of the state increment computational procedure with various useful modifications. Many interesting examples are given in detail and a final chapter gives a development of the method of successive approximations. The author is to be congratulated on a notable and well-presented contribution to the art and craft of dynamic programming.

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