

Mathematics of Computation

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Edited by

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New Translations of the

INTRODUCTION TO THE THEORY OF LINEAR NONSELFADJOINT OPERATORS

By I. C. Gohberg and M. G. Kreĭn

Translations of Mathematical Monographs, Volume 18

396 Pages; List Price \$21.40; Member Price \$16.05

The theory of nonselfadjoint operators in Hilbert space is a recent branch of functional analysis. In recent times, it has attracted the ever increasing attention of mathematicians and physicists, and sometimes of engineers also. The aim of this book is to present a number of achievements in this field, most of them related to the theory of completely continuous operators.

The authors discuss the well-known results of the general theory of bounded non-selfadjoint operators; the theory of symmetrically-normed ideals of the ring of bounded operators in a Hilbert space; the theory of perturbation of determinants and some of its applications; the various theorems on the completeness of the system of root (eigen- and associated) vectors of a completely continuous operator (operator bundle); the study of the spectral properties of a selfadjoint quadratic bundle; the simplest tests for the existence of a basis (of one kind or another) made up of the root vectors of a given linear operator.

The presentation in this book is carried out in the spirit of the abstract theory of operators. It is illustrated by various applications to the theory of integral equations. The reader who has some experience in the theory of boundary value problems for differential equations, or an acquaintance with the theory of linear vibrating systems with a finite or infinite number of degrees of freedom, will easily discover how many of the results discussed here find immediate application in each of these fields.

MAHLER'S PROBLEM IN METRIC NUMBER THEORY

By V. G. Sprindžuk

Translations of Mathematical Monographs, Volume 25

200 Pages; List Price \$12.70; Member Price \$9.53

This book deals with the solution of a group of questions related both to the general theory of transcendental numbers and to the metrical theory of diophantine (and also algebraic) approximations. The fundamental problem in this field has been known in the literature since 1932 as Mahler's conjecture. The main result of this book is a proof of Mahler's conjecture and some analogous theorems.

In Part I, the "classical" case of Mahler's conjecture, dealing with real and complex numbers, is considered. This part should be comprehensible to anyone who knows the elements of measure theory and possesses sufficient perseverance in overcoming purely logical difficulties. Part II is concerned with locally compact fields with nonarchimedean valuation. This part requires a general familiarity with the structure of fields with non-archimedean valuation. All the necessary information is given in the text with references to the sources.

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GEOMETRIC THEORY OF FUNCTIONS OF A COMPLEX VARIABLE

By G. M. Goluzin

Translations of Mathematical Monographs, Volume 26

684 Pages; List Price \$35.50; Member Price \$26.63

The first edition of Goluzin's monograph was published in 1952, shortly after the author's death. In the last decade, an extensive literature has appeared on themes closely related to the content of this monograph, and many of these results were obtained in the works of Goluzin's pupils. A survey of this literature is given in a special supplement by N. A. Lebedev, G. U. Kuzmina, and Ju. E. Alenicyn.

The text of the book has undergone only slight modifications. Three bibliographic lists have been added. One of these corresponds to references made in the main text, another to the supplement. In addition, a complete list is given of Goluzin's works.

GEODESIC FLOWS ON CLOSED RIEMANN MANIFOLDS OF NEGATIVE CURVATURE

By D. V. Anosov

Proceedings of the Steklov Institute of Mathematics, Number 90

240 Pages; List Price \$15.20; Member Price \$11.40

The methods and results of this monograph are chiefly based on the fact that a geodesic flow on a closed Riemannian manifold of negative curvature satisfies a so-called (U)-condition, roughly expressible as follows: near an arbitrary fixed trajectory of the dynamical system, the behavior of the neighboring trajectories is similar to that of trajectories close to a saddle. Numerous examples are given of (U)-flows (continuous time) and (U)-cascades (discrete time). Most important among the many results is the theorem that every (U)-system is structurally stable in the sense that for an arbitrary, sufficiently small perturbation, there exists a homeomorphism of the phase space which is close to the identity and takes the trajectories of the unperturbed system into those of the perturbed system.

EXTREMAL PROBLEMS OF THE GEOMETRIC THEORY OF FUNCTIONS

Edited by Ju. E. Alenicyn

Proceedings of the Steklov Institute of Mathematics, Number 94

176 Pages; List Price \$11.80; Member Price \$8.85

This volume is a collection of papers on various problems in the geometric theory of functions of a complex variable. For the most part, the papers are the work of students of Gennadii Mihailovič Goluzin and are related to problems with which he has been concerned. The authors are Ju. E. Alenicyn, S. A. Gel'fer, E. G. Goluzina, G. V. Kuz'mina, N. A. Lebedev, I. A. Aleksandrov, I. M. Milin, M. I. Revjakov, G. A. Skotnikova, and N. M. Gol'dina. The work of the last three authors is based on theses written at Leningrad State University.

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The problems of continuum physics have been a fruitful source of mathematical ideas and techniques ever since Daniel Bernoulli invented Fourier series to describe the motion of a vibrating string in 1728. For more than two centuries, mathematicians and physicists relied primarily on the methods of separation of variables and superposition of solutions to solve the (usually linear) systems of partial differential equations embodying the laws of physics. To make the most of the opportunities provided by high-speed digital computers, a new generation of numerical analysts has applied many ingenious mathematical considerations to improve the accuracy of the approximate models and their numerical treatment. The present volume contains a cross-section of recent work in this field, representing a broad spectrum of activity. This ranges from largely empirical (or "phenomenological") numerical simulation of meteorological problems to highly theoretical considerations of families of difference approximations to large classes of partial differential equations which include nonlinear problems. The applications of piecewise polynomial and other spline-like approximations to implement variational approaches have also not been neglected.

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