MATHEMATICS OF COMPUTATION, VOLUME 24, NUMBER 111, JULY, 1970

Remark on a Conjecture of Erdös on Binomial Coefficients

By Andrzej Makowski

Abstract. A conjecture attributed to Erdös concerning the Diophantine equation

$$2\binom{x+n-1}{n} = \binom{y+n-1}{n}$$

is shown to be false.

M. Wunderlich [2] attributes the following conjecture to P. Erdös:

The equation

$$2\binom{x+n-1}{n} = \binom{y+n-1}{n} \tag{1}$$

has only one solution in positive integers: $x = n, y = n + 1$.

Because (1) has infinitely many solutions for $n = 2$ (cf. [1, p. 30]) the assumption $n \geqq 3$ must surely be added. But that does not suffice.

Observe that for $b - a \geqq 3$ the equality

$$s\binom{a}{2} = t\binom{b}{2} \tag{2}$$

implies

$$s\binom{b-2}{b-a} = t\binom{b}{b-a}.$$

Because (2) has infinitely many solutions in integers a, b for $s = 2, t = 1$, we obtain infinitely many counterexamples to the conjecture of Erdös, viz. $n = b - a$, $x = a - 1, y = a + 1$, where

$$2\binom{a}{2} = \binom{b}{2}.$$

For example,

$$2\binom{19}{6} = \binom{21}{6}$$

is a solution of (1).

Probably the conjecture is true when we require $y - x \geqq 3$.

Institute of Mathematics
University of Warsaw
Warsaw, Poland

1. L. E. Dickson, *History of the Theory of Numbers.* Vol. II, reprint, Chelsea, New York, 1952.

2. M. Wunderlich, "Certain properties of pyramidal and figurate numbers," *Math. Comp.*, v. **16**, 1962, pp. 482–486. MR **26** #6115.

Received August 21, 1969, revised November 10, 1969.
AMS Subject Classifications. Primary 1013; Secondary 0505.
Key Words and Phrases. Diophantine equation.