REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS

The numbers in brackets are assigned according to the indexing system printed in Volume 22, Number 101, January 1968, page 212.

49 [2.00, 3, 4, 5, 6, 7, 8].—CARL-ERIK FROBERG, *Introduction to Numerical Analysis*, Addison-Wesley Publishing Co., Reading, Mass., 1969, xii + 433 pp., 24 cm. Price $11.95.

This is an expansion and revision of the 1st edition of 1965 which in turn was a translation and minor revision of the 1962 Swedish original. The first edition covered a wide range of topics within its 340 pages: nonlinear equations, linear systems and matrix inversion, matrix eigenvalue problems, approximation, interpolation and numerical quadrature, ordinary and partial differential equations, Monte Carlo, and linear programming. Moreover, within each of these broad headings, the author tried to say something about a great variety of methods and subtopics with the result that a large portion of the book is either written in a terse style that the uninitiated will find tough going or is so condensed as to be almost useless (e.g., two pages on boundary value problems for ODE, one page on systems of nonlinear equations, a half page on the LR method). Although this revision amplifies a few discussions (e.g., stability of multistep methods), the bulk of the additional 100 pages goes to new topics including the optimum ω for SOR, the QR method, Hyman's method, the Adams-Bashford-Moulton methods, and two new chapters on linear integral equations and special functions. But, again, some of this new material is so condensed that its value is questionable (e.g., one page on Fredholm equations of the first kind with no mention of the intrinsic difficulties associated with this problem).

Moreover, such serious deficiencies of the first edition as lack of discussion of interchanges with Gaussian elimination have not been corrected.

Nevertheless, the reviewer feels that this book is one of the better introductions available and, in the hands of an experienced instructor willing to amplify, clarify, and edit, can be a satisfactory text for both one and two semester undergraduate courses.

J. M. O.

50 [2.00, 3, 4, 5, 13. 35].—SHAHEN A. HOVANESSIAN & LOUIS A. PIPES, *Digital Computer Methods in Engineering*, McGraw-Hill Book Co., New York, 1969, xvi + 400 pp., 24 cm. Price $14.50.

This book covers most of the standard problem areas expected in an introduction to numerical analysis, but also contains short chapters on linear and dynamic programming and the fast Fourier transform. Many methods are illustrated by Fortran programs and each chapter ends with exercises, mostly of elementary mathematical type.

With the exception of three or four examples and a few exercises, however, the book is not especially directed towards engineers and must be judged in relation to the many standard numerical analysis texts which now exist. In this comparison, it fares quite badly as a few examples will indicate: In the chapter on eigenvalue/vector computation, the stress is put on methods which first find the characteristic polynomial (Krylov, Danilevsky). Although the power method is also discussed, no mention is made of the Givens-Householder method nor the LR/QR methods. In the discussion of Gaussian elimination for linear equations, no mention is made of the need for interchanges except when a pivot is zero. In various places throughout the book, a problem is reduced to a system of linear equations $Ax = b$ (e.g., in the least squares problem of Chapter 3) and this is followed by a directive to form $x = A^{-1}b$ where "the inverse matrix can be obtained, for example, by the augmented matrix method described in Chapter 1" (p. 149). The chapter which covers numerical integration says nothing about Gaussian quadrature or Romberg integration.

The above omissions would not be so bad if adequate references to the literature were provided. However, although 156 pages are devoted to linear equations, eigenvalues, and roots of polynomials, and 16 references are given, no mention is made of Wilkinson. Similarly, Chapters 6, 7, and 8 on differential equations contain references to several books as well as papers, but no mention of the standard works by Henrici, Varga, and Forsythe and Wasow.

All in all, this reviewer unfortunately must conclude that the book is very uneven and does little justice to the advances made in numerical analysis in the last twenty years.

J. M. O.

51 [2.00, 3, 4, 8, 13.00].—Brice Carnahan, H. A. Luther & James O. Wilkes, *Applied Numerical Methods*, John Wiley & Sons, Inc., New York, 1969, xvii + 604 pp., 29 cm. Price $14.95.

This new, rather unevenly written and organized, compendium is a good reference book for the practicing engineer. The contents include interpolation, approximation, numerical integration, solution of polynomial equations, matrices, systems of equations, approximate solution of ordinary differential equations, approximate solution of partial differential equations, and statistical methods. Documented Fortran programs, experienced remarks on computational procedures, and meaningful applied illustrative problems are distributed abundantly throughout.

Pedagogically, however, the book presents severe problems. The sensitivity, depth, and yet low key presentation of the material on interpolation, ordinary differential equations, and parabolic equations conflicts radically with, for example, the material on matrices, in which an entire matrix course, without the more complex proofs but with selected short ones, has been compacted into a single chapter. The authors' habit of doing *all* the problems in the book on a computer, even when the exact solution is attainable by methods from high school algebra (see, e.g. p. 173), is one *not* worth passing on to readers and students. And the omission of such important contemporary topics as analysis of roundoff error in algebraic processes, spline interpolation, integral equations, Monte Carlo techniques, boundary value problems for ordinary differential equations, hyperbolic partial differential equations, the Navier-Stokes equations, linear and nonlinear programming methods, and discrete model theory give the book a sense of age and weight similar to that associated with dictionaries and encyclopedias, to which one rarely turns for inspiration and vitality.

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52 [2.05].—AMERICAN MATHEMATICAL SOCIETY TRANSLATIONS, *Fourteen Papers on Series and Approximation*, American Mathematical Society, Providence, R. I., 1968, iv + 266 pp., 25 cm. Price $13.60.

Except for a paper of I. M. Vinogradov (Estimation of Trigonometric Sums), motivated by additive number theory, all papers of this volume belong to the theory of approximation or to related branches of analysis (orthogonal series). Short reviews follow.

Balasov has very neat theorems about series of Rademacher functions and about series of the form $\sum a_k f(n_k x)$.

Osipov generalizes work of Ul'janov and R. P. Agnew and shows that if $\sum_{n=1}^{\infty} a_n^2 = \infty$, and if f is measurable on (0, 1), then there exists an orthonormal system ϕ_n for which $\sum_{n=1}^{\infty} a_n\phi_n(x)$ converges everywhere to $f(x)$ for any rearrangement of its terms. The paper of *Jastrebova* deals with Walsh-Fourier series.

Among the papers on Fourier series, *Bojanić* and *Tomić* deal with the absolute convergence of Fourier series with gaps for which $n_{k+1} - n_k \geqq$ const. *M. F. Timan* discusses the approximation in spaces L^p of f by the λ-means of its Fourier series, where λ stands for many classical summability matrices. *Berdysev* estimates $\sup_f |a_n(f)|$, $\sup_f ||f - s_n(f)||_\infty$, when the modulus of continuity of f is given. Two papers deal with the degree of approximation, in a Banach function space X, of a function f by trigonometric polynomials. *Cyganok* has generalizations of Jackson's estimate (involving moduli of continuity of the function f or of its derivatives) for the degree of approximation of f in an Orlicz space norm. *A. V. Efimov* relates the lower estimate for the degree of approximation of a class $M \subset X$ to the supremum of $||\phi||_x$, where ϕ are all functions of M which are "cos nx-symmetric," and finds this supremum for several classes M.

Teljakovskii answers positively a question proposed by this reviewer, and proves that for $f \in C^r[-1, 1]$ there exists a sequence of algebraic polynomials P_n for which

$$|f(x) - P_n(x)| \leqq C((1 - x^2)^{1/2}/n)^r\omega(f^{(r)}, (1 - x^2)^{1/2}/n), \qquad n \geqq r.$$

G. C. Tumarkin in 2 papers treats the possibility of approximation, in the norm of L_σ^p, of a function by rational functions with prescribed poles. *Lizorkin* has inequalities of Bernštein type for fractional derivatives. Finally, *Suetin* discusses uniqueness properties of interpolation series for certain analytic functions.

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53 [2.10, 4, 5, 6, 12, 13.15].—H. GREENSPAN, C. N. KELBER & D. OKRENT, Editors, *Computing Methods in Reactor Physics*, Gordon & Breach Science Publishers, New York, 1968, xi + 589 pp., 24 cm. Price $18.00 cloth, $12.00 paper.

The intent of this compendium is imparted on the front flap: "—The main function—is to help graduate students, faculty members and workers in the field [understand] formal methods [in the] formidable body of computational codes [which] have become the tools of the trade—Major numerical problems in reactor physics [are treated] in sufficient detail that the reader might gain an understanding of the fundamental methods used and the nature of the solutions." This objective has for the most part been achieved, thus making this a worthwhile addition to the literature on reactor computation.

A detailed review is precluded by space and reviewer ability limitations. A multiple-authored book might better be handled by multiple-reviewer critiques. This could, however, compound the deficiencies of such endeavors which are the inevitable change in style and level of approach, duplication, and inconsistencies arising from multiple authorship. The editors are to be commended in that these shortcomings are less noticeable here than in many other comparable endeavors.

A brief summary of each chapter with a few criticisms follows:

Chapter 1. *One Dimensional Diffusion Theory*. M. K. Butler and J. M. Cook.

The book starts with a reduction of the transport equation to the diffusion theory equation, and this is perhaps too sophisticated a beginning which might have been better placed as an appendix or as a part of Chapter 8. This is followed by a well-written introductory description of diffusion theory with a historical review.

Section 1.4 contains a concise description of the theory underlying numerical solution of 1-D diffusion problems, including some of the positivity analysis and the matrix factorization technique. There is no mention of Wielandt's fractional iteration which is the major tool for solving few-group 1-D diffusion problems, nor is there any discussion of the "stabilized march technique."

Chapter 2. *Diffusion Theory in Two- and Three-Dimensions*. A. Hassitt.

Numerical methods for solving two-space-dimension diffusion theory problems have an extensive theoretical foundation, and Hassitt does a reasonably good job of sketching some of the important areas of analysis. Limited space permits little more than a cursory look at each topic.

Difference equations are developed for various geometries, properties of the resulting matrix equations are discussed, and commonly used solution techniques are described and compared in terms of projected application.

I noted a few minor inaccuracies in this presentation: Property P7 on p. 120 is false. The matrix H on the bottom of p. 136 should include positive diagonal elements from the discretization of $-\partial^2/\partial x^2$. On the top of p. 138 it is implied that the sum of two singular matrices cannot be positive definite. This is wrong. The eigenvector deficiency in Gauss-Siedel iteration, discussed on pp. 132–133, occurs with the so-called "normal ordering" but not for the "σ_1-ordering."

Chapter 3. *Transport Theory—The Method of Discrete Ordinates*. B. G. Carlson and K. D. Lathrop.

A well-rounded description of the Sn method is given in sufficient detail to be useful with little supplementary reading. The equations are developed in terms of discrete variables and then directly from the transport equation in conservation form. Angular quadrature coefficients are discussed and various approximations and simplifications are examined. Solution techniques are evaluated, numerical examples given, and extensions to integral transport theory and adjoint calculations are described.

Chapter 4. *Spherical Harmonics Methods.* E. M. Gelbard.

Gelbard does an admirable job of presenting a wealth of material. The bulk of the chapter is restricted to monoenergetic problems for simplicity, although multigroup extensions are cited. The P_L and double P_L approximations are derived and interface continuity and boundary conditions are considered.

The connection between Sn and P_L methods in terms of numerical quadrature formulas is developed in lucid fashion.

Solution techniques are described for the coupled first order differential equations of P_L theory. Convergence and stability are examined for various procedures. Methods for casting equations in diffusion theory form for solution are developed and the FLIP code is described in this context.

Discontinuities introduced at $\mu = 0$ by double P_L approximations in spherical coordinates are treated at length. Methods are described for avoiding anomalies which arise when one replaces a cell in a repeating array by a cylinder with reflecting boundaries. Analytic methods for solving the P_L and double P_L equations are discussed for slab-geometry. (Gelbard has advised me that the basic ideas in Section 4.5 should be credited to Aronson.)

Chapter 5. *Monte Carlo Methods in Reactor Computations.* M. H. Kalos, F. R. Nakache and J. Celnik.

This chapter describes the salient features of Monte Carlo techniques in reactor computations. No prior experience with Monte Carlo is required for an understanding of the material.

The analog procedure in which a stochastic process is simulated by history tracing of particles is described first.

A nonanalog viewpoint is then presented. A clear exposition of how history simulation and scoring enables probabilistic evalaution of integrals sets the stage for a description of "importance sampling" techniques. Subsequently, a variational approach is used to indicate how approximate adjoint functions can yield low variance results.

Miscellaneous devices such as various scoring techniques, stratification of the random variables, antithetic variates and correlated sampling for related configurations are outlined.

Some of the applications to reactor design calculations mentioned are criticality calculations, neutron thermalization, slowing down through resonances, and temperature coefficient evaluation. Salient features of pertinent programs are given.

Appendices on generating random variables and on problems of geometry round out the chapter. Unfortunately, Coveyou's elegant Fourier analysis of certain pseudo random numbers was not included here.

Chapter 6. *Reactor Kinetics Calculation.* H. P. Flatt.

The bulk of this chapter is devoted to numerical solution of the reactor kinetics equations, although some analytic and analog techniques are mentioned. The treatment is broad and informative. I would have preferred a few more introductory paragraphs on the significance of delayed neutrons with numerical values for the various time constants and sketches of flux levels as a function of time. Such fundamental concepts as prompt and delay critical are not discussed, for example. The reader should have some prior exposure to reactor kinetics.

The kinetics equations are expressed in integral form and various techniques for solution are considered. The method of collocation is described, and illustrated with quadratic and exponential trial functions, each of which is appropriate for a different range of criticality. An error analysis is given relating "defect" in satisfaction of the integral equation to error in the neutron number density.

Feedback effects are discussed briefly. I had hoped to find a more detailed discussion of feedback in the next chapter, but this was not the case.

Flatt points out that incentives are high for developing and evaluating numerical techniques for solution of space-dependent kinetics problems. Unfortunately, this chapter was written too soon to include recent developments along these lines. This is one of the current frontiers in reactor analysis.

Chapter 7. *Coupled Neutronic-Dynamic Problems.* R. B. Lazarus, W. R. Stratton and T. H. Hughes.

I found this to be a fascinating summary of an aspect of reactor computations with which I had no prior experience, i.e., reactor excursions. The governing equations, models and methods of solution bear little relation to the rest of the book. Nevertheless, the methodology is described with sufficient clarity for a novice, like this reviewer, to comprehend its salient features.

Excursions have been analyzed by the reactor kinetics techniques of Chapter 6, the Bethe-Tait perturbation technique, or by numerical solution of appropriate partial differential equations. The governing equations are described. They are basically motion, continuity, energy and equation of state relationships. A monoenergetic one-space-dimension mockup of neutron flux is used with a separated exponential time dependence multiplying a more slowly varying (normalized) function of space and time. The flux is obtained by solving a sequence of steady-state neutron problems.

Rigorous analysis of stability is preempted by the nonlinearity but conditions for stability of linearized equations for small perturbation of the flow variable are described.

Comparison with experiment is discussed and a flowchart is given for a sample code.

Chapter 8. *Mathematical Foundations.* J. M. Cook.

Each section in this presentation could be expanded into a full chapter, and this is the most difficult chapter in the book. Cook indicates areas for research into the mathematical foundations of reactor computations, gives a well rounded bibliography for such an effort, and sketches some of the pertinent lines of thought.

The section headings indicate the scope of the treatment: The Stochastic Process, Partially Ordered Linear Spaces, The Forward Kolmogorov Equation, The Integro-Differential Transport Equation, The Integral Transport Equation, The Multigroup Diffusion Equation, Adjoint Fluxes, Ergodicity, Existence and Uniqueness of the Stochastic Process.

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54 [2.15].—D. S. MITRINOVIC, R. S. MITRINOVIC & S. S. TURAJLIC, "A table of coefficients for numerical differentiation," *Publ. Fac. Élect. Univ. Belgrade* (*Série: Math. et Phys.*), No. 247–No. 273, 1969, pp. 115–122.

This table appears in the publication numbered 264 in this compilation of recent papers in mathematics and physics by members of the faculty of the University of Belgrade. It consists of exact (rational) values of the coefficients $A_{r,n}$ for $r = 1(1)30$, $n = r(1)30$ in Markoff's formula for the rth derivative in terms of forward differences. These values were computed (partly on an Olivetti desk calculator and partly on an IBM 1130 system) by use of the formula $A_{r,n} = r!S_n^r/n!$, where S_n^r is a Stirling number of the first kind.

In the preparation of this extensive table the authors confirmed the accuracy of the table of Lowan, Salzer and Hillman [1], which was previously the most extensive in the literature.

J. W. W.

1. A. N. LOWAN, H. E. SALZER & A. HILLMAN, "A table of coefficients for numerical differentiation," *Bull. Amer. Math. Soc.*, v. 48, 1942, pp. 920–924.

55 [2.35, 8, 13. 25, 13.35].—M. T. WASAN, *Stochastic Approximation*, Cambridge Univ. Press, New York, 1969, viii + 202 pp., 22 cm. Price $9.50.

A stochastic approximation method is an iterative procedure by means of which successive values of a parameter are generated and corresponding random samples are observed. For example, the parameter may be the dosage of a new drug and the observations are then the responses. A scheme is set up so that these values converge stochastically in some sense to the desired quantity, such as the mean of an unknown quantity (the critical dosage). A typical scheme, due to Monro-Robbins (1951) and having its origin in bioassay, looks like this:

$$x_{n+1} = x_n - cn^{-1}[y(x_n) - \alpha].$$

Older methods go back to Newton in numerical solutions of equations and to the nonstochastic up-and-down method. Kiefer and Wolfowitz considered a stochastic method for determining the location of the maximum of a regression function. A lot of work was done from the fifties down to recent years. This monograph collects the results of many authors, discusses (i) the conditions under which the method gives a valid approximation to the required solution, (ii) methods for optimal choice of parameters to hasten convergence, and (iii) comparisons with other techniques. The stochastic process involved is usually a nonstationary Markov chain of a very special kind, and the mathematical content of the results boils down to upper and lower bounds for moments and related quantities, and sometimes their asymptotic behavior. The prototypes of such inequalities, given by the reviewer to establish asymptotic normality, and ramified by Burkholder, Derman, Sacks, and others, are collected with detailed proofs in an appendix. There are many examples, applications and references to sources. Although the book is not for browsing, by the nature of the subject, the author has written a readable account for readers who are ready to plow through the abundant formulas. There are a few minor misprints including the spelling of Loeve on p. 155 and some little *o*'s for big *O*'s on p. 176. But, on the whole, the printing is pleasant to look at.

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56 [7].—MURRAY GELLER & EDWARD W. NG, "A table of integrals of the exponential integral," *J. Res. Nat. Bur. Standards—B. Mathematical Sciences*, v. 73B, 1969, pp. 191–210.

The main part of this exhaustive compilation consists of a total of 202 definite and indefinite integrals of products of the exponential integral with both elementary and transcendental functions.

Also included in this paper is a brief introduction enumerating the procedures followed in obtaining the tabular entries and citing applications in diffusion theory, transport problems, astrophysics, and quantum mechanics. This is followed by sections giving, respectively, a glossary of relevant functions and notations, the definition, special values, and integral representations of the exponential integral. A list of 15 references is appended; these include several standard collections of integrals, as well as publications relating specifically to the exponential integral and its applications.

J. W. W.

57 [7].—EDWARD W. NG & MURRAY GELLER, "A table of integrals of the error functions," *J. Res. Nat. Bur. Standards—B. Mathematical Sciences*, v. 73B, 1969, pp. 1–20.

The greater part of the definitive table in this paper consists of the systematic tabulation of a total of 179 definite and indefinite integrals of products of the error function and its complement with both elementary and transcendental functions.

Preliminary sections of the paper include an introduction enumerating the procedures followed in obtaining these integrals and citing several applications, a glossary of pertinent functions and notation, and analytic definitions and integral representations of the error function and related functions.

Appended to the main table is a table of 16 relevant integrals of elementary functions. A list of 16 references includes several standard tables of integrals from which many of the tabular entries were taken and a number of papers relating to applications in atomic physics, astrophysics, and statistical analysis.

J. W. W.

58 [7].—G. Blanch & D. S. Clemm, *Mathieu's Equation for Complex Parameters, Tables of Characteristic Values*, U. S. Government Printing Office, Washington, D.C., 1969, xix + 273 pp., 27 cm. Price $4.50.

Mathieu's equation

$$d^2y/dx^2 + (a - 2q \cos 2x)y = 0$$

admits of periodic solutions of period π and 2π for four denumerable sets of characteristic values $a(q)$ for each assigned value of q. There are extensive tabulations for real values of q.* But very little is available in the complex plane.

The present work provides a tabulation in the complex plane (Tables I and II) for $q = \rho \exp(i\varphi)$, $\varphi = 5°(5°)90°$, $\rho = 0(0.5)25$, 4D and additional values for $\varphi = 90°$ for ρ up to 100, 8D (Tables III, IV and V). The data in Tables I and II is sufficient to depict the eigenvalues over the range covered though the mesh lengths in φ and ρ are not sufficiently small for satisfactory interpolation. As the authors correctly remark, excessive computation just to provide for easy interpolation is no longer of paramount importance in view of the availability of high speed computing equipment. Indeed a shorter table that presents a general overview of the function may be more serviceable. An exception was made on the 90° ray due to its importance in both theoretical and applied problems. For this ray, the tables are interpolable and extend beyond the singular point (if there is one) for each order, and always at least up to $\rho = 100$. In one of the sets, the last entry is for $\rho = 130$.

Power series expansions for the characteristic values are known, but the radii of convergence have been for the most part an unknown quantity since these depend on a knowledge of the multiple eigenvalues. Mulholland and Goldstein** gave six sets of characteristic values for $q = is$, $s \leqq 2$. They found that $a_0(q)$ and $a_2(q)$ have a common value when $s = 1.468$. Bouwkamp*** gave the more accurate value 1.468769. The present authors have confirmed that the latter singular point is nearest the origin in Euclidean distance. In addition, the double points in the complex plane for orders $r \leqq 15$ are tabulated to 8D (Table VI). It is believed that all tabular values are accurate to within a unit in the last place.

An introduction gives the background of the problem, derivation of the basic equations, method of computation, useful auxiliary functions near a singular point, method of computing the singular points and remarks on interpolation. There is a complete bibliography and some reliefs of the data.

The table typography is clear but not uniform. As the tables were prepared by offset from machine printout, the nonuniformity appears due to a poor ribbon. This slight imperfection aside, the tables are a definitive and marked contribution to the literature.

Y. L. L.

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* See National Bureau of Standards, *Tables Relating to Mathieu Functions*, Second Edition AMS 59. U. S. Govt. Printing Office, Washington, D. C., 1967. See also *Math Comp.*, v. 22, 1968, p. 466.

** H. P. Mulholland and S. Goldstein, "The characteristic numbers of the Mathieu equation with purely imaginary numbers," *Philos. Mag.*, v. 8, 1929, pp. 834–840.

*** C. J. Bouwkamp, "A note on Mathieu functions," *Kon. Nederl. Akad. Wetensch. Proc.*, v. 51, 1948, pp. 891–893.

59 [8].—I. J. GOOD, T. N. GOVER & G. J. MITCHELL, *Tables of Smoothed but Almost Exact Distributions of X^2 and of the Likelihood-Ratio Statistic for the Equiprobable Multinomial Distribution*, ms. of 580 computer output sheets (unnumbered) deposited in the UMT file.

Consider a multinomial distribution of t categories for which the physical probability of each category is $1/t$ (the "equiprobable multinomial distribution"). Let a sample of size N be drawn and tlet he frequencies in the t cells be $n_1, n_2, \cdots, n_t$.

Let

$$X^2 = \frac{t}{N} \sum_{i=1}^{t} \left(n_i - \frac{N}{t}\right)^2$$

and

$$\Lambda = 2N \ln t - 2N \ln N + \sum_{i=1}^{t} 2n_i \ln n_i .$$

Both X^2 and Λ have asymptotically chi-squared distributions, but for small samples the chi-squared approximation is sometimes poor. Let $P_1(a)$ and $Q_1(a)$ be the smoothed tail-area probabilities of the exact distributions of X^2 and Λ, that is, the smoothings of $P(X^2 > a)$ and $P(\Lambda > a)$, where the smoothing is performed in the following manner.

Each distribution is a step function, forming a "staircase," and this is first replaced by another "staircase" function obtained by drawing a graph of the negative of the logarithm of the right-hand tail areas. Each step has a horizontal and vertical segment. We bisect all these segments and join adjacent bisecting points by new straight-line segments, each of which thus connects the midpoint of one of the original horizontal segments to the midpoint of one of the adjacent vertical segments. The new segments form, of course, a continuous approximation to the original distribution; however, the derivative is discontinuous at each point of bisection.

At the midpoint between two jumps in the original distribution, the smoothed tail-area probability is equal to the exact unsmoothed one, whereas at a jump it is equal to the geometric mean of the two tail-area probabilities just to the left and just to the right of the jump. At other points the logarithm of the smoothed tail-area probability is determined by linear interpolation, as a consequence of the smoothing procedure described.

The present tables consist of 4D values of the common logarithms of $P_1(a)$ and $Q_1(a)$ for $t = 3(1)6$, $N = 3(1)12$; $t = 6(1)14$, $N = 6(1)2t$; $t = 15(1)18$, $N = 6(1)28$. In both tables, these values appear in columns headed LOGP and the values of a are given in columns headed A. The notations a, P_1, and Q_1 are those used by the authors in a paper [1] for which these tables were computed. These symbols are replaced, respectively, by A, P, and again P in the computer printout.

The tables were computed on the British Science Research Council's Atlas Computer at Chilton, Didcot, Berkshire, England, following many preliminary computations on the CDC 1604 system at the Communications Research Division of the Institute for Defense Analyses, Princeton, New Jersey.

AUTHORS' SUMMARY

1. I. J. GOOD, T. N. GOVER & G. J. MITCHELL, "Exact distribution for X^2 and for the likelihood-ratio statistic for the equiprobable multinomial distribution," *J. Amer. Statist. Assoc.*, v. 65, 1970, pp. 267–283.

60 [9.10].—MORRIS NEWMAN, *The Number of Partitions into Primes*, National Bureau of Standards, November 1969. Nine pages of computer output deposited in the UMT file.

To test a new set of subroutines with an interesting problem, the author extends the function called $q(n)$ in the recent table of Chawla and Shad [1]. They computed $q(n)$ for n to 150, but earlier O. P. Gupta and S. Luthra had computed $q(n)$ for $n = 1(1)300$. The latter table gave

$$q(300) = 62737270. \tag{1}$$

The present table lists $q(n)$ for $n = 1(1)500$. It confirms (1) and continues to

$$q(500) = 414270104287.$$

The computation required only 30 seconds on a UNIVAC 1108.

Since $p(500)$ is approximately $2.3 \cdot 10^{21}$, the ratio $\log q(500)/\log p(500)$ is down to 0.5438 here—see the discussion in the review of [1].

D. S.

1. *Math. Comp.*, v. 24, 1970, p. 490, RMT **38**.

61 [12].—DON SECREST & JURG NIEVERGELT, Editors, *Emerging Concepts in Computer Graphics*, W. A. Benjamin, Inc., New York, 1968, ix + 418 pp., 24 cm. Price $20.00.

The book consists of a compilation of 16 of the total of 19 papers presented at the Conference on Emerging Concepts in Computer Graphics at the University of Illinois in November 1967.

This reviewer was especially impressed by the paper of K. C. Knowlton, entitled "Computer-Animated Movies." It is liberally illustrated with excellent figures; in addition, it contains a comprehensive list of references, and a detailed description of various uses of computer-produced movies.

Perhaps the editorial shortcomings afflicting so many of the other papers can be explained by a note from the publisher, facing the title page. Therein we find the statements: "This volume was printed directly from a typescript prepared by the editors, who take full responsibility for its content and appearance. The Publisher has not performed his usual functions of reviewing, editing, typesetting, and proofreading the material prior to publication. The Publisher fully endorses this informal and quick method of publishing conference proceedings, and he wishes to thank the editors for preparing the material for publication."

In the light of these remarks of the publisher and the specialized contents of the book, the reviewer found the description on the dust jacket inaccurate and misleading. For example, the book does not appear to be in any sense an "indispensable guide," nor does it "delineate the basic problems" of computer graphics. Indeed, it is neither a textbook nor a handbook; it is merely the proceedings of a conference.

Some of the conference presentations have necessarily suffered in the process of being transcribed into the book. For example, films accompanied the presentations of T. O. Ellis & W. I. Sibley ("On the Problem of Directness in Computer Graphics") and of R. Resch ("Experimental Structures"), but these, of course, have not been reproduced in the book. The article corresponding to the first of these presentations suffers particularly from this omission.

Several of the papers contain too much detail peculiar to their respective technical specialties to be easily understood by the layman. Included in this category are the papers by D. L. Bitzer & H. G. Slottow ("The Plasma Display Tube—A New Device for Direct Display of Graphics"), M. Faiman ("ARTRIX—A Hybrid Graphical Processor"), C. Levinthal, C. D. Barry, S. A. Ward & M. Zwick ("Computer Graphics in Macromolecular Chemistry"), and B. Herzog ("Computer Graphics for Designers").

In addition to the previously mentioned paper of Knowlton, this reviewer found particularly interesting and informative the papers of C. Levinthal et al., R. Resch, W. F. Miller & A. C. Shaw ("A Picture Calculus"), S. H. Chasen ("Experience in the Application of Interactive Graphics"), C. W. Beilfuss ("Automated Graphics in an Industrial Environment"), and W. A. Fetter ("Computer Graphics").

Levinthal and his associates describe a project for graphical display with simulated manipulation and rearrangement of molecular models by a computer, with the objective of freeing the investigator of molecular structures from the constraints imposed by physical models.

Resch discusses the use of a computer graphic console in the design of three-dimensional geometric structures.

Miller and Shaw attempt to analyze and synthesize "artificial pictures" (i.e., well-defined or structured pictures) by means of a "picture calculus."

Chasen describes the use of computer graphics in the aircraft design process.

Beilfuss discusses problems involved in an attempt to make computer graphics feasible and economical in the preparation of structural steel shop drawings.

In the concluding paper, Fetter describes the application of computer graphics to the choice of location of radar stations for air traffic control by plotting the radar visibility of aircraft and to the drawing of animated human figures.

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62 [13.35].—Michael A. Arbib, *Theories of Abstract Automata*, Prentice-Hall, Inc., Englewood Cliffs, N. J., 1969, xiii + 412 pp., 24 cm. Price $14.95.

The aim of this book is to provide an advanced textbook for graduate study in automata theory. Its scope and depth attest to this purpose. The book is divided into three sections: I. Background, II. An Introduction to Automata Theory, III. Selected Topics.

The background section contains the motivation and mathematical preliminaries for studying automata theory. The author states that "automata theory is the pure mathematics of computer science" by which he further interprets as meaning "automata theory is a branch of mathematics ... asking questions about biological and electronic computers." The mathematical preliminaries are appropriately terse and provide the reader an adequate refreshing of definitions and results needed to understand the remainder of the book.

Part II contains finite automata theory, Turing Machine computability and Post systems. Finite automata are given the most detailed treatment and include the author's view of a *system* of which an automata is a special case. The Winograd [1], [2] and Spira [3] results on minimum computation time and the Minsky and Papert [4] results on perceptrons are featured topics exhibiting a main theme throughout the book, namely, to illustrate the power and complexity of computational systems. The Turing Machine chapter includes the demonstrations of equivalence of multi-head multi-tape TMs to ordinary TMs. The final chapter of this section covers Post systems and includes the theory of phrase-structure grammars with emphasis on CF languages.

The last section of chapters are the author's candidates for important research areas. A preliminary section introduces some recursive function theory, and then computational complexity, algebraic decomposition theory, stochastic automata, and self-reproducing automata are given a chapter. The treatment of the existence of a universal self-reproducing automata [5], [6] follows the author's own [7] module design which while more complex than von Neumann's leads to sizable reduction in the construction of the universal constructor automata.

No other automata theory text has such a broad scope and while this is the book's main strength, it is also its main weakness. It is very difficult to give a fluid insightful treatment to so many topics. Invariably, one must paraphrase and condense from previous sources losing their original elegance. An example of this is some of the results on threshold functions which can be found in *Perceptrons* [4]. The instructor armed with the book's excellent bibliography can, where necessary, flesh out the text treatment and make good use of its organization.

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2. S. WINOGRAD, "On the time required to perform multiplication," *J. Assoc. Comput. Mach.*, v. 14, 1967, pp. 793–802.

3. P. M. SPIRA, "The time required for group multiplication," *J. Assoc. Comput. Mach.*, v. 16, 1969, pp. 235–243.

4. M. MINSKY & S. PAPERT, *Perceptrons*, M.I.T. Press, Cambridge, Mass., 1969.

5. J. VON NEUMANN, *Theory of Self-Reproducing Automata* (editor A. W. Burks), University of Illinois Press, Urbana, Illinois, 1966.

6. J. W. THATCHER, *Universality in the von Neumann Cellular Model*, University of Michigan Report, 1964.

7. M. A. ARBIB, "A simple self-reproducing universal automation," *Information and Control*, v. 9, 1966, pp. 177–189.