REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS

The numbers in brackets are assigned according to the indexing system printed in Volume 22, Number 101, January 1968, page 212.

63[1, 7].—HERBERT BUCHHOLZ, *The Confluent Hypergeometric Function, with Special Emphasis on its Applications*, translated by H. Lichtblau & K. Wetzel, Springer-Verlag, New York, 1969, xviii + 238 pp., 24 cm. Price $16.00.

This translation of the original 1953 edition published in German is a valuable book on the subject. For some reason, the original was not reviewed in these annals, and so we present a more detailed review. One must bear in mind that a great deal of research has been done in this area since 1953. In this connection, one should consult references [1]–[6] and other pertinent references given in these sources.

As is perhaps well known, there are several notations in use for confluent hypergeometric functions. The principal functions investigated are those of Whittaker, $M_{k,m}(z)$ and $W_{k,m}(z)$, though the author frequently prefers

$$M_{k,m/2}(z) = [\Gamma(m + 1)]^{-1} M_{k,m/2}(z).$$

These functions are often called parabolic functions, which should not be confused with parabolic cylinder functions, as the latter are special cases of the Whittaker functions.

In Chapter I, the Kummer series ${}_1F_1$ is introduced as a limiting form of Gauss' ${}_2F_1$ series. The differential equation satisfied by the ${}_1F_1$ is transformed into Whittaker's form and various properties of its solutions are developed. These include Wronskians, derivatives, and contiguous relations (the translation comes out as "circuital relations"). Related differential equations and separation of the wave equation in coordinates of the parabolic cylinder and of the paraboloid of revolution are also studied.

Integral representations of Whittaker functions and their products are treated in Chapter II while indefinite and definite integrals (e.g., Laplace and Mellin transforms) of these functions are studied in Chapter IV.

Chapter III takes up asymptotic expansions of the Whittaker functions when one of the three quantities z, k or m is large, the others being fixed, or when k and m are both large but $k - m$ is fixed.

A number of polynomials including those of Laguerre and Hermite are special cases of $M_{k,m/2}(z)$ and are studied in Chapter V. Numerous series and integrals involving these polynomials are listed.

Chapter VII is devoted to integral representations and expansions in series of Whittaker functions for various types of waves in mathematical physics.

Zeroes of $M_{k,m}(z)$ as a function of z and of k are considered in Chapter VIII. Eigenvalue problems involving parabolic functions are also studied.

Appendix I summarizes special cases of the parabolic functions. These include Bessel functions, the incomplete gamma functions and related functions, parabolic cylinder functions, Coulomb wave functions and the polynomials already noted.

Appendix II is an impressive list of references. A notation and symbol index at the beginning of the book and a subject index at the end of the book are of considerable help to the user. The intervening 17 years have not diminished the stature of this important treatise.

Y. L. L.

1. A. ERDÉLYI ET AL., *Higher Transcendental Functions*, Vols. 1 and 2, McGraw-Hill, New York, 1953. (See *MTAC*, v. 11, 1957, pp. 114–116.)

2. F. G. TRICOMI, *Funzioni Ipergeometriche Confluenti*, Edizioni Cremonese, Rome, 1954.

3. L. J. SLATER, *Confluent Hypergeometric Functions*, Cambridge Univ. Press, New York, 1960. (See *Math. Comp.*, v. 15, 1961, pp. 98–99.)

4. L. J. SLATER, *Generalized Hypergeometric Functions*, Cambridge Univ. Press, New York, 1966. (See *Math. Comp.*, v. 20, 1966, pp. 629–630.)

5. A. W. BABISTER, *Transcendental Functions Satisfying Nonhomogeneous Linear Differential Equations*, Macmillan, New York, 1967. (See *Math. Comp.*, v. 22, 1968, pp. 223–226.)

6. Y. L. LUKE, *The Special Functions and Their Approximations*, Vols. 1 and 2, Academic Press, New York, 1969.

64[2, 3, 4, 5, 13].—ROBERT L. KETTER & SHERWOOD P. PRAWEL, JR., *Modern Methods of Engineering Computation*, McGraw-Hill Book Co., New York, 1969, xiv + 492 pp., 23 cm. Price $15.50.

The book is intended to provide an introductory numerical analysis text for second- or third-year students of engineering and applied science. Some familiarity with computer programming is assumed.

After two introductory chapters on engineering problems and digital computers, the authors devote five chapters on matrix computation. Among the topics included are determinants, matrices, linear algebraic systems, matrix inversion, and the eigenvalue problem. Surprisingly, there is no mention of pivotal strategies in connection with Gauss elimination. Nonlinear equations are treated next, and topics related to interpolation, numerical differentiation and integration, least squares approximation, are collected in a chapter entitled "Miscellaneous Methods." There follow two chapters on the numerical solution of ordinary and partial differential equations, and a final chapter on optimization.

The discussion is verbose and discursive, throughout, and there are numerous instances of lax terminology and factual inaccuracies. The reviewer does not believe, therefore, that the book adequately fills the needs of the students for which it is intended.

W. G.

65[2, 4, 12].—W. A. WATSON, T. PHILIPSON & P. J. OATES, *Numerical Analysis—The Mathematics of Computing*, American Elsevier Publishing Co., New York, 1969, v. 1, xi + 224 pp.; v. 2, x + 166 pp., 23 cm. Price $4.50 and $5.50, respectively (paperbound).

This attractive textbook in two volumes was written specifically as an introduction to numerical analysis in the sixth form of British secondary schools and for more advanced students. Nevertheless, it should serve equally well as a lucid introduction to this subject in other school systems, such as that in this country.

Volume 1 provides in the space of nine chapters a very readable introduction to such topics as the use of hand-calculating machines; rounding errors; flow charts; curve tracing and the graphical solution of equations; iterative methods for the solution of equations in one or more variables; differences of a polynomial and their application in locating and correcting tabular errors; solution of linear simultaneous equations by the methods of elimination, triangular decomposition, and Gauss-Seidel iteration; numerical solution of polynomial equations; linear interpolation; and numerical integration by the trapezoidal, mid-ordinate, and Simpson rules.

Volume 2 treats equally clearly and concisely in eight chapters such topics as the interpolation formulas of Gregory-Newton, Bessel, and Everett (including throwback); inverse interpolation; Lagrange interpolation (including Aitken's method); numerical integration using differences; numerical differentiation; numerical solution of ordinary differential equations of the first and second orders; curve fitting by least squares; and the summation of slowly convergent series by Euler's method and the Euler-Maclaurin formula.

Each volume is well supplied with illustrative examples as well as with exercises (and answers) for the student. Also included are short bibliographies of material for further reading and study.

J. W. W.

66[2.10].—F. G. LETHER & G. L. WISE, *Ralston Quadrature Constants*, Tables appearing in the microfiche section of this issue.

An n-point quadrature rule of the form

$$\int_{-1}^{1} f(x)\, dx \simeq \sum_{i=2}^{n-1} a_i f(x_i) + a_1(f(-1) - f(1))$$

which is of polynomial degree $2n - 4$ is termed a Ralston Quadrature Rule. A list of weights and abscissas for $n = 3(1)9$ is given, together with coefficients e_1 and e_2 which may be used to bound the approximation error in terms of bounds on the first or second derivatives of $f(x)$.

Rules of this type may be used in cytolic integration. Because $a_1 = -a_n$ and $x_1 = -x_n = -1$, if the integration interval is divided into N equal panels and the n point rule used in each, only $N(n - 2) + 2$ distinct function values are required for a result of polynomial degree $2n - 4$. This may be compared with $N(n - 2)$ distinct function values using a Gauss Legendre formula to obtain a result of polynomial degree $2n - 5$.

The weights and abscissas are given to between nine and eleven significant figures. The authors also list the coefficients in the polynomials whose roots are the abscissas. This information may be useful both to users and to theoreticians, and I am happy to see its inclusion with the tables.

J. N. L.

67[2.55].—E. HANSEN, Editor, *Topics in Interval Analysis*, Oxford Univ. Press, London, 1969, viii + 130 pp., 24 cm. Price $8.00.

This book is a collection of papers presented at a symposium on Interval Analysis at Oxford University Computing Laboratory, 1967. The authors are Ramon Moore, Karl Nickel, Eldon Hansen, Jean Meinguet, F. Krückeberg and Michael Dempster. The papers are divided into two sections—algebraic problems and continuous problems—and topics include linear algebraic equations, zeros of polynomials, estimation of significance, two-point boundary value problems, ordinary and partial differential equations, and linear programming. There is also a brief description of Triplex-Algol, an extension of ALGOL 60 which facilitates the writing of programs involving interval arithmetic, and a short chapter by Hansen containing a proof of Moore's "centered form" conjecture.

The book is a useful addition to the literature of this field, and in fact is a suitable continuation of R. Moore's *Interval Analysis*, (Prentice-Hall, N. J., 1966). Hansen has done an excellent job of editing, so that the style of writing is uniform throughout. The papers are all reasonably self-contained, provided the reader is familiar with the Moore book. The paper on Triplex-Algol is the only English language discussion of this subject of which the reviewer is aware.

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68[3].—BEN NOBLE, *Applied Linear Algebra*, Prentice-Hall, Inc., Englewood Cliffs, N. J., 1969, xvi + 523 pp., 24 cm. Price $9.95.

One could quibble over certain features of this book (e.g., the style tends toward the prolix), but this reviewer knows of none on the market that could really compete. The topics covered are adequate for most applications; the applications are interspersed within the theory, but so separated that they can be included or omitted at the instructor's discretion; there are many exercises and a number of concrete examples; the bibliography is extensive (though theorems are not generally attributed); there is attention to numerical methods, including pitfalls; and there is a reasonably detailed index. Any competent instructor can supplement where he feels it necessary, but little would be needed. In most courses omissions would very likely be necessary, but the organization is such that this would not cause much trouble.

There are fourteen chapters, ending with one on "Abstract vector spaces," and including linear programming. The Jordan form is not neglected. Also, there is a chapter on "Norms and Error Estimates" (the thirteenth).

The text is now being used at the University of Tennessee, and quite successfully.

A. S. H.

69[6, 7, 13.15, 13.35].—PIERRE VIDAL, *Non-linear Sampled-Data Systems*, Gordon and Breach Science Publishers, New York, 1969, xv + 346 pp., 24 cm. Price $24.50.

This volume is concerned with the analysis of nonlinear systems with discrete information where the sampling is caused by, or inherent in, the selected method of control. The book is a practical working tool for engineers and research workers, and, although it leans heavily on numerous mathematical concepts, it is not, nor is it meant to be, a thorough-going, rigorous, and mathematically sophisticated treatment of the subject. Numerous block diagrams and figures are presented to illustrate the various automatic control problems and their mathematical analogs. Unfortunately, it is assumed that the reader is already familiar with the ideas such diagrams and related notations convey, as no definitions are provided. Indeed, a glossary of terms would have considerably enhanced the book. The volume does contain numerous examples and this should add to its usefulness.

Chapter 1 takes up the fundamentals of the calculus of finite differences. Topics treated include the Carson transform (the Laplace transform multiplied by p) and its inverse. Emphasis is on step-like functions. Table 1.1 gives the transform of numerous step functions and Table 1.2 gives some commonly occurring difference equations which can be solved with the aid of such transforms. The stability of solutions of difference equations is considered. The z-transform and its application for the solution of linear difference equations is the subject of Chapter 2. Applications to certain nonlinear difference equations and linear difference equations with periodic coefficients are given. A modified z-transform is introduced. Table 2.1 gives the Laplace transform, z-transform and modified z-transform of some commonly occurring transcendents.

The solution of a system of two nonlinear difference equations by the so-called discrete phase plane method is taken up in Chapter 3.

Chapters 4 and 5 consider the analysis of nonlinear difference equations by various methods. The stability of certain nonlinear sampled-data systems and related functions are treated in Chapters 6, 7, 8 and 9.

There is a notation index. The printing and typography are satisfactory save that pages 162, 163, 166, 167, 170. 171, 174 and 175 of my copy are blank.

Y. L. L.

70[7].—HENRY E. FETTIS & JAMES C. CASLIN, *Tables of Toroidal Harmonics*, II: *Orders* 5–10, *All Significant Degrees*, Report ARL 69–0209, Aerospace Research Laboratories, Office of Aerospace Research, United States Air Force, Wright-Patterson Air Force Base, Ohio, December 1969, iv + 179 pp., 28 cm. [Copies obtainable from the Clearinghouse, U. S. Department of Commerce, Springfield, Virginia 22151 at $3.00 each.]

The tables comprising this report are a continuation of the two main tables in a previous report [1] by the same authors.

Thus, in this second report on toroidal harmonics we find a table of 11S values of $Q^m_{n-1/2}(s)$ for $m = 5(1)10$, $s = 1.1(0.1)10$, and n varying from 0 through consecutive integers to a value ranging from 35 to 160 for which the value of the function relative to that when n is zero is less than 10^{-21}.

Also, as in the first report, this is immediately followed by the tabulation of the same function to the same precision and for the same orders, m, but for arguments $s = \cosh \eta$, where $\eta = 0.1(0.1)3$. Here the upper limit for the degree, n, varies from 34 to 450.

No explanatory text accompanies these tables; accordingly, the user should consult the first report for a mathematical discussion of these functions and the various methods used in the preparation of the tables, as well as for additional references.

J. W. W.

1. Henry E. Fettis & James C. Caslin, *Tables of Toroidal Harmonics*, I: *Orders* 0–5, *All Significant Degrees*, Report ARL 69–0025, Aerospace Research Laboratories, Office of Aerospace Research, United States Air Force, Wright-Patterson Air Force Base, Ohio, February 1969. (See *Math Comp.*, v. 24, 1970, p. 489, RMT 35.)

71[7].—K. A. Karpov & E. A. Ĉhistova, *Tablitsy Funktsiĭ Vebera*, Tom III (*Tables of Weber Functions*, v. III), Computing Center, Acad. Sci. USSR, Moscow, 1968, xxiv + 215 pp., 27 cm. Price 2.05 rubles.

Weber functions are defined as solutions of the differential equation

$$\frac{d^2y}{dz^2} + (p + \tfrac{1}{2} - \tfrac{1}{4}z^2)y = 0. \tag{1}$$

Whittaker's solution $D_p(z)$ of (1) may be defined by the initial values

$$D_p(0) = \frac{2^{p/2}\sqrt{\pi}}{\Gamma\left(\frac{1-p}{2}\right)}, \qquad D_p'(0) = -\frac{2^{p/2}(2\pi)^{1/2}}{\Gamma\left(-\frac{p}{2}\right)},$$

and is characterized by the asymptotic behavior

$$D_p(z) \sim e^{-z^2/4}z^p \quad \text{as } z \to \infty \text{ in } |\arg z| < \pi/2.$$

If p is not an integer, then $D_p(z)$, $D_p(-z)$ and $D_{-p-1}(iz)$, $D_{-p-1}(-iz)$ are pairs of linearly independent solutions of (1).

The function $D_p(z)$ for real p and $z = x(1 + i)$, x real, has been tabulated in two earlier volumes [1], [2]. The present volume tabulates $D_p(z)$ for z real and purely imaginary, and p real, and completes the tabulation of Weber functions undertaken by the Computing Center of the U.S.S.R. Academy of Sciences.

There are three principal tables in the present volume. The first gives $D_p(x)$ for $0 \leqq x < \infty$; the second, $\exp(-x^2/4)D_p(x)$ for $-\infty < x \leqq 0$; and the third, the real and imaginary parts of $\exp(-x^2/4)D_p(ix)$ for $0 \leqq x < \infty$. The tabular interval in x is 0.01 for $|x| \leqq 5$, and 0.001, or 0.0001, in $y = 1/x$ for $|x| > 5$. The range in p is $-1(0.1)1$ throughout, but can be extended with the aid of recurrence relations. All tabular entries are given to 7D, if less than 1 in absolute value; otherwise they are given to 8S.

The introduction contains detailed comments on interpolation and on methods for extending the tabular range. Many worked examples are included, as well as auxiliary tables. Also included are eight graphs and three reliefs illustrating the behavior of the functions tabulated.

W. G.

1. I. E. Kireeva & K. A. Karpov, *Tablitsy Funktsiĭ Vebera*, v. I, Computing Center, Acad. Sci. USSR, Moscow, 1959. Edition in English, Pergamon Press, New York, 1961. (See *Math. Comp.*, v. 16, 1962, pp. 384–387, RMT **38**.)

2. K. A. Karpov & E. A. Chistova, *Tablitsy Funktsiĭ Vebera*, v. II, Computing Center, Acad. Sci. USSR, Moscow, 1964.

72[7].— Dzh. Ch. P. Miller, *Tablitsy Funktsiĭ Vebera* (J. C. P. Miller, *Tables of Weber Functions*), Computing Center, Acad. Sci. USSR, Moscow, 1968, cxvi + 143 pp., 27 cm. Price 1.69 rubles.

Weber functions (or parabolic cylinder functions) in Whittaker's standardization are solutions of

$$\frac{d^2y}{dx^2} - (\tfrac{1}{4}x^2 + a)y = 0 \tag{1}$$

or solutions of

$$\frac{d^2y}{dx^2} + (\tfrac{1}{4}x^2 - a)y = 0. \tag{2}$$

Although the second equation may be obtained from the first by simultaneous replacement of a by $-ia$ and x by $xe^{i\pi/4}$, it is convenient to consider each equation separately when dealing with the real-variable theory of these equations.

Both equations arise naturally in the solution of Helmholtz's equation upon separation of variables in parabolic cylinder coordinates. They also occur in the asymptotic theory of second-order differential equations with turning points. For special values of a the solutions of (1) are related to the normal error function and its repeated integrals and derivatives.

One owes to J. C. P. Miller [1] a thorough mathematical treatment of Weber functions, covering both equations (1) and (2), and the first attempt at systematic tabulation of the solutions of (2) for real x and a.

The volume under review is a Russian translation of [1] by M. K. Kerimov. All tables and mathematical formulas appear to have been reproduced photographically from the original.

A supplementary section added by the translator contains additional material on Weber functions, mostly of recent origin. In particular, one finds an account of the asymptotic theory of these functions and their zeros as developed by Olver in the late 1950's miscellaneous results such as integral representations, limit relations, addition theorems, infinite series and integrals involving Weber functions, as well as a survey of recent tables and computer programs. The bibliography of cited references contains some 200 items.

W. G.

1. NATIONAL PHYSICAL LABORATORY, *Tables of Weber Parabolic Cylinder Functions*, Computed by Scientific Computing Service Limited, Mathematical Introduction by J. C. P. Miller, Editor. Her Majesty's Stationery Office, London, 1955. (See *MTAC*, v. 10, 1956, pp. 245–246, RMT **101**.)

73[7].—A. R. KERTIS, *Volnovye Funktsii Kulona* (A. R. CURTIS, *Coulomb Wave Functions*), Computing Center, Acad. Sci. USSR, Moscow, 1969, li + 209 pp., 27 cm. Price 2.23 rubles.

This is a translation into Russian of the Royal Society Mathematical Tables of Coulomb wave functions [1]. The original has been reviewed in this journal (v. 19, 1965, pp. 341–342, RMT **46**).

The tables, as well as all formulas and mathematical characters contained in the preface, appear to have been reproduced photographically from the original. A few minor blemishes in the original printing have been corrected. According to the editor, all tabular entries were checked by differencing, and no errors were found.

The translator, M. K. Kerimov, has added a supplementary section in which he gives further relationships for the Coulomb wave functions (in the notation of the NBS tables [2]) and a comprehensive survey of published tables in the field.

W. G.

1. A. R. CURTIS, *Coulomb Wave Functions*, Royal Society Mathematical Tables, Volume 11, Cambridge Univ. Press, New York, 1964.
2. NATIONAL BUREAU OF STANDARDS, *Tables of Coulomb Wave Functions*, Volume I, Applied Mathematics Series, No. 17, U. S. Government Printing Office, Washington, D. C., 1952.

74[7].—ANNE E. RUSSON & J. M. BLAIR, *Rational Function Minimax Approximations for the Bessel Functions $K_0(x)$ and $K_1(x)$*, Report AECL-3461, 1969, Atomic Energy of Canada Limited, Chalk River, Ontario. Price $1.50.

Consider

$$x^{-r}\left[K_r(x) + (-1)^r \ln x I_r(x) - \frac{r}{x}\right] = F_r(x^2),$$
$$x^{-r} I_r(x) = G_r(x^2),$$
$$x^{1/2} e^x K_r(x) = H_r(z), \qquad z = x^{-1},$$

where $r = 0$ or 1. Let $F_r(x^2)$ and $G_r(x^2)$ be approximated by $P_n(x^2)/Q_m(x^2)$ where $P_n(x^2)$ and $Q_m(x^2)$ are polynomials in x^2 of degree n and m respectively. For the range $0 \leqq x \leqq 1$, the coefficients in these polynomials corresponding to the 'best' approximation in the Chebyshev sense are tabulated for $m = 0$, $n = 1(1)8$, $m = 1$, $n = 2(1)6$ and $m = 3$, $n = 3,4$. Define precision as $P = -\log$ |maximum error in the range|. Then P ranges from about 3 to 23. Similarly coefficients for the 'best' Chebyshev approximation for $H_r(x)$ in the form $P_n(z)/Q_m(z)$ are given for the range $0 \leqq z \leqq 1$, where $m = 1(1)12$, $n = m - 1$ and $n = m$ if $r = 0$; and where $m = 1(1)12$, $n = m$ and $n = m + 1$ if $r = 1$. Again P ranges from about 3 to 23.

Y. L. L.

75[7].—D. A. TAGGART & F. W. SCHOTT, *Mathematical Tables of Integrals Involving Spherical Bessel Functions*, Electrical Sciences and Engineering Department, School of Engineering and Applied Science, University of California, Los Angeles, California, ms. of 16 typewritten pp. deposited in the UMT file.

Let $j_n(r)$ be the usual notation for the spherical Bessel function of order n. Herein are given six one-page tables, respectively, of 5S values (in floating-point form) of the integrals:

$$I_1 = \int_0^{r_0} j_t(ar)j_n(br)\,dr,$$

$$I_2 = \int_0^{r_0} \frac{\partial}{\partial r}[rj_t(ar)]\frac{\partial}{\partial r}[rj_n(br)]\,dr,$$

$$I_3 = \int_0^{r_0} j_t(ar)\frac{\partial}{\partial r}[rj_n(br)]\,dr,$$

$$I_4 = \int_0^{r_0} rj_t(ar)j_n(br)\,dr,$$

$$I_5 = \int_0^{r_0} rj_t(ar)\frac{\partial}{\partial r}[rj_n(br)]\,dr,$$

$$I_6 = \int_0^{r_0} r^2j_t(ar)j_n(br)\,dr.$$

Here $ar_0 = \mu_{np}$, the pth zero of $j_n(ar)$, and p is limited to 1, 2, and 3 throughout the tables. Also, these zeros are tabulated to 4 or 5S.

In the first three tables the orders t and n are restricted to 2 and 4. In Tables 4 and 5, $n = 2$ and 4, whereas $t = 1$, 3, and 5. Finally, in Table 6, $n = 1(1)5$ and $t = 1$, 3, and 5.

In the prefatory text the authors describe the computational procedure followed in the preparation of these tables, including several checks on the accuracy of the results to 5S.

Related tables to which the authors refer are those of Butler & Pohlhausen [1].

Y. L. L.

1. T. BUTLER & K. POHLHAUSEN, *Tables of Definite Integrals Involving Bessel Functions of the First Kind*, WADC Technical Report 54-420, Wright Air Development Center, 1954. (See *MTAC*, v. 9, 1955, p. 79, RMT 50.)

76[7].—HENRY E. FETTIS & JAMES C. CASLIN, *A Table of the Complete Elliptic Integral of the First Kind for Complex Values of the Modulus*, Part I, Report ARL 69-0172, Aerospace Research Laboratories, Office of Aerospace Research, United States Air Force, Wright-Patterson Air Force Base, Ohio, December 1969, iv + 298 pp., 27 cm. [Released to the Clearinghouse, U. S. Department of Commerce, Springfield, Virginia 22151.] and

HENRY E. FETTIS & JAMES C. CASLIN, *A Table of the Complete Elliptic Integral of the First Kind for Complex Values of the Modulus*, Part II, Report ARL 69–0173, Aerospace Research Laboratories, Office of Aerospace Research, United States Air Force, Wright-Patterson Air Force Base, Ohio, 1969, iv + 250 pp., 27 cm. [Released to the Clearinghouse, U. S. Department of Commerce, Springfield, Virginia 22151.]

These reports tabulate

$$K(k) = \int_0^{\pi/2} (1 - k^2 \sin^2 \varphi)^{-1/2} \, d\varphi, \qquad k = Re^{i\theta},$$

$$K(k') = K'(k) \quad \text{and} \quad iK'(k)/K(k), \qquad k' = (1 - k^2)^{1/2}$$

to 11D for $\theta = 0(1°)90°$ and $R = 0(0.01)1.0$. In Part I, the tables are arranged with θ as the parameter and R as the variable while in Part II, the tables are arranged with R as the parameter and θ as the variable. The introductory portion to Part I gives a discussion of the complete elliptic integrals of the first and second kinds, their relation to the Gaussian hypergeometric function and Legendre functions, and other properties such as analytic continuation, Jacobi's nome, and the Gauss or Landen transformation formulas. The latter were employed to produce the tables. This introduction is not given in Part II.

An errata sheet is included for Part I. This has to do with the introductory comments noted above. There is also an errata page for Part II which calls attention to the fact that the column of numbers headed by K'/K should be headed by iK'/K.

Y. L. L.

77 [8].—U. NARAYAN BHAT & IZZET SAHIN, *Transient Behavior of Queueing Systems $M/D/1$, $M/E_k/1$, $D/M/1$ and $E_k/M/1$-Graphs and Tables*, Technical Memorandum No. 135, Department of Operations Research, Case Western Reserve University, Cleveland, Ohio, January 1969, iii + 323 pp., 28 cm. One copy deposited in the UMT file.

This report presents tables [to 5D] and graphs for the imbedded Markov-chain behavior of the infinite-source, single-server queueing systems with (i) Poisson arrivals and constant service time ($M/D/1$); (ii) Poisson arrivals and Erlangian service times ($M/E_k/1$), with $k = 1(1)5(5)15$; (iii) regular arrivals and exponential service times ($D/M/1$); and (iv) Erlangian arrivals and exponential service times ($E_k/M/1$), with $k = 1(1)5(5)15$. The characteristics considered are the busy-period distribution and its mean, transition probabilities and the time-dependent mean and variance of queue length, effective steady-state and time-dependent measures of utilization and effectiveness.

The graphs and tables follow a discussion of the need for the construction of such tables in the study of the behavior of queueing systems and some details of the method employed in constructing the tables.

AUTHORS' SUMMARY

78[8].—ROBERT R. BRITNEY & ROBERT L. WINKLER, *Tables of nth Order Partial Moments about the Origin for the Standard Normal Distribution*, $n = 1(1)6$, ms. of four typewritten pp. + 10 computer sheets depositied in the UMT file.

These unpublished tables consist of 11S floating-point decimal values of the integral $(2\pi)^{-1/2} \int_{-\infty}^{z} x^n e^{-x^2/2}\, dx$ for $z = 0(0.01)5$ and $n = 1(1)6$. The underlying extended-precision computer calculations utilized data from the 15D NBS tables [1] of the normal probability function.

The introductory text cites several applications of such tables, with corresponding references to the literature.

These tables supersede the corresponding 7D table of Pearson [2], which is not mentioned by the authors.

J. W. W.

1. NATIONAL BUREAU OF STANDARDS, *Tables of Normal Probability Functions*, Applied Mathematics Series, v. 23, U. S. Government Printing Office, Washington, D. C., 1953.

2. K. PEARSON, EDITOR, *Tables for Statisticians and Biometricians. Part I*, third edition, Biometric Laboratory, University College, London, 1930, pp. 22–23 (Table 9).

79[8].—IRWIN GREENBERG, *Tables of the Compound Poisson Process with Normal Compounding*, ms. of 10 pp. + 15 computer sheets, deposited in the UMT file.

These manuscript tables give the cumulative distribution function of a compound Poisson process with normal compounding. This c. d. f. may be expressed as

$$F(z) = e^{-\lambda} + \sum_{n=1}^{\infty} \frac{\lambda^n}{n!} e^{-\lambda} N(z \mid 0, n)$$

for $z \geqq 0$, where $\lambda > 0$ and $N(z \mid 0, n)$ denotes the c. d. f. of a normally distributed random variable Z with mean 0 and variance n. For $z < 0$, the relationship $F(z) = 1 - F(-z)$ holds. The tables give $F(z)$ to 5D for 15 values of λ (1(1)5, 10, 15, 20, and their reciprocals) with $z = 0.00(.01)4.99$.

The manuscript describes some properties of the probability function and gives two approximation formulas. A brief table indicates that for selected values of z and λ a simple approximation to the c. d. f. gives values which differ from the exact values by less than 0.01. Two errors were found in this table. For $z = 5.0$ and $\lambda = 20$, the approximation formula gives 0.8682 (not 0.8708) and the exact value is 0.8708 (not 0.8683).

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80[10].—C. J. BOUWKAMP, A. J. W. DUIJVESTIJN & P. MEDEMA, *Table of c-Nets of Orders* 8 *to* 19, *Inclusive*, Philips Research Laboratories, Eindhoven, Netherlands, 1960. Ms. of trimmed and bound computer output sheets in two volumes each of 206 pp., 24 × 30 cm., deposited in the UMT file.

This table is a by-product of the research of the authors on squared rectangles [1], [2], [3]. A *c*-net, as the term is used here, is another name for a 3-connected planar graph; the number of edges is called the order of the *c*-net. Since 3-connected planar graphs are isomorphic with convex polyhedra, the table is also a table of convex polyhedra with up to 19 edges. The method of deriving the table and the program are given in [3], which also lists the nets with 15 and 16 edges. Drawings of the nets with up to 14 edges are given in [1]. (Two missing drawings can be constructed from the last line of data on page 71 of that reference.) The table is referred to in [5] and [6].

The two parts of the table give the same data in two different arrangements. In each part the listing is first by order; in Part I further arrangement is by the "complexity," and in Part II by an identification number. The following is the arrangement of the tabular columns, from left to right.

1. *Order*. In each part orders 8 to 14 are on one page each; order 15, pages 8 and 9; order 16, pages 10–14; order 17, pages 15–28; order 18, pages 29–72; and order 19, pages 73–206.

2. *Complexity*. This is the common absolute value of the cofactors of a certain incidence matrix of the graph, which plays a role in the theory of squared rectangles. It is also equal to the number of spanning trees of the graph. Two different graphs can have the same complexity.

3. *Identification number*. This number is given in octal; if converted to binary, it specifies the upper triangular part of the vertex-vertex incidence matrix of the graph. For further details see [3].

4. *Symmetry*. The degree of symmetry is indicated by the numerals 1, 2, or 3; a blank indicates no symmetry.

5. *Duality*. The letter *S* indicates that the graph is self-dual, otherwise this column is blank. Only one of a dual pair of nets is listed in the table; when the number of faces is different, the one with the greater number is the one included. In Part II the arrangement by the identification number brings together, under each order, the nets with the same number of faces.

6. *Code of the c-nets*. Each vertex of the *c*-net is lettered A, B, C, etc., and the code gives the circuit of each face of the net, repeating at the end the first letter given, with the face circuits separated by spaces.

The correction of an erroneous listing (due to the incorrect replacement of a torn punched card and discovered by J. Haubrich in the preparation of [4]) has been entered in ink on page 37 of Part I and page 34 of Part II.

AUTHORS' SUMMARY

1. C. J. BOUWKAMP, "On the dissection of rectangles into squares," *Proc. Acad. Sci. Amst.*, v. 49, 1946, pp. 1176–1188; v. 50, 1947, pp. 58–78, 1296–1299. (Same as *Nederl. Akad. Wetensch. Indag. Math.*, v. 8, 1946, pp. 724–736; v. 9, 1947, pp. 43–63, 622–625.)

2. C. J. BOUWKAMP, A. J. W. DUIJVESTIJN & P. MEDEMA, *Tables Relating to Simple Squared Rectangles of Orders Nine through Fifteen*, Technische Hogeschool, Eindhoven, Netherlands, 1960. (See *Math. Comp.*, v. 15, 1961, p. 315, RMT 84.)

3. A. J. W. DUIJVESTIJN, *Electronic Computation of Squared Rectangles*, Thesis, Technische Hogeschool, Eindhoven, Netherlands, 1962. (Also, in *Philips Res. Rpts.*, v. 17, 1962, pp. 523–613.)

4. C. J. BOUWKAMP, A. J. W. DUIJVESTIJN & J. HAUBRICH, *Catalogue of Simple Perfect Squared Rectangles of Orders* 9 *through* 18, ms. of 12 volumes, 3090 pp., containing 154490 squared rectangles, Philips Research Laboratories, Eindhoven, Netherlands, 1964.

5. B. Grundbaum, *Convex Polytopes*, Interscience (Wiley), New York, 1967, pp. 48, 289, 431.
6. P. J. Federico, "Enumeration of polyhedra: the number of 9-hedra," *J. Combinatorial Theory*, v. 7, 1969, pp. 155–161.

81[10].—Frank Harary, Editor, *Proof Techniques in Graph Theory*, Academic Press, New York, 1969, xv + 330 pp., 25 cm. Price $14.50.

This book contains the proceedings of the Second Ann Arbor Graph Theory Conference, held in February 1968, and comprises the following papers.

F. Harary, The Four Color Conjecture and other Graphical Diseases.

L. W. Beineke and J. W. Moon, Several Proofs of the Number of Labeled 2-Dimensional Trees.

G. Chartrand and J. B. Frechen, On the Chromatic Number of Permutation Graphs.

B. Descartes, The Expanding Unicurse. [A poem.]

P. Erdös, Problems and Results in Chromatic Graph Theory.

D. Geller, Forbidden Subgraphs.

D. Geller and S. Hedetniemi, A Proof Technique in Graph Theory.

R. P. Gupta, Independence and Covering Numbers of Line Graphs and Total Graphs. [Abstract only.]

R. K. Guy, The Decline and Fall of Zarankiewicz's Theorem.

F. Harary, On the Intersection Number of a Graph. [Taken from pages 19, 20 and 25 of F. Harary, *Graph Theory*, Addison-Wesley, Reading, Mass., 1969.]

Z. Hedrlín, On Endomorphisms of Graphs and their Homomorphic Images.

T. A. Jenkyns and C. St. J. A. Nash-Williams, Counterexamples in the Theory of Well-Quasi-Ordered Sets.

W. Kuich and N. Sauer, On the Existence of Certain Minimal Regular n-Systems with Given Girth.

B. Manvel, Reconstruction of Unicyclic Graphs.

A. Mowshowitz, The Group of a Graph whose Adjacency Matrix has all Distinct Eigenvalues.

U. S. R. Murty, Extremal Nonseparable Graphs of Diameter 2.

E. A. Nordhaus, A Class of Strongly Regular Graphs.

E. M. Palmer, The Exponentiation Group as the Automorphism Group of a Graph.

G. Ringel and J. W. T. Youngs, Remarks on the Heawood Conjecture. [Comments on the same authors' solution of the Heawood map-coloring-problem, *Proc. Nat. Acad. Sci.* U.S.A., v. 60, 1968, pp. 438–445.]

F. S. Roberts, Indifference Graphs.

R. W. Robinson, Enumeration of Euler Graphs. [The solution of problem 8 on F. Harary's third list of unsolved graphical enumeration problems—see *Graph Theory and Theoretical Physics*, edited by F. Harary, Academic Press, New York, 1967, p. 30.]

J. Turner, A Graph-Theoretical Model for Periodic Discrete Structures. [This is a shortened version of a paper with the same title appearing in *J. Franklin Inst.*, v. 285, 1968, pp. 52–58.]

W. T. Tutte, Even and Odd 4-Colorings.

M. E. Watkins, A Theorem on Tait Colorings with an Application to the Generalized Peterson Graphs. [This is a shortened version of a paper with the same title appearing in *J. Combinatorial Theory*, v. 6, 1969, pp. 152–164.]

H. S. Wilf, The Möbius Function in Combinatorial Analysis and Chromatic Graph Theory. [This is an expanded version of H. S. Wilf, "Hadamard determinants, Möbius functions, and the chromatic number of a graph," *Bull. Amer. Math. Soc.*, v. 74, 1968, pp. 960–964.]

J. Turner, Key-Word Indexed Bibliography of Graph Theory.

This book is the seventh graph theory symposium proceedings to appear in recent years (see the list on page ix) and like its predecessors is a valuable collection for research workers in graph theory. (In spite of the title of the book, however, a number of the papers contain no proofs.) Probably the most valuable paper is Turner's bibliography which occupies almost half the book. This contains some 1800 entries, arranged both by author and by key words, and is current up to July 1968. (Mention should be made of the more recent bibliography of N. Deo, *An Extensive English Language Bibliography on Graph Theory and its Applications*, Technical Report 32–1413, Jet Propulsion Laboratory, California Institute of Technology, October 15, 1969. This contains some 2200 entries, arranged by author, and is current to April 1969. Only English language articles are listed. Both Turner's and Deo's bibliographies are based on that of A. A. Zykov, in the *Theory of Graphs and Its Application*, edited by M. Fiedler, Academic Press, New York, 1964, although they appear to have been compiled independently. Finally, a long survey article on graph theoretic work in the Soviet Union will be published by J. Turner and W. H. Kautz in the *SIAM Rev.* in 1970. This is based on the Soviet entries in Turner's bibliography in the book under review.)

The following misprints are noted.

p. 4, Theorem 7: *for* $(p - 1)k$ *read* $(p - 1)/2$.

p. 37, line 17: *for* $K_{[(n+1)/2],[(n+1)/2]}$ *read* $K_{[(n+2)/2],[(n+2)/2]}$.

p. 37, line 23: *for* $K_{[n+1/2],[n+1/2]}$ *read* $K_{[(n+2)/2],[(n+2)/2]}$.

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