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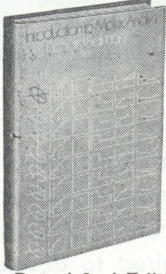
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also a symmetry operation of the whole configuration. The honeycomb is said to be twisted if it is not symmetrical by reflection; thus a twisted honeycomb, like a screw, occurs in right-handed and left-handed varieties. The subject originated in 1933, when Weber and Seifert considered two one-celled honeycombs, each consisting of a regular dodecahedron with opposite faces identified. The work involves a combination of geometry and group theory, assisted by quaternions.

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