REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS

The numbers in brackets are assigned according to the indexing system printed in Volume 22, Number 101, January 1968, page 212.

13[1].—CONSTANCE REID, *Hilbert*, Springer-Verlag, New York, 1970, xii + 290 pp., 24 cm. Price $8.80.

No better review could serve for this eloquently written book than to quote Richard Courant's foreword to the book:

"David Hilbert was one of the truly great mathematicians of his time. His work and his inspiring scientific personality have profoundly influenced the development of the mathematical sciences up to the present time. His vision, his productive power and independent originality as a mathematical thinker, his versatility and breadth of interest made him a pioneer in many different mathematical fields. He was a unique personality, profoundly immersed in his work and totally dedicated to his science, a teacher and leader of the very highest order, inspiring and most generous, tireless and persistent in all of his efforts.

"To me, one of the few survivors of Hilbert's inner circle, it always has appeared most desirable that his biography should be published. Considering, however, the enormous scientific scope of Hilbert's work, it seemed to me humanly impossible that a single biographer could do justice to all the aspects of Hilbert as a productive scientist and to the impact of his radiant personality. Thus, when I learned of Mrs. Reid's plan for the present book I was at first skeptical whether somebody not thoroughly familiar with mathematics could possibly write an acceptable book. Yet, when I saw the manuscript my skepticism faded, and I became more and more enthusiastic about the author's achievement. I trust that the book will fascinate not only mathematicians but everybody who is interested in the mystery of the origin of great scientists in our society."

E. I.

14[2, 3, 4, 5, 6, 7, 8, 11].—KAREL REKTORYS, Editor, *Survey of Applicable Mathematics*, translated from Czech by R. Vyborny, MIT Press, Cambridge, Mass., 1969, 1369 pp., 25 cm. Price $16.95.

As mathematical activities proliferate, and mathematical sciences increasingly interact with applied sciences, it is important that the line of communication between mathematicians and all those who wish to use mathematics be kept open. The volume under review is a notable contribution to this effort. It presents to the applied scientist, in a language which he can easily understand, a variety of mathematical subjects likely to be relevant to his work. The material is consistently presented in a format of definitions and theorems. Instead of proofs, there are explanatory remarks and illustrative examples. The reader, therefore, can quickly get a survey of any particular subject and learn what concepts are in use and what results are available. For a deeper study, he is referred to appropriate books and journal articles. The subjects treated range from college mathematics through calculus to advanced topics in analysis. Approximate methods are emphasized throughout; their treatment, however, reflects the state of knowledge in the early sixties. (The original work was published in 1963.) A listing of chapter headings and respective authors follows.

1. Arithmetic and algebra (V. Vilhelm), 2. Trigonometric and inverse trigonometric functions. Hyperbolic and inverse hyperbolic functions (V. Vilhelm), 3. Some formulae (V. Vilhelm), 4. Plane curves and constructions (K. Drábek), 5. Plane analytic geometry (M. Zelenka), 6. Solid analytic geometry (F. Kejla), 7. Vector calculus (F. Kejla and K. Rektorys), 8. Tensor calculus (V. Vilhelm), 9. Differential geometry (B. Kepr), 10. Sequences and series of constant terms. Infinite products (K. Rektorys), 11. Differential calculus of functions of a real variable (K. Rektorys), 12. Functions of two or more variables (K. Rektorys), 13. Integral calculus of functions of one variable (K. Rektorys), 14. Integral calculus of functions of two or more variables (K. Rektorys), 15. Sequences and series with variable terms (K. Rektorys), 16. Orthogonal systems. Fourier series. Some special functions (K. Rektorys), 17. Ordinary differential equations (K. Rektorys), 18. Partial differential equations (K. Rektorys), 19. Integral equations (K. Rektorys), 20. Functions of a complex variable (K. Rektorys), 21. Conformal mapping (J. Fuka), 22. Some fundamental concepts from the theory of sets and functional analysis (K. Rektorys), 23. Calculus of variations (F. Nožička), 24. Variational methods for solving boundary value problems of differential equations (M. Prager), 25. Approximate solution of ordinary differential equations (O. Vejvoda and K. Rektorys), 26. Solution of partial differential equations by infinite series (K. Rektorys), 27. Solution of partial differential equations by the finite-difference method (E. Vitásek), 28. Integral transforms (J. Nečas), 29. Approximate solution of Fredholm integral equations (K. Rektorys), 30. Numerical methods in linear algebra (O. Pokorná and K. Korvasová), 31. Numerical solution of algebraic and transcendental equations (M. Fiedler), 32. Nomography and graphical analysis. Interpolation. Differences (V. Pleskot), 33. Probability theory (J. Hájek), 34. Mathematical statistics (J. Hájek), 35. Method of least squares. Fitting curves to empirical data. Elements of the calculus of observations (O. Fischer).

W. G.

15[2, 4, 5, 6, 13.05, 13, 15].—R. SAUER & I. SZABÓ, *Mathematische Hilfsmittel des Ingenieurs*, Teil II, Springer-Verlag, Berlin, 1969, xx + 684 pp., 24 cm. Price $37.40.

[For reviews of Volumes I and III of this four-volume sequence, see *Math. Comp.*, v. 23, 1969, pp. 208–209 and *Math. Comp.*, v. 24, 1970, pp. 475–476.]

Volume II of this encyclopedic work is devoted to the theory and practical solution of differential equations, and thus takes up a topic which is of vital concern not only to the engineer, but also to the scientist in general. Accordingly, the subject is treated in considerable depth and on the advanced mathematical level which it demands. While the style of presentation is necessarily concise, numerous examples are included throughout for clarification and illustration.

The material is organized into two large sections, D and E, of which the first

(292 pages) is concerned with initial-value problems and the second (377 pages) with boundary and eigenvalue problems. Both sections include ordinary and partial differential equations, as well as systems of such equations.

Section D, written by W. Törnig, begins with a chapter on ordinary differential equations. An outline of the basic existence and uniqueness theory, and an exposition of some elementary integration methods is followed by a more detailed treatment of linear equations, particularly of Fuchs' class, and of systems of such equations, where problems with periodic coefficients are given special attention. There is also a discussion of numerical methods of the one-step and multi-step type. Partial differential equations are taken up in Chapter II, which is devoted to single equations of the first order. Here, one has a complete geometric theory of integration, based on characteristics, which is expounded first for linear and quasilinear equations in two and more variables, and then for general nonlinear equations. Chapter III proceeds to hyperbolic equations and first discusses the characteristic as well as the Cauchy initial value problem for weakly nonlinear and linear second-order equations in two variables. The wave equation and associated initial and initial-boundary value problems are examined next in more detail. This is followed by a discussion of hyperbolic systems of the first order, where the equations of gas dynamics figure prominently among the applications. Numerical methods, particularly the method of characteristics and finite difference methods, are then discussed in considerable detail. Chapter IV gives a similar treatment to parabolic equations. The discussion revolves mainly around the heat and diffusion equation, but includes also the nonlinear equations of boundary layer theory and equations of mixed type. Numerical methods again receive due attention.

Section E has eight chapters, the fourth being authored by L. Collatz, the others by R. Nicolovius. Chapter I deals with boundary-value problems for ordinary differential equations, first with linear problems, and in this connection also with the theory of linear integral equations, and then with nonlinear problems. Much space is allotted to existence and uniqueness results, and to questions of monotonicity and two-sided approximation. Chapter II treats boundary-value problems for partial differential equations, generally in N-space, with special emphasis on the three types of boundary conditions associated with elliptic second-order equations. Among the topics discussed are the concepts of fundamental solutions and Green's function, various uniqueness, existence, and alternative theorems, the maximum principle, and monotonicity properties of partial differential operators. Some of these topics are also discussed in the context of nonlinear equations. A separate chapter is devoted to problems in the theory of the potential and to other problems and equations of mathematical physics, including the reduced wave equation, the problem of minimal surfaces, the Navier-Stokes equations in hydrodynamics, and the equations of elasticity theory. Chapter IV then turns to eigenvalue problems for differential and integral equations. Some general theorems on selfadjoint completely definite problems, concerning in particular the minimum properties of eigenvalues and expansions in eigenfunctions, are followed by inclusion principles for eigenvalues and a presentation of Ritz's method for their approximation. Other approximate methods are also briefly considered, among them the difference method, collocation, perturbation methods, and Weinstein's method of intermediate problems. Variational problems, already touched upon in Chapter IV, receive a systematic treatment in the next chapter, where one finds not only a formulation of the basic problems and methods of the calculus of variations, but also a detailed discussion of how to construct a variational problem from a given boundary-value, eigenvalue, or integral equations problem, and a thorough exposition of direct methods (due to Ritz, Galerkin, Friedrichs, Trefftz, Synge, and others) for solving variational problems. The remaining three chapters are largely method-oriented, but draw frequently upon the problems discussed earlier for illustration. Chapter VI begins with closed-form solutions by means of series (power series, orthogonal and asymptotic expansions), and then illustrates some general principles and approaches toward the numerical treatment of problems. The method of finite differences for differential equations, and the quadrature method for integral equations, of course, hold a central position in the context of this section, and are therefore discussed very thoroughly in Chapter VII. Iteration methods, finally, are the subject of Chapter VIII, which contains general contraction and fixed point theorems as well as a formulation of Newton's method and the method of false position in a Banach space.

W. G.

16[2.05, 2.10, 2.15, 2.20, 2.35, 2.40, 2.55, 3, 4].—CHARLES B. TOMPKINS & WALTER L. WILSON, JR., *Elementary Numerical Analysis*, Prentice-Hall, Inc., Englewood Cliffs, N. J., 1969, xvi + 396 pp., 24 cm. Price $10.50.

The book is directed to a wide audience of beginning students and offers them a sound introduction into the techniques and underlying philosophies of numerical analysis. A commendable effort has been made to motivate all subjects discussed, and to emphasize general principles involved. While the selection of topics is fairly standard, it is an unusual feature of the book that all formulas are displayed in a one-line format not unlike that of present-day computer outputs. Table of contents: 1. Introduction, 2. Taylor's formula: truncation error, 3. Iteration processes: Newton's method, 4. Systems of linear equations, 5. Eigenvalues and eigenvectors, 6. Finite differences, 7. Interpolation, 8. Least squares estimates, 9. Numerical differentiation, 10. Numerical integration, 11. Difference equations, 12. Numerical solution of differential equations.

W. G.

17[2.05, 7].—GÉZA FREUD, *Orthogonale Polynome*, Birkhäuser Verlag, Basel, Switzerland, 1969, 294 pp., 25 cm. Price Sfr. 42.00.

To indicate the general character of this important book and how it relates to earlier monographs on the subject, it is best to quote (in free translation) from the author's preface.

"This book is concerned with the general theory of orthogonal polynomials relative to a nonnegative measure on the real line. For prerequisites, it is assumed that, beyond the usual basic analysis courses, the reader has completed an introductory course in real analysis. Only the last chapter requires some knowledge of complex analysis. I hope to offer something useful to every reader, regardless of whether he is interested in applications of finished results, in lecture material, or in further research in the subject. To the expert, too, I hope to say much which is new to him.

"Since the appearance of G. Szegö's monograph, thirty years have passed. In these three decades, his excellent book has served as a guiding line for further research. The second edition of Szegö's work, published in 1959, contained relatively few additions. More recent publications, such as the books of F. Tricomi and G. Sansone, as well as the relevant portions of the 'Bateman Project', are mostly concerned with special orthogonal polynomials. The monograph of Ja. L. Geronimus treats, exclusively, Szegö's theory. An up-to-date survey of the general theory of orthogonal polynomials was not available. I hope, with my book, to be able to fill this gap. By 'general theory', we mean that all the results are derived from the two facts that we are dealing with polynomials, and that the sequence of these polynomials forms an orthogonal system relative to a given measure. I hope to convince the reader that in the framework of this general theory, it is possible to prove many theorems concerning special orthogonal polynomials (e.g., on the convergence of interpolation processes and series of orthogonal polynomials) much simpler, and in a logically more transparent manner, than by considering these polynomials as special functions.

"I was not satisfied with merely grouping together new theorems, but attempted to provide a new framework for the whole theory. In this framework, a whole chapter was devoted to the moment problem (in a form free of continued fractions). To the great classical investigators of orthogonal polynomials, like Chebyshev and Stieltjes, the close connection between the moment problem and orthogonal polynomials was still self-evident. This connection was reinforced also through more recent investigations in this century. It suffices to point to the beautiful theorem of M. Riesz characterising all measures $d\alpha$ for which the orthogonal polynomials are complete in $L^2_{d\alpha}$. However, I penetrated into the theory of the moment problem only as far as seemed useful to the applications in this book. The exposition was simplified with the aid of some results of my own."

The book has five chapters. Chapter I develops the basic properties of orthogonal polynomials $p_n(d\alpha; x)$ relative to a measure $d\alpha$. Many results, such as the Gauss-Jacobi quadrature formula and the Markov-Stieltjes inequalities, are formulated in terms of the more general "quasi-orthogonal" polynomials of M. Riesz, $\psi_n(x, \xi) = p_{n-1}(\xi)p_n(x) - p_n(\xi)p_{n-1}(x)$, where ξ is an arbitrary real number. The chapter also contains a brief account of Chebyshev and Legendre polynomials and some elementary estimates for orthogonal polynomials. Chapter II is devoted to the Hamburger-Stieltjes moment problem. It begins with Hamburger's fundamental result on the solubility of the moment problem, and then proceeds to develop the uniqueness theory due to Hamburger, M. Riesz, and others. Related questions of one-sided approximation by polynomials, and the completeness in $L^2_{d\alpha}$ of orthogonal polynomials, are also discussed. Chapter III first treats the convergence of interpolatory quadrature formulas and the convergence in $L^2_{d\alpha}$ of interpolation polynomials, when the nodes are zeros of orthogonal polynomials, or zeros of certain related polynomials. There are also results on pointwise and uniform convergence. For measures $d\alpha$ with support in $[-1, 1]$, there follows a discussion of the behaviour of $p_n(d\alpha; z)$ for large n and for z in the complex plane cut along the segment $[-1, 1]$. One finds, in particular, the result of P. Erdös and P. Turán, according to which $\lim_{n\to\infty} [p_n(d\alpha; z)]^{1/n} = z + (z^2 - 1)^{1/2}$, if the support of $d\alpha$ coincides with $[-1, 1]$ and

$\alpha'(x) > 0$ a.e. in $[-1, 1]$. This is then applied to discuss interpolation of analytic functions at the zeros of orthogonal polynomials. The chapter concludes with a result on the equidistribution of zeros (more precisely, their projections on the unit circle) of orthogonal polynomials. In Chapter IV, the author then turns his attention to the convergence and summability theory of orthogonal series, assuming measures with support in $[-1, 1]$. The final Chapter V presents Szegö's theory, i.e., the theory of orthogonal polynomials on the unit circle, and contains further important asymptotic results on orthogonal polynomials, Christoffel numbers, and the distance between consecutive zeros.

Each chapter is followed by a collection of exercises, which form an integral part of the book. These in turn are followed by historical notes. In an epilogue, the author points toward certain parts of the theory which are not as yet completely developed and lists a series of important open problems. This should be especially valuable for the young mathematician seeking research problems in the area of orthogonal polynomials.

It is impossible, in a brief review, to convey the extraordinary wealth and beauty of the results presented. Any reader who seriously studies this book will find his efforts richly rewarded.

It is only to be regretted that the printing is not up to the high standards one has come to expect from the publisher and that there are a disturbing number of typographical errors.

W. G.

18[2.35].—J. M. Ortega & W. C. Rheinboldt, *Iterative Solution of Nonlinear Equations in Several Variables*, Academic Press, New York, 1970, xx + 572 pp., 24 cm. Price $24.00.

This mature presentation is said by the authors to be the outgrowth of their research and graduate teaching over the last five years. Their "aim is to present a survey of the basic theoretical results about nonlinear equations in n dimensions as well as an analysis of the major iterative methods for their numerical solution—to provide here a text for graduate numerical analysis courses—to make the work useful as a reference source".

The authors succeed admirably. They supply numerous exercises to extend the text, many with references to research articles by other workers. Another outstanding feature is the addition of a supplement, called "Notes and Remarks", to each section. This auxiliary material gives pertinent literature citations, and valuable extensions of the text which give the reader a feeling for the state of our current knowledge in this field. They supply a two page annotated list of basic reference texts and a thirty-five page comprehensive bibliography.

The authors deal almost exclusively with methods that involve, at most, first-order derivatives. They divide the book into five parts, with each part containing two or more sections.

Part I—Background Material—contains:

Section 1—Sample Problems—an interesting collection of problems from ordinary differential, partial differential, and integral equations, along with minimization and variational problems. This section gives the motivation for studying the subject.

Section 2—Linear Algebra—and

Section 3—Analysis—good summaries of multivariate calculus and linear algebra, with references to the basic texts.

The remainder of the book is now developed within the background of Part I. A listing of the remaining section headings may suffice to indicate the scope of the book:

Part II—Nonconstructive Existence Theorems,
  Section 4—Gradient Mappings and Minimization,
  Section 5—Contractions and the Continuation Property,
  Section 6—The Degree of a Mapping;

Part III—Iterative Methods,
  Section 7—General Iterative Methods,
  Section 8—Minimization Methods;

Part IV—Local Convergence,
  Section 9—Rates of Convergence—General,
  Section 10—One-Step Stationary Methods,
  Section 11—Multistep Methods and Additional One-Step Methods;

Part V—Semilocal and Global Convergence,
  Section 12—Contractions and Nonlinear Majorants,
  Section 13—Convergence under Partial Ordering,
  Section 14—Convergence of Minimization Methods.

An analysis of asymptotic convergence rates and appropriate indications of how to use the general theory in the presence of round-off errors are given. The book is not only well conceived mathematically, but it is beautifully written and "as self-contained as possible".

E. I.

19[2.45, 2.55, 12].—John K. Rice & John R. Rice, *Introduction to Computer Science*, Holt, Rinehart & Winston, Inc., New York, 1969, xv + 463 pp., 24 cm. Price $12.95.

Among those concerned with teaching computer science, there is a view, currently in the ascendant, that one ought to teach principles of algorithm construction rather than mere programming technique. The authors of this book hold this view, and have ably constructed a textbook, based upon it, for a first course in computer science. They have written a textbook as textbooks ought to be written: clear prose, ample and relevant exercises, clean organization, but with a great deal of inherent flexibility, and pleasing typography and layout, with two-color printing. It is a triumph of the pedagogue's craft.

Nevertheless, I disagree with the basic principle upon which this book is constructed: that one can, in a meaningful sense, study algorithms outside of the context of a programming language. The difficulty, as this reviewer sees it, is that only the necessity of writing a program for an algorithm can force one to define that algorithm rigidly. Any abstract notion of what an algorithm is implies an underlying notion of what a machine is, be it a Turing machine, an Algol machine, or an IBM 360/65. One can exhort the student to be more specific in reducing a vaguely specified computational process to an algorithm; but the only way to answer the question "Have I got an algorithm?" is to try to write a program in some specific programming language. Furthermore, many of the more vexing problems in algorithm construction are concerned with representational problems, and these are easily glossed over in the absence of the demand that a program be written.

The book consists of eight chapters plus three appendices: one on ALGOL, one on FORTRAN, and one on Programs, Compilers, and Systems. The eight chapters are:

1. The Formulation of Problems.
2. The Structure of Algorithms.
3. Languages for Algorithms.
4. Computer Organization.
5. The Construction of Familiar Algorithms.
6. The Representation of Information.
7. The Classification of Solution Methods.
8. The Nature of Errors and Uncertainty.

In keeping with their philosophy, the authors place all material concerning specific programming languages into the Fortran and Algol appendices, and the body of the book is almost entirely language independent. In Chapter 3, they introduce a "natural programming language" which introduces a number of notational conveniences, including normal mathematical symbolism. This natural programming language is not rigidly defined; rather, it is intended for communication with the reader, and according to the authors:

> The natural programming language used in this book is primarily intended for communication with the reader. The criterion for acceptability of a statement in this language is: Can a person understand what is meant? In particular, the reader should not attempt to learn this language in the way that he would learn algorithmic languages like Algol and Fortran.

Most algorithms given later in the book are given in natural programming language; often they are also given in Fortran and Algol.

There are places where, I feel, some distortion has crept into the material because of the authors' conception of it. For example, the first chapter is on problem formulation. The general procedure suggested by the authors is to form a model, get some answers from it, and if the answers are not reasonable, modify the model. This procedure has two very dubious underlying assumptions: that any problem can be modelled by simple equations, and that reasonable answers are necessarily correct answers. A specific example is given: how long should one keep a car before trading it in in order to minimize average annual cost. After many arbitrary assumptions, e.g., that the depreciation in a given year is 30% of the value at the beginning of the year, a conclusion is reached: that "Big Shot" should trade in his car after 2 years, that "Average Joe" can trade it in anytime after 3 years, and that "Penny-Pincher" should keep it for 12 years. Only linear equations are used, and constants (like the 30%) are chosen at will. The results are accepted because they are reasonable. And one of the exercises, requiring an analysis of this sort, is:

> You are a girl's advisor in a major college. A young lady comes to you for advice about the field in which she should major. Work out a schedule.

I submit that examples such as these do not lead to good judgment about when mathematical models should be used.

The book contains some excellent chapters on matters not usually treated adequately. For instance, there is a chapter on the representation of information, which shows the student how to encode information in a form in which a computer can deal with it. The problem is treated on several levels: the physical representation of information, the representation of alphabetic information numerically, the use of arrays and lists, the representation of graphical relationships, and the Polish prefix representation of algebraic expressions. There is also a chapter on errors: where they come from, how to classify them, and how to safeguard (as far as possible) against them. More generally, the book contains a great deal of material designed to give the student a "world view" of what is going on in the computer field.

In summary, this is an excellent textbook for those instructors who are in sympathy with the philosophy that a first computer science course should teach students about algorithms rather than about programming. I happen not to be in sympathy with that philosophy.

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20[4, 5, 13.15].—John R. Radbill & Gary A. McCue, *Quasilinearization and Nonlinear Problems in Fluid and Orbital Mechanics*, American Elsevier Publishing Co., Inc., New York, 1970, xxiii + 228 pp., 24 cm. Price $14.00.

In Chapter 1 of this book the authors summarize some elementary results on ordinary differential equations. In Chapter 2 they present the "quasilinearization method," which, as far as can be determined from this book, is, in fact, Newton's method. The discussion of the validity and applicability of this method is best described as minimal. Chapters 3 through 9 present a number of applications, drawn mainly from the theory of hydrodynamic stability and boundary layer theory, but including also the computation of electrostatic probe characteristics (Chapter 5) and optimum orbital transfer with "bang bang" control (Chapter 8). A computer program is given in Chapter 9. Some of these problems are difficult and important, but the presentation is too sketchy to be understood without extensive prior knowledge. One cannot make sense of, say, the Orr-Sommerfeld equation, with so few mathematical tools. The level of the mathematical discussion throughout the book is extremely low, and this is particularly true of the numerical aspects which the authors claim to emphasize. The prose, always undistinguished, is sometimes incomprehensible.

In summary, it is difficult to imagine for what kind of reader this book has been written; the beginning mathematician or engineer should be referred to standard textbooks on numerical analysis or hydrodynamics, few of which, by the way, he will find in the authors' short bibliography.

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21[7].—CHIH-BING LING & JUNG LIN, *A Table of Sine Integral* Si $(n\pi/2)$, Virginia Polytechnic Institute and State University, Blacksburg, Virginia, and Tennessee Technological University, Cookeville, Tennessee, November 1970, ms. of ii + 4 pp. deposited in the UMT file.

The table announced in the title of this manuscript actually consists of 25D values of $(2/\pi)$ Si $(n\pi/2)$ for $n = 1(1)200$.

For values of n not exceeding 7, full accuracy to 25D was attained on an IBM 1620 computer by use of the standard power series for the sine integral. The corresponding asymptotic series sufficed to yield the desired accuracy for values of n exceeding 35. For the intermediate values of n, recourse was had to power-series evaluation on an IBM 360 system, using a multi-precision arithmetical package supplied by Dr. T. C. Ting.

As a partial check, the values of Si $(m\pi)$ for $m = 1(1)3$ were deduced and successfully compared with the corresponding 15D values in a W.P.A. table [1] of the sine and cosine integrals.

J. W. W.

1. W. P. A., NEW YORK MATHEMATICAL TABLES PROJECT, *Table of Sine, Cosine and Exponential Integrals,* v. 2, 1940, pp. 206–207.

22[7].—T. S. MURTY, *Tables of the Conical Functions* $K_p(x)$, Marine Sciences Branch, Department of Energy, Mines and Resources, Ottawa, Ontario, Canada, ms. of 2 pp. + 120 computer sheets deposited in the UMT file.

The body of this manuscript consists of two tables: the first consists of 8S values of $K_p(x)$, or $P_{-1/2+ip}(x)$ in the standard notation of Legendre functions, for $p = 0.1(0.1)10$ and $x = 1(0.1)10$; the second gives 8S values of the zeros, the corresponding first derivatives, and the coordinates of the bend points of $K_p(x)$, for $p = 0.9(0.1)10$.

An introductory note briefly describes the formulas used in calculating these tables by double-precision computer arithmetic, thereby insuring complete accuracy of the final tabular data, according to the author.

These tables were calculated in connection with the theoretical determination of frequencies for an annular regime of liquid in rotating paraboloidal basins.

J. W. W.

23[9].—K. Y. Choong, D. E. Daykin & C. R. Rathbone, *Regular Continued Fractions for π and γ*, University of Malaya Computer Centre, September 1970. Computer output deposited in the UMT file.

The main table here lists the first 21230 partial quotients a_n in

$$\pi = 3 + \frac{1}{a_1 +} \frac{1}{a_2 +} \cdots \frac{1}{+ a_{21230} +} \cdots .$$

The paper [1] describing this computation appears elsewhere in this issue. This main table is printed on eight sheets of computer paper, with five blocks of ten lines each on each page. The last index n is listed after each block. There follows a statistical table that lists the number of n here such that $a_n = k$ for each $k = 1, 2, 3, \cdots, 2000$. The first missing k is $k = 103$. (An unsolved problem! Who will settle it?)

An earlier computation by R. S. Lehman [2] went to a_{1986}; the tables agree to that limit.* Lehman's discovery,

$$a_{431} = 20776,$$

remains the largest partial quotient up to $n = 21230$ (see Table 2 of [1]). Table 2 of [1] lists all ten $a_n > 2000$ up to 21230. The authors do not comment on the fact that this count of 10 seems a little low, since the Gauss-Kuzmin law predicts

$$\frac{21230 \ln (2002/2001)}{\ln 2} = 15.3$$

for almost all real numbers.

Anyone planning to check or extend this computation could check his output against Table 2 of [1] if the complete table is not available to him.

There is also deposited the first 3470 a_n in

$$\gamma = \frac{1}{a_1 +} \frac{1}{a_2 +} \cdots \frac{1}{+ a_{3470} +} \cdots .$$

Here the largest quotient is $a_{528} = 2076$. Again, a statistical table is given.

The previous computation [3] of γ to a_{371} is not mentioned in [1]; nor is [4], which presumably was the source of the decimal value of γ used.

For a good bibliography up to 1959 see Lehman [2].

D. S.

1. K. Y. Choong, D. E. Daykin & C. R. Rathbone, "Rational approximations to π," *Math. Comp.*, v. 25, 1971, pp. 387–392.

2. R. Sherman Lehman, *A Study of Regular Continued Fractions*, BRL Report 1066, Aberdeen Proving Ground, Maryland, February 1959.

3. Donald E. Knuth, "Euler's constant to 1271 places," *Math. Comp.*, v. 16, 1962, pp. 275–281.

4. Dura W. Sweeny, "On the computation of Euler's constant," *Math. Comp.*, v. 17, 1963, pp. 170–178.

* Subsequently, I obtained a copy of the unpublished continued fraction for π to a_{10053} that was computed on the Illiac II in the summer of 1963. This computation was by an NSF project for undergraduates under the direction of Norman T. Hamilton. The partial quotients agree with those deposited here.

24[12].—T. D. Sterling & S. V. Pollack, *Computing and Computer Science, A First Course with* PL/I, The Macmillan Co., New York, 1970, xvi + 414 pp., 24 cm. Price $9.95.

This attractively bound hard cover book is quite suited to a basic course in computer science, though not to a course in PL/I programming which, for all the unsuspecting reader is aware, might be the subject of the book. However, it is a systematic treatment of the basic ideas including numbers, Turing machines, algorithms, etc. In fact, for the first seven chapters, no mention of PL/I is made at all. The eighth chapter quickly goes through the basic repertoire of PL/I, followed in the next three chapters by a treatment of the remainder of the repertoire. In the few instances where a complete PL/I segment is shown, key PL/I words are shown in one kind of type while the remainder are printed in bigger type, making for an unusual appearance although the purpose is quite clear.

The rest of the book is an analysis of metalanguages and more complex topics of computer science. Except for the chapters explaining the repertoire of PL/I, the text is absolutely identical to its companion book of the series "A First Course with Fortran IV", written by the same authors.

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25[12].—T. D. Sterling & S. V. Pollack, *Computing and Computer Science, A First Course with Fortran* IV, The Macmillan Co., New York, 1970, xvi + 398 pp., 24 cm. Price $9.95.

This attractively bound hard cover book is quite suited to a basic course in computer science, though not to a course in Fortran IV programming which, for all the unsuspecting reader is aware, might be the subject of the book. However, it is a systematic treatment of the basic ideas including binary numbers, Turing machines, algorithms, etc. In fact, for the first seven chapters, no mention of Fortran is made at all. The eighth chapter quickly goes through the basic repertoire of Fortran IV, followed in the next three chapters by a treatment of the remainder of the repertoire. In the few instances where a complete Fortran IV segment is shown, key Fortran words are shown in one kind of type while the remainder are printed in bigger type, making for an unusual appearance although the purpose is quite clear.

The rest of the book is an analysis of metalanguages and more complex topics of computer science. Except for the chapters explaining the repertoire of Fortran IV, the text is absolutely identical to its companion book of the series "A First Course with PL/I", written by the same authors.

Henry Mullish