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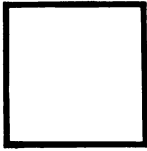
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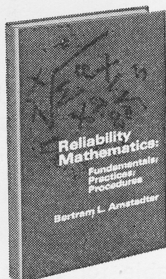
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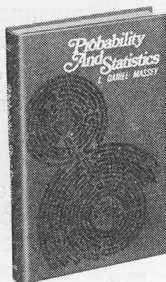
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Difficulties beyond our control have caused a delay in publishing the late 1970 and the early 1971 issues. It is expected that the journals will be issued on schedule before the end of 1971.

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