REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS

The numbers in brackets are assigned according to the indexing system printed in Volume 22, Number 101, January 1968, page 212.

26[2.10].—JAMES L. KINSEY, *Tables for Gaussian Quadrature of*

$$\int_0^\infty 2x^2 \exp(-x^2)f(x)\,dx,$$

6 pages of tables and 4 pages of explanatory text, reproduced on the microfiche card attached to this issue.

The abscissas and weights for n-point Gaussian quadrature of the integral in the title are tabulated to 20S for $n = 2(1)18$.

A second table gives, to the same precision, the coefficients α_n and β_n in the recurrence formula $P_{n+1}(x) = (x + \alpha_n)P_n(x) + \beta P_{n-1}(x)$ for the polynomials orthogonal on $(0, \infty)$ with respect to the Maxwellian weighting factor $2x^2 \exp(-x^2)$, as well as the normalization integral γ_n for $P_n(x)$. The corresponding error coefficients $d_n = \gamma_n/(2n)!$ are given to 6S in the first table.

Details of the underlying calculations on a CDC 3600 system at the University of Wisconsin are also furnished.

J. W. W.

27[3].—S. J. HAMMARLING, *Latent Roots and Latent Vectors*, The University of Toronto Press, 1970, xi + 172 pp., 26 cm. Price $13.50.

This book gives a descriptive account of methods for the numerical solution of eigenvalue problems. It begins with elementary properties of eigenvalues and their applications; this is not without some minor errors, notably stating a weak form of Gerschgorin's theorem (p. 9) and giving an oversimplified discussion of stability of the Crank-Nicolson method (Section 2.3). After this, the author proceeds to discuss in order the Danilevsky and Krylov methods, eigenvalues of tridiagonal matrices, the Givens and Householder methods, Lanczos' method, the power method, and finally QR.

Most of the discussion is dated, and can be found in either Wilkinson's or Householder's book; indeed the author appeals to one or the other continually for details and rigor. Besides this, there are some glaring omissions: the Givens and Householder methods are only described for symmetric matrices and, in fact, the Hessenberg form for nonsymmetric matrices is barely mentioned. Also, there is very little on inverse iteration for eigenvectors, and the author does not attach enough importance to the QR method (referred to as the method of Francis); only a cursory description is given and its use for symmetric tridiagonal matrices is not even mentioned.

Moreover, much is made of Danilevsky's method requiring only $O(n^3)$ operations to find the characteristic polynomial. However, the possibility of extreme loss of accuracy in the roots due to ill-conditioned polynomial coefficients, even when the eigenvalues are well-conditioned, is only hinted at. Indeed, the author takes the whole matter of "stability" or "instability" of a method with regard to numerical computation much too lightly. One gets the impression that the author's primary experience and concern is with methods for hand computation, hardly appropriate in this day and age.

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28[3, 4, 5].—A. S. HOUSEHOLDER, *KWIC Index for Matrices in Numerical Analysis*, Volume I: Primary Authors A–J, viii + 124 pp., Volume II: Primary Authors K–Z, vii + 151 pp., 1969, Oak Ridge National Laboratory, Oak Ridge, Tennessee, 28 cm. Available from National Technical Information Service, U. S. Department of Commerce, Springfield, Virginia 22151. Price: Printed copy $3.00, Microfiche $0.65, each volume.

Here are listings of papers and books which Professor Householder has compiled during the last ten years. Subjects included are numerical linear algebra, theory of real and complex matrices, difference schemes for differential equations. For the most part, the subjects of infinite matrices, Banach spaces, Hilbert spaces, matrices over arbitrary fields, combinatorial and functional analysis are not represented.

The 2600 items are listed alphabetically by author and also in a KWIC (Key Word in Context) Index. The authors are also listed separately.

All people who work in the field of matrix computations should be grateful to Professor Householder for making available to us this valuable information retrieved from the passing flood of scientific publications.

A third volume will contain more recent titles and also foreign titles which have not yet been translated.

B. N. P.

29[3, 4, 5, 8, 13.35].—R. V. GAMKRELIDZE, EDITOR, *Probability Theory, Mathematical Statistics, and Theoretical Cybernetics*, translated from Russian, Plenum Press, New York, 1969, vii + 112 pp., 24 cm. Price $15.00.

This book is a peculiar combination including, as it does, two papers entitled: "Markov Processes and Differential Equations" by M. I. Freidlin and "Discrete Problems in Mathematical Programming" by A. A. Korbut and Yu. Yu. Finkel'shtein. As such, two subject matters, entirely and fundamentally disparate, are presented, and the likelihood of finding readers, let alone reviewers, interested in the contents or competent to judge the merits of both, is nil.

This reviewer's competence extends only to the second paper. The first is devoted largely to a survey of the Russian literature (viz., on p. 2, "A great deal of work represented in the survey comes from papers by Soviet authors. Indeed, this reflects the true state of affairs, that our mathematicians stand at the forefront in the application of probabilistic methods to differential equations."), and may or may not be a complete guide to the Soviet contributions to the subject. The extent to which it does summarize Russian work measures its usefulness, since it is full of "... it turns out that ..." and "... it has been proved that ...".

The second paper, however, is of no particular use. It is a survey of integer programming which makes passing reference to some Russian work but emphasizes the Western contributions to the subject. This is unfortunate, for one learns almost nothing about what the Russians are doing in this field. The inference is that they are doing very little, indeed. On the other hand, what remains of the survey seems to be—by and large—gleaned from survey papers published in the West (notably the papers of Dantzig (*Econometrica*, 1960), Balinski (*Management Science*, 1965), and Beale (*Operational Research Quarterly*, 1965)).

The mystery remains: why does the book exist in English?

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30[7].—HENRY E. FETTIS & JAMES C. CASLIN, *A Table of the Complete Elliptic Integral of the First Kind for Complex Values of the Modulus*: III. *Auxiliary Tables*, Aerospace Research Laboratories, Office of Aerospace Research, United States Air Force, Wright-Patterson Air Force Base, Ohio, 1970, iv + 162 pp., 27 cm. Copies may be obtained from the Defense Documentation Center, Cameron Station, Alexandria, Virginia 22314.

For the review of Parts I and II, see *Math Comp*., v. 24, 1970, pp. 993–994, RMT 76.

Let

$$K(k) = K(R, \theta) = \int_0^{\pi/2} (1 - k^2 \sin^2 \lambda)^{-1/2} \, d\lambda, \qquad k = Re^{i\theta},$$

$$K'(k) = K(k'), \qquad k' = (1 - k^2)^{1/2} = \rho e^{-i\varphi}.$$

This report gives tables of the auxiliary functions

$$K(R, \theta) - F(R, \theta) = \left\{1 + \frac{2}{\pi} K'(R, \theta)\left[\ln \frac{4}{\rho} - 1 + i\varphi\right]\right\}, \qquad K'(R, \theta),$$

and

$$K'(R, \theta) - F'(R, \theta) = \left\{1 + \frac{2}{\pi} K(R, \theta)\left[\ln \frac{4}{R} - 1 - i\theta\right]\right\}, \qquad K(R, \theta),$$

for

$$R = 0.700(0.001)1.0, \quad \theta = 1°(1°)10°, \quad 10\ \mathrm{D}$$

and

$$R = 0.01(0.01)0.35, \quad \theta = 1°(1°)90°, \quad 10\ \mathrm{D}$$

respectively. The functions are interpolatable in the regions tabulated, and second central differences are provided.

Y. L. L.

31[9].—Alan Forbes & Mohan Lal, *Tables of Solutions of the Diophantine Equation* $x^2 + y^2 + z^2 = k^2$, Memorial University of Newfoundland, St. John's, Newfoundland, Canada, July 1969, x + 200 pp.

Table 2 lists all solutions $0 < x \leqq y \leqq z$ for all $k = 3(2)701$. Table 1 lists the number of such solutions, for each k, and the number of primitive solutions. These tables are an extension of an earlier table [1] which went to $k = 381$. (See the earlier review for more detail.)

The introduction here reports a few errors in the earlier table [1].

In the earlier review I noted that (empirically) if k is a prime p, written as $8n \pm 1$ or $8n \pm 5$, then there are exactly n solutions here. Here is a proof: By Gauss, (see *History of the Theory of Numbers* by L. E. Dickson, Vol. 2, Chapter VII, Item 20) the number of proper (that is, primitive) solutions of $m \equiv 1 \pmod 8$ as

$$m = x^2 + y^2 + z^2,$$

counting all possible permutations and changes of sign, and allowing x, y, or z to be 0, is

$$3 \cdot 2^{\mu+2} H,$$

where m is divisible by μ primes, and H is the number of properly primitive classes of binary quadratic forms of determinant $-m$ that are in the principal genus. For $m = p^2$, this becomes

$$6(p - (-1/p)) \tag{1}$$

proper solutions.

Each solution

$$p^2 = 0^2 + x^2 + y^2 \tag{2}$$

is counted 24 times by Gauss, but is omitted here. Each solution

$$p^2 = x^2 + x^2 + y^2 \tag{3}$$

is counted 24 times by Gauss and once here. Each solution

$$p^2 = x^2 + y^2 + z^2$$

is counted 48 times by Gauss and once here. Now examine

$$p = 8n \pm 1 \quad \text{and} \quad p = 8n \pm 5$$

separately, and allowing for the value of $(-1/p)$ in (1), and whether representations (2) and (3) do or do not exist, one finds that the $6(p - (-1/p))$ counts of Gauss become a count of n here in all four cases. Neat.

D. S.

1. Mohan Lal & James Dawe, *Tables of Solutions of the Diophantine Equation* $x^2 + y^2 + z^2 = k^2$, Memorial University of Newfoundland, St. John's, Newfoundland, Canada, February 1967. (See *Math. Comp.*, v. 22, 1968, p. 235, RMT **23**.)

32[9].—Herbert Hauptman, Emanuel Vegh & Janet Fisher, *Table of All Primitive Roots for Primes less than* 5000, Naval Research Laboratory Report 7070, April 1970, Washington, D. C., v + 587 pp. May be purchased from U. S. Government Printing Office, Washington, D. C. Price $4.25 (paperbound).

For each of the 668 odd primes $p < 5000$ there is listed the factorization of $p - 1$, the values of $\varphi(p - 1)$ and $\varphi(p - 1)/(p - 1)$, and each of the $\varphi(p - 1)$ positive primitive roots less than p.

There is a very brief introduction giving definitions, methods, references, and, primarily, conjectures concerning the distribution of the primitive roots. Thus: it is known that infinitely many primes have two consecutive primitive roots, but it has not been proven that all sufficiently large primes do. The authors suggest that one use of this volume is to study empirically such distribution questions.

The function $\varphi(p - 1)/(p - 1)$ is the density of the primitive roots in these modular systems, and I note that the lowest value attained here is 0.2078 for the two primes

$$2311 = 1 + 2\cdot 3\cdot 5\cdot 7\cdot 11 \quad \text{and} \quad 4621 = 1 + 4\cdot 3\cdot 5\cdot 7\cdot 11.$$

Queries. What is the distribution of these densities? Is there a positive lower bound?

The table was computed in 14 minutes. The photographic reproduction here is not perfect; a number of pages here and there are rather blurred.

For related reviews in this journal, see Osborn, RMT **18**, v. 16, 1962, p. 252, and Western & Miller, RMT **51**, v. 23, 1969, pp. 683–685.

D. S.

33[10].—Robert Riley, *FORTRAN Program for the Computation of the First Integral Homology Group of* 3-*Dimensional Manifolds, Considered as Branched Covering Spaces of the* 3-*Sphere*, ms. of about 4 typewritten pages and about 12 computer sheets (reduced), deposited in the UMT file.

This is the homology routine described in Section 5 of [1]. The typewritten pages contain instructions for the use of the routine, and suggestions on how to modify the program for special purposes. In normal use, nothing will have to be modified except the specification of the array sizes, and the once-for-all setting of the largest permitted integer in the Fortran of the machine on which the program will be run.

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1. W. Burnside, *Theory of Groups of Finite Order*, Cambridge Univ. Press, Cambridge, 1897.

34[10].—HILLEL KUMIN, *Some Enumeration Tables for Rooted Trees by Height and Diameter*, University of Oklahoma, Norman, Oklahoma, undated ms. of 21 typewritten sheets deposited in the UMT file.

Let r_{ph} be the number of rooted trees with p points and height h, and let t_{pd} be the number of trees with p points and diameter d. Then these manuscript tables give r_{ph} and t_{pd} for $p = 1(1)35$, $h = 1(1)34$, and $d = 2(1)34$, respectively.

All the calculations were performed on a CDC 1604 system, and the results confirm and extend the similar tables of Riordan [1], for which the upper limit of p is 20 and that for h and d is 19.

The introductory text gives definitions of the relevant graph-theoretic terms and the combinatorial formulas used in the calculations, as well as a list of seven references.

J. W. W.

1. J. RIORDAN, "The enumeration of trees by height and diameter," *IBM J. Res. Develop.*, v. 4, 1960, pp. 473–478.

35[12].—C. T. FIKE, *PL/I for Scientific Programmers*, Prentice-Hall, Inc., Englewood Cliffs, N. J., 1970, xiii + 241 pp., 24 cm. Price $7.50.

Mr. Fike has made a worthy contribution to the literature with his hard-cover book *PL/I for Scientific Programmers*. In it he clearly states the important differences between PL/I and Fortran in an attempt to help experienced Fortran programmers to adapt to the language as quickly as possible.

The treatment of the various facets of PL/I is excellently handled and while it is by no means a comprehensive treatment of the vast PL/I repertoire, it does, nevertheless, provide the interested reader with the necessary tools in a quick and palatable manner.

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36[12].—J. M. FOSTER, *Automatic Syntactic Analysis*, American Elsevier Publishing Co., Inc., New York, 1970, 65 pp., 24 cm. Price $4.00.

This is a rather simple introductory book to the field of syntactic analysis. It explains what a context-free (BNF) grammar is, and then goes on to explain some of the principal parsing methods. None of the material is covered in great depth, but the explanations of what is covered are generally clear. There are unfortunately some misprints that may lead to confusion, e.g., on the top line of p. 38, the = should be $\leqq$. Though such difficulties would not deter the experienced reader, they could well cause problems for the novice. And an experienced reader would do far better to read the article by Feldman and Gries in *Comm. ACM*, February 1968.

The first three chapters explain why parsing is necessary, its role in the compilation process, the formal definition of a context-free grammar, and a generalized description of the parsing process that is independent of the particular method chosen. The fourth chapter discusses the unmodified top-to-bottom and bottom-to-top methods. The fifth chapter considers how parsing can be speeded up by working with a restricted class of grammars. Precedence methods are treated in this chapter. The sixth chapter examines how a grammar may be transformed to a restricted form, and also explains how associated semantic actions must be manipulated. There are also short appendices on elementary list processing and on a particular top-to-bottom algorithm given in list processing notation.

This is not a particularly substantive book, but it may prove useful to one desiring a rapid tutorial overview of its subject.

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