REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS

The numbers in brackets are assigned according to the indexing system printed in Volume 22, Number 101, January 1968, page 212.

17[2.05, 2.20, 2.35, 3, 4, 5, 6].—L. Collatz, G. Meinardus, H. Unger & H. Werner, Editors, *Iterationsverfahren, Numerische Mathematik, Approximationtheorie*, Birkhäuser Verlag, Basel, 1970, 257 pp., 25 cm. Price sFr. 36.—.

This book contains the proceedings of three conferences held at the Mathematics Research Institute, Oberwolfach, Germany. The three conferences were:

I. Nonlinear Problems in Numerical Analysis, November 17–23, 1968, organized by L. Collatz and H. Werner,

II. Numerical Methods in Approximation Theory, June 8–14, 1969, organized by L. Collatz and G. Meinardus, and

III. Iterative Methods in Numerical Analysis, November 16–22, 1969, organized by L. Collatz and H. Unger.

A list of the titles of the papers (in translation) and short reviews of each follows.

I. 1. Brosowski, B., Hofmann, K.-H., Schafer, E., and Weber, H.: *Continuity of metric projections.* Some conditions on a normed linear space X are given to assure that a (set-valued) metric projection P_V onto a subspace V is continuous in an appropriate sense. Also, some conditions are found on X, V and $P_V(f)$ which guarantee that the classical Kolmogorov criterion characterizing best approximations holds for some best approximation.

2. Brosowski, B., Hofmann, K.-H., Schafer, E., and Weber, H.: *Metric projections on linear subspaces of* $C_0[Q, H]$. Continuity of (set-valued) metric projections of spaces of continuous H-valued functions on a locally compact Hausdorff space is considered (where H is a real pre-Hilbert space). Sets of points in Q where best approximations coincide are involved.

3. Frehse, J.: *On the convergence of difference and other approximation methods for nonlinear variational problems.* Some methods for approximately minimizing $\int_a^b f(x, u, u')\, dx = 0$ over an appropriate space are considered. In particular, convergence is established for some Ritz, difference, and interval perturbation methods. Variational problems in multidimensional spaces are considered.

4. Laasonen, P.: *On a method for solving nonlinear systems of equations.* In order to overcome the drawback that a matrix of derivatives is needed in applying Newton's method to a system of nonlinear equations, a scheme is devised which requires no derivatives. It is shown to be quadratically convergent under appropriate assumptions.

5. Lancaster, P.: *Spectral properties of operator functions.* Results are obtained for the eigenvalues of operators of the form $D(\lambda) = A_0\lambda^l + \cdots + A_{l-1}\lambda + A_l$, where λ is a complex number and $A_0, A_1, \cdots, A_l$ are bounded linear operators mapping a Banach space into itself (and A_0 has a bounded inverse.)

6. Mayer, H.: *Estimates for the defect vector in the solution of linear systems with inaccuracies in the data and their numerical evaluation.* Methods for estimating δ_x in

$(A + R)(x + \delta x) = b + f$ with $|r_{ij}| \leqq \epsilon$, $|f_i| < \delta$ in terms of ϵ, δ, A (or A^{-1}) and b are developed. Numerical methods avoiding the use of A^{-1} are presented.

7. Nitsche, J.: *Convergence of the Ritz-Galerkin method for nonlinear operator equations.* Error bounds for $x - x_n$ are obtained where $Ax = f$ (A nonlinear on a Hilbert space H) and x_n is obtained by the Ritz-Galerkin method. With appropriate assumptions, it is shown that if x_n is chosen from a subspace H_n then it is almost best in the sense that $||x - x|| \leqq \phi(||f||) \inf ||x - \xi||$.

8. Nixdorff, K.: *Nonlinear computational methods in course finding.* Two methods for analyzing sounding measurements using several waves of equal frequencies are discussed. The methods are appropriate for use with digital and analog computers.

9. Nixdorff, K.: *Remarks on the application of the harmonic balance.* This is a short description of the intent of the lectures.

10. Reimer, M.: *Semidefinite Peano kernels of stable difference forms.* The kernel K in difference forms

$$Ly = \sum_{\nu=0}^{k} \sum_{\mu=0}^{m} h^{\mu} a_{\nu}^{(\mu)} y_{\nu}^{(\mu)} = h^{p+1} \int_0^k y^{(p+1)}(x + h + t) K(t)\, dt$$

is investigated.

11. Wetterling, W.: *On minimal conditions and Newton iteration in nonlinear optimization.* Conditions are obtained assuring the existence of a local minimum for a constrained optimization problem: minimize $F(x)$ over $x \in R^n$ such that $f_j(x) \leqq 0$, $j = 1, \cdots, m$. F and f_j are not assumed to possess convexity properties. A result on the applicability of the Newton method is included.

12. Zeller, K.: *Newton-Chebyshev approximation.* The problem of determining a rational starting function $f_0(x)$ in Heron's method for approximating the function $\sqrt{x}$, so that after n steps $||f_n(x)/\sqrt{x} - 1||$ is minimized, is discussed.

II. 1. Anselone, P. M.: *Abstract Riemann integrals, monotone approximations, and generalizations of Korovkin's theorem.* A method of extending the Riemann integral from the continuous functions to a larger class is generalized to a procedure for extending other positive linear functionals or operators from (partially) ordered linear spaces to larger ones. Results on sets of uniform convergence for such functionals and operators are obtained as well as a Korovkin-type theorem for arbitrary partially ordered Banach spaces.

2. Cheney, E. W., and Price, K.: *Minimal interpolating projections.* Let $X = C(T)$ where T is a compact Hausdorff space, and let Y be an n-dimensional subspace. Given $t_1, \cdots, t_m \in T$ and $y_1, \cdots, y_n \in Y$, $Px = \sum x(t_i)y_i$ is called an interpolating projection of X onto Y. A generalized Kolmogorov-criterion for characterizing the minimal interpolating projections from X onto Y (i.e. those with minimal $||P||$) is obtained. As a corollary, it is shown that, if Y is a Haar subspace and P is minimal, then $||P|| = 1$ or $\sum |y_i|$ has $n + 1$ critical points.

3. Collatz, L.: *Approximation theory and applications.* Some examples of practical problems involving differential and integral equations are presented to illustrate how many of the standard questions in theoretical approximation theory are too special for applications, while some of the very general investigations seem to have no applicability. For example, the usual nonlinear exponential approximation problem occurs relatively seldom in practice, while a reasonable approximation process for

the boundary-value problem $\Delta u = e^u$ in a square, with $u = c$ on the boundary, leads to a 2-dimensional exponential approximation problem.

4. GILBERT, R. P.: *Integral operator methods for approximating solutions of Dirichlet problems.* Approximate methods for solving the Dirichlet problem associated with $\Delta u(x) - P(r^2)u(x) = 0$ for $x \in D$, an appropriate domain in E^n, are considered. (Here $r = ||x||$ and P is assumed to be nonnegative and in $C'[0, a]$, $a = \sup_{x \in D} ||x||$.) Several of the classical methods for solving Laplace's equation (the case when $P \equiv 0$) such as the methods of Fredholm integral equations, balayage, Rayleigh-Ritz, and images are extended, leading to natural procedures in approximation theory for estimating solutions. An appendix by K. Atkinson discusses some numerical methods.

5. HAUSSMANN, W.: *Multidimensional Hermite interpolation.* A representation for two-dimensional (0, 1)-Hermite polynomial interpolation is obtained which is used to obtain convergence results. It is shown that for certain grids on $E = [-1, 1] \times [-1, 1]$, called ρ-normal, the Hermite polynomial converges uniformly to any $f \in C(E)$. The rate of convergence of the Fejér operators (Hermite interpolation at the zeros of the Chebyshev polynomial) is determined for $f \in \text{Lip } \alpha$.

6. LOCHER, F., and ZELLER, K.: *Approximation on gridpoints.* Many numerical approximation schemes involve fitting a function f on a domain A by functions in a class V by choosing $p \in V$ such that p and f agree on some grid of points in A. Some of the relevant questions, such as form of the grid, representation of the function, *a priori* error estimates, *a posteriori* error estimates and exchange methods for improving the approximation, are discussed.

7. LUPAS, A.: *On the approximation by linear positive operators.* The Szasz-Mirakyan operator is a positive linear operator defined on a class of functions W with domain $[0, \infty)$. It is shown that these operators preserve the convexity (or nonconvexity) of any $f \in W$, and that if f is a polynomial of degree m, then so is sf. Moreover, a kind of r-order variation diminishing property is defined, and the general Baskakov operators are shown to possess it.

8. SMITH, L. B.: *Using interactive graphical computer systems on approximation problems.* A review of on-line graphical computer systems for mathematical problems and a discussion of their advantages for approximation theory is given. A least squares system is discussed in some detail, and some areas where on-line systems would be useful are suggested.

III. 1. DÖRING, B.: *A theorem on a class of iterative methods considered by Grebenjuk.* A simple general principle for deriving several classes of higher order methods for the solution of nonlinear operator equations in Banach spaces is presented. A class of methods studied by Grebenjuk using majorants is examined in a new way to produce existence, uniqueness and convergence theorems under simpler conditions and with much improved error estimates.

2. GEKELER, E.: *Relaxation methods for a class of nonlinear systems.* Two theorems are established on the convergence of relaxation methods applied to the nonlinear systems arising in the numerical solution of Hammerstein integral equations. The theorems apply, for example, to the systems generated by the method of Theodorsen and Wittich applied to a certain conformal mapping problem.

3. NIETHAMMER, W.: *Acceleration of the convergence of one-step iterative methods by summation.* A k-step iterative method for solving a system of linear equations is

obtained from a given one-step method by averaging the last k-iterates. Results on the domains of convergence and acceleration are obtained, both of which may be much larger than the domain of convergence of the original method.

4. WACKER, H. J.: *A method for nonlinear boundary value problems.* To solve the operator equation $T(y) = 0$, the problem is embedded in a family $T(s, y) = 0$ with $0 \leqq s \leqq 1$, and such that $T(0, y) = 0$ is easily solvable, and such that $T(1, y) = T(y)$. For a sequence of s's, the solution for s_i can be used as a starting value for the computation at s_{i+1}.

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18[2.10].—V. I. KRYLOV & A. A. PAL'TSEV, *Tables for Numerical Integration of Functions with Logarithmic and Power Singularities*, translated from Russian, Israel Program for Scientific Translations, Jerusalem, 1971, iv + 172 pp., 25 cm. Price \$10.—.

The original edition of these tables was published in 1967 by the "Nauka i Tekhnika" Publishing House in Minsk.

Herein are tabulated the elements of four Gaussian quadrature formulas involving the respective weight functions $x^\alpha \ln(e/x)$, $x^\beta \ln(e/x) \ln[e/(1 - x)]$, $\ln(1/x)$, and $x^\beta e^{-x} \ln(1 + x^{-1})$. The range of integration for the first three is the interval (0, 1), while that for the fourth is $(0, \infty)$. The tabular points (nodes) and corresponding weight coefficients are uniformly presented to 15S in floating-point format, and the number of points extends from 1 to 10, inclusive. In Table 1 the exponent α assumes the values $-0.9(0.01)0(0.1)5$, while in Tables 2 and 4 the exponent β assumes the values $0(1)5$.

Only the material in Table 3 appears to have been published elsewhere. An 8S table was given by Anderson [1] and an extensive 30S table appears in the book of Stroud & Secrest [2], which confirms the accuracy of Table 3.

Two examples of the application of Table 1 are presented, and interpolation with respect to α in that table is discussed in detail.

A bibliography of six items contains a reference to the paper of Anderson but not to the work of Stroud & Secrest, which presumably was not available to the authors.

J. W. W.

1. D. G. ANDERSON, "Gaussian quadrature formulae for $\int_0^1 - \ln x f(x)\, dx$," *Math. Comp.*, v. 19, 1965, pp. 477–481.

2. A. H. STROUD & DON SECREST, *Gaussian Quadrature Formulas*, Prentice-Hall, Englewood Cliffs, N.J., 1966. (See *Math. Comp.*, v. 21, 1967, pp. 125–126, RMT **14**.)

19[2.20].—B. DEJON & P. HENRICI, Editors, *Constructive Aspects of the Fundamental Theorem of Algebra*, John Wiley & Sons, New York, 1969, vii + 337 pp., 23 cm. Price \$9.95.

These papers are the published proceedings of a symposium held on June 5–7, 1967, at the IBM Research Laboratory in Rüschlikon, Switzerland. The spectrum of ideas treated ranges from the presentation of an ALGOL algorithm for finding a real zero of a function $f(x)$ (that changes sign), to the specification of a mathematical algorithm for finding (to any given accuracy) all of the zeros of a polynomial with complex coefficients, to a discussion of iterative methods for numerically solving certain polynomial matrix equations, to a treatment of a logical constructive proof of the fundamental theorem for polynomials with algebraic coefficients. The state of the theory and the art in this fundamental field of logic and numerical analysis are well presented in the clearly written papers listed below:

DEJON, B., and NICKEL, K.: A Never Failing, Fast Convergent Root-Finding Algorithm; DEKKER, T. J.: Finding a Zero by Means of Successive Linear Interpolation; FORSYTHE, G. E.: Remarks on the Paper by Dekker; FORSYTHE, G. E.: What is a Satisfactory Quadratic Equation Solver?; FOX, L.: Mathematical and Physical Polynomials; GOODSTEIN, R. L.: A Constructive Form of the Second Gauss Proof of the Fundamental Theorem of Algebra; HENRICI, P., and GARGANTINI, L.: Uniformly Convergent Algorithms for the Simultaneous Approximation of all Zeros of a Polynomial; HERMES, H.: On the Notion of Constructivity; HOUSEHOLDER, A. S., and STEWART, G. W., III: Bigradients, Hankel Determinants, and the Padé Table; JENKINS, M.A., and TRAUB, J. E.: An Algorithm for an Automatic General Polynomial Solver; KUPKA, I.: Die numerische Bestimmung mehrfacher und nahe benachbarter Polynomnullstellen nach einem verbesserten Bernoulli-Verfahren; LEHMER, D. H.: Search Procedures for Polynomial Equation Solving; OSTROWSKI, A. M.: A Method for Automatic Solution of Algebraic Equations; PAVEL-PARVU, M., and KORGANOFF, A.: Iteration Functions for Solving Polynomial Matrix Equations; RUTISHAUSER, H.: Zur Problematik der Nullstellenbestimmung bei Polynomen; SCHRODER, J.: Factorization of Polynomials by Generalized Newton Procedures; SPECKER, E.: The Fundamental Theorem of Algebra in Recursive Analysis.

E. I.

20[3].—HAROLD W. KUHN, Editor, *Proceedings of the Princeton Symposium on Mathematical Programming*, Princeton Univ. Press, Princeton, New Jersey, 1970, vi + 620 pp., 24 cm. Price $12.50 (paperbound).

The field of mathematical programming has existed for less than 25 years. In that time, it has experienced phenomenal growth. The Princeton Symposium on Mathematical Programming, held at Princeton University on August 14–18, 1967, is one of a series of symposia held every three years since 1949. My major comment on these published Proceedings is that their value has been greatly diminished by the more than three years that it has taken to publish them. In fact, these Proceedings did not appear until after the 1970 Symposium had been held at the Hague.

From the more than 90 papers and addresses presented at the conference, 33 papers, in their entirety, and 48 abstracts are included in this volume. Of particular note are two bibliographies, one of 128 references on large-scale systems in the paper by Dantzig and one of 232 references on integer programming at the end of the first two papers by Balinski. These two papers form a very well written comprehensive survey of integer programming. The authors and titles of the 33 complete papers included in these Proceedings are listed below:

Part I. Large Scale Systems

J. Abadie and M. Sakarovitch: Two Methods of Decomposition for Linear Programming

E. M. L. Beale: Matrix Generators and Output Analyzers

R. H. Cobb and J. Cord: Decomposition Approaches for Solving Linked Programs

G. B. Dantzig: Large Scale Systems and the Computer Revolution

Part II. Programming Under Uncertainty

M. Avriel and D. J. Wilde: Stochastic Geometric Programming

M. J. L. Kirby: The Current State of Chance-Constrained Programming

A. Prekopa: On Probabilistic Constrained Programming

D. W. Walkup and R. J. B. Wets: Stochastic Programs with Recourse: Special Forms

A. C. Williams: Nonlinear Activity Analysis and Duality

Part III. Integer Programming

E. Balas: Duality in Discrete Programming

M. L. Balinski: Integer Programming: Methods, Uses, Computation

M. L. Balinski: On Recent Developments in Integer Programming

M. L. Balinski: On Maximum Matching, Minimum Covering and Their Connections

P. Huard: Programmes Mathématiques Nonlinéaires à Variables Bivalentes

C. Zoutendijk: Enumeration Algorithms for the Pure and Mixed Integer Programming Problem

Part IV. Algorithms

A. Ben-Israel: On Newton's Method in Nonlinear Programming

H. D. Mills: Extending Newton's Method to Systems of Linear Inequalities

J. D. Roode: Interior Point Methods for the Solution of Mathematical Programming Problems

Part V. Applications

P. Bod: The Solution of a Fixed Charge Linear Programming Problem

A. Charnes and K. Kortanek: On Classes of Convex Preemptive Nuclei for N-Person Games

A. J. Hoffman and T. J. Rivlin: When is a Team "Mathematically" Eliminated?

D. McFadden: On the Existence of Optimal Development Plans

Part VI. Theory

O. L. Mangasarian: Optimality and Duality in Nonlinear Programming

E. L. Peterson and J. G. Ecker: Geometric Programming: Duality in Quadratic Programming and l_p-Approximation. I

R. T. Rockafellar: Conjugate Convex Functions in Nonlinear Programming

Part VII. Nonlinear Programming

A. R. Colville: A Comparative Study of Nonlinear Programming Codes

D. G.

21[3].—B. N. Pshenichnyi, *Necessary Conditions for an Extremum*, Marcel Dekker, Inc., New York, 1971, xviii + 230 pp., 24 cm. Price $11.50.

Dr. B. N. Pshenichnyi is one of the most prolific contributors to the literature on optimization. He has written highly regarded papers on optimality conditions, on optimization algorithms, on optimal control problems, on minimax problems and on games. The breadth of his research experience has contributed substantially to the well-balanced perspective and maturity of this beautifully executed monograph on optimality conditions. Dr. Pshenichnyi avoids cumbersome results. Because of this, his monograph does not include an exhaustive study of constraint qualifications, nor the most general discrete maximum principle, nor the Pontryagin maximum principle for the general case, nor the most general possible optimality conditions for constrained optimization problems. Instead, by masterfully introducing a few simplifying, but not particularly constraining, assumptions, Dr. Pshenichnyi manages to present in an elegant fashion most of the important optimality conditions without getting bogged down in the very messy analysis which is required to treat the most general case. The result is an excellent and most readable middle-level text. The quality of the translation, carried out by Dr. K. Makowski under the supervision of the translation editor, Dr. L. W. Neustadt, both of the University of Southern California, is impeccable, and they deserve a commendation.

As to the actual contents of the monograph, which consists of an introduction and five chapters, Dr. Pshenichnyi starts out in the introduction with a few basic concepts of functional analysis and with some properties of convex sets and of convex functionals. Chapter I continues with more advanced properties of convex functionals defined on Banach spaces, and, in particular, it presents their directional derivatives.

Chapter II derives optimality conditions for convex programming problems in Banach space, with and without differentiability assumptions. In particular, the Kuhn-Tucker conditions are obtained.

Chapter III is primarily concerned with formulas for the directional derivatives of functions of the form $\mu(x) = \max_{\alpha \in Z} \varphi(x, \alpha)$, with x and α elements of Banach spaces, which occur in minimax problems.

Chapter IV obtains a necessary condition of optimality for general mathematical programming problems in Banach space. This condition is quite similar to, though not as general as, the condition in [1]. It is then related to a very general optimality condition due to Dubovitskii and Milyutin [2].

Finally, Chapter V applies the general optimality condition obtained in Chapter IV to a number of specific problems to obtain specialized optimality conditions. These problems include the classical mathematical programming problem, mathematical programming problems with a continuum of constraints, minimax problems, Chebyshev approximation problems, linear optimal control problems with state space constraints, problems in convex inequalities, the moment problem and a discrete maximum principle.

The book concludes with an annotated bibliography.

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1. H. Halkin & L. W. Neustadt, "General necessary conditions for optimization problems," *Proc. Nat. Acad. Sci. U.S.A.*, v. 56, 1966, pp. 1066–1071.
2. A. Y. Dubovitskii & A. A. Milyutin, "Extremum problems with constraints," *Soviet Math. Dokl.*, v. 4, no. 2, 1963, pp. 452–455.

22[4].—Leon Lapidus & John H. Seinfeld, *Numerical Solution of Ordinary Differential Equations*, Academic Press, New York, 1971, xii + 299 pp., 24 cm. Price $16.50.

This book provides an excellent survey of the most important methods for the numerical solution of initial value problems for ordinary differential equations.

After an introductory chapter, a chapter is devoted to each of the following topics: Runge-Kutta and allied single-step methods; stability of multi-step and Runge-Kutta methods; predictor-corrector methods; extrapolation methods; methods for stiff equations. The treatment of each topic is up-to-date (the references run through 1969). For example, the hybrid methods due to Butcher (1965), Danchick (1968), and Kohfeld and Thompson (1967) are all described in detail.

Much of the recent work on numerical methods for ordinary differential equations has been evolutionary in the sense that existing methods have been mutated, usually by the introduction of additional parameters, so as to improve certain desirable characteristics such as stability. This evolutionary process is brought out very clearly in the present book where the different mutations are compared and the more successful ones are noted as worthy of further development.

The authors pay great attention to the question of which methods are best computationally. The numerical results in the literature are discussed. In addition, almost all the methods described were used to solve test problems and the results are summarized. On the basis of all this information, the authors recommend highly the fourth order Adams-Bashforth-Adams-Moulton predictor-corrector, the third order semi-implicit Runge-Kutta method of Rosenbrock, and a fifth order Runge-Kutta method of Butcher.

There are some errors and omissions. On page 2, it is stated that every system of m first-order differential equations is equivalent to one mth order equation, which is not true. In the discussion of Runge-Kutta methods, only scalar equations are treated in detail and the impression is given that the results can always be extended to systems, although Butcher has given an example where this is not the case. The coverage of the theoretical aspects of stability is a little too superficial, and the definition of A-stability is not quite right. There seems to be no mention in the text of the work of Butcher proving that there is no explicit n-step Runge-Kutta method of order n if $n \geqq 5$. Finally, there is no mention of the following topics: statistical estimates for round-off error; rigorous error bounds using interval arithmetic; methods using Chebyshev series; methods using splines; methods using Lie series; and methods for differential equations with right-hand sides which have singularities or discontinuities. However, these are minor complaints about an otherwise excellent book.

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23[5,6].—V. S. VLADIMIROV, *Equations of Mathematical Physics*, Marcel Dekker, Inc., New York, 1971, vi + 418 pp., 24 cm. Price $19.75.

This book is a translation into English of a textbook which first appeared in Russian in 1967 and which is being used at Moscow University. It contains a comprehensive treatment of the standard boundary value problems for second order partial differential equations. Its most distinguishing feature is its consistent use of distribution theory. The presentation is elegant, thorough and yet easily accessible. The author has succeeded in integrating the distribution theory into the analysis of the boundary value problems of mathematical physics in a natural and coherent way.

The book consists of six chapters. The first chapter introduces the necessary background material from analysis, including a brief presentation, partly without proofs, of the basic facts of Lebesgue integration and of operator theory in the Hilbert space L_2. It then describes the physical interpretation of the common second order partial differential equations and discusses their classification. The second chapter presents distribution theory, including Fourier transformation of tempered distributions. A number of specific concrete distributions which are needed in the remainder of the text are analyzed here in detail. The third chapter treats the concept of a fundamental solution in distribution terminology, with special application to the initial value problem for hyperbolic and parabolic equations. The fourth chapter develops the theory of integral equations, in particular, the Fredholm theorems and the Hilbert-Schmidt theory. The fifth chapter, which is the longest, deals with elliptic equations. Among the topics covered are eigenvalue problems and expansion theorems, the Sturm-Liouville problem and its reduction via Green's function to an integral equation, harmonic functions with the mean value property and the maximum principle, and special functions occurring in connection with special domains. The final chapter on mixed problems for hyperbolic and parabolic equations covers the method of separation of variables and uniqueness and stability results by means of energy and maximum norm a priori estimates.

Although over the last few years a number of treatments of partial differential equations have appeared, I have found it hard to find any particular book which is an ideal beginning graduate level text. Particularly in the subject under consideration, it is very hard to find the right balance between too much and too little, both in sophistication and in quantity of material. I think this book is what I have been waiting for. The translation is not perfect.

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24[7].—HENRY E. FETTIS & JAMES C. CASLIN, *Ten Place Tables of the Jacobian Elliptic Functions: Part* III, Report ARL 71-0081, Aerospace Research Laboratories, Air Force Systems Command, United States Air Force, Wright-Patterson Air Force Base, Ohio, May 1971, iv + 449 pp., 28 cm. Copies obtainable from the National Technical Information Service, Operations Division, Springfield, Virginia 22151. Price $3.00.

This report has been designed to supplement Part I [1] of these tables in the vicinity of $k^2 = 1$. Specifically, herein are tabulated 10D values of the Jacobian elliptic functions $am(u, k)$, $sn(u, k)$, $cn(u, k)$, and $dn(u, k)$, as well as the elliptic integral $E(am(u), k)$, for $k^2 = 0.950(0.001)0.999$ and $u = 0(0.01)K(k)$. Also, the headings of the tables include corresponding 10D values of $K(k)$ and $E(k)/K(k)$, where $K(k)$ and $E(k)$ conventionally represent the complete elliptic integrals of the first and second kinds, respectively.

As in the preparation of [1], the underlying calculations of these extensive tables were performed on an IBM 7094 system.

J. W. W.

1. HENRY E. FETTIS & JAMES C. CASLIN, *Ten Place Tables of the Jacobian Elliptic Functions, Part* 1, Report ARL 65-180, Aerospace Research Laboratories, Office of Aerospace Research, United States Air Force, Wright-Patterson Air Force Base, Ohio, September 1965. (See *Math. Comp.*, v. 21, 1967, pp. 264–265, RMT **25.**)

25[7].—SWARNALATA PRABHU, *Tables of the Incomplete Beta Function for Small Values of the Parameters*, Indian Institute of Science, Bangalore, India, v + 250 pp., 27 cm. (paperbound). Copy deposited in the UMT file.

These tables consist of 6S values of the incomplete Beta function $B_x(p, q)$ for $x = 0.01(0.01)0.50$ and $p, q = 0.02(0.02)0.50$, together with 6 or 7S values of $B(p, q)$ for the same range of p and q.

The underlying calculations were performed to 8S on a National Elliott 803 electronic computer, and the results corresponding to $p = 0.5$ and $q = 0.5$ were successfully compared with the corresponding entries in the well-known 7S tables of Pearson [1], for which these values of p and q constitute the lower limit of the parameters. Accordingly, the present tables have been designed to supplement Pearson's tables in this respect.

The author points out in the introduction that values of $B_x(p, q)$ for $x = 0.50(0.01)0.99$ and the tabular values of p and q can be readily derived from the tables by means of the known relation $B_x(p, q) = B(p, q) - B_{1-x}(q, p)$.

Also explained in the introduction are the methods followed in computing the tables and the procedure for interpolating therein by means of an eight-point Lagrange formula.

These tables, prepared in connection with a study of shock-wave propagation and radiation gas dynamics, have many other applications in physics and engineering and constitute a unique contribution to the tabular literature for the incomplete Beta function.

J. W. W.

1. KARL PEARSON, *Tables of the Incomplete Beta-Function*, Cambridge Univ. Press, 1934; reissued in 1968 in a revision edited by E. S. Pearson & N. L. Johnson.

26[7, 13.05].—I. M. KUNTSEVICH, N. M. OLEKHNOVICH & A. U. SHELEG, *Tables of Trigonometric Functions for the Numerical Computation of Electron Density in Crystals*, translated from Russian, Israel Program for Scientific Translations, Jerusalem, 1971, 218 pp., 25 cm. Price $12.—.

These unique tables (originally published in Minsk in 1967) consist of 4D tables of the functions $a_{h,k,l}(x, y, z)$ and $b_{h,k,l}(x, y, z)$, which are defined as the trigonometric sums $\sum \cos 2\pi hx \cos 2\pi ky \cos 2\pi lz$ and $\sum \sin 2\pi hx \sin 2\pi ky \sin 2\pi lz$ extended over all the permutations of h, k, l for each set of values of x, y, and z. The coordinates x, y, z are herein expressed as integer multiples of $1/60$, and are limited to the ranges $0 \leqq x \leqq 15/60$, $0 \leqq y \leqq x$, $0 \leqq z \leqq y$, while h, k, l are integers in 27 triads ranging from (0, 0, 0) to (8, 0, 0). A total of 816 independent points are tabulated, so arranged that the trigonometric sums for each of four points appear on each page. Values of $a_{h,k,l}(x, y, z)$ and $b_{h,k,l}(x, y, z)$ for all other points of the cubical lattice can then be derived by means of 38 formulas listed in the introduction. Use of the tables is facilitated by a detailed index of nearly four pages.

As stated in the introduction, these tables have been designed to simplify the computation of the electron density distribution in crystals on the basis of X-ray diffraction data. It is also stated that they can be used to compute the distribution of electrons with uncompensated spins and the potential distribution in crystals from appropriate neutron and electron diffraction data.

A foreword presents further details of the scientific background of these tables (including a bibliography of 15 items), and reveals that they were computed on the URAL and MINSK-2 computers and that in the past decade they have been in daily use in the X-Ray and Neutron Diffraction Laboratories of the Institute of Solid-State and Semiconductor Physics of the Belorussian Academy of Sciences.

J. W. W.

27[9].—SOL WEINTRAUB, *Distribution of Primes between* 10^{14} *and* $10^{14} + 10^8$, 6 pages of computer output deposited in the UMT file together with a text of 3 pages, 1971.

The number of primes between 10^{14} and $10^{14} + 10^8$ is 3102679. (Riemann's formula gives the estimate 3102104.)

For each $k = 2(2)600$, these tables list four quantities:

COUNTS		RATIOS to $k = 2$	
GAPS	PAIRS	ACTUAL	THEORY

GAPS are the number of p_i in this interval such that $p_{i+1} - p_i = k$. PAIRS are the number of p here such that $p + k$ is prime (whether or not it is the next prime). ACTUAL is the ratio

$$\frac{\text{PAIRS } (k)}{\text{PAIRS } (2)}$$

and THEORY is that ratio according to the Hardy-Littlewood Conjecture.

Here are several observations. The most popular gap is for $k = 6$ (237524 specimens). The average gap is, of course, $\ln 10^{14} = 32+$. The number of twins ($k = 2$) is 127084. The first missing gap is $k = 332$. The largest gap is 414 and follows the prime $10^{14} + 13214473$. The most popular pairs are, obviously, for $k = 210$ and 420, namely, 408552 and 406950 specimens, respectively. "Actual" and "Theory" agree closely.

The brief text also mentions triples and quadruples.

See the following references for related tables.

D. S.

1. D. H. LEHMER, UMT 3, *MTAC*, v. 13, 1959, pp. 56–57.
2. F. GRUENBERGER & G. ARMERDING, UMT 73, *Math. Comp.*, v. 19, 1965, pp. 503–505.
3. M. F. JONES, M. LAI & W. J. BLUNDON, UMT 20, *Math. Comp.*, v. 21, 1967, p. 262.