

REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS

The numbers in brackets are assigned according to the indexing system printed in Volume 22, Number 101, January 1968, page 212.

42[2.10].—W. ROBERT BOLAND, *Coefficients for Product-Type Quadrature Formulas*, Department of Mathematics, Clemson University, Clemson, South Carolina. Ms. of 16 pp. (undated) deposited in the UMT file.

Product-type quadrature formulas have been introduced by the author [1] in collaboration with C. S. Duris for the numerical approximation of definite integrals of the form $\int_a^b f(x)g(x) dx$. Such a formula is said to be "product-interpolatory" if it is derived by integrating $p_n(x) \cdot q_m(x)$, where $p_n(x)$ is the polynomial interpolating $f(x)$ at the nodes x_i , $i = 0(1)n$, and $q_m(x)$ similarly is the polynomial interpolating $g(x)$ at the nodes y_j , $j = 0(1)m$. In this case the author proves in [1] that the coefficients in the corresponding quadrature formula

$$\int_a^b f(x)g(x) dx \approx \sum_{i=0}^n \sum_{j=0}^m a_{ij} f(x_i)g(y_j)$$

are given by $a_{ij} = \int_a^b l_i(x)L_j(x) dx$, where $l_i(x)$ and $L_j(x)$ are, respectively, the i th and j th Lagrange interpolation coefficients for the nodes x_i and y_j .

In the present tables the range of integration is taken to be $(-1, 1)$, and the parameters n and m are restricted to the ranges $n = 1(1)5$ and $m = 1(1)5$.

Table 1 consists of the exact (rational) values of the coefficients for the corresponding product Newton-Cotes formulas; Table 2 consists of 16S values (in floating-point form) of the coefficients for the corresponding product Gauss formulas; and Table 3 gives 16S values of the coefficients for the corresponding product Gauss-Newton-Cotes formulas.

The tabular values were calculated on an IBM 360/75 system, using double-precision arithmetic, and the author believes they are correct to at least 15S. As partial confirmation, a spot check by this reviewer revealed no errors exceeding 5 units in the least significant digit. However, in Table 1 a serious printing error was discovered; namely, the least common denominator of the coefficients corresponding to $n = m = 3$ should read 840 instead of 1.

For an appropriate error analysis of such quadrature formulas, the user of these tables should consult [1], where he will also find some comments on their applicability, in particular to the study of the Fredholm integral equation of the second kind.

J. W. W.

1. W. R. BOLAND & C. S. DURIS, "Product type quadrature formulas," *Nordisk Tidskr. Informationsbehandling (BIT)*, v. 11, 1971, pp. 139–158.

43[2.10].—PAUL F. BYRD & DAVID C. GALANT, *Gauss Quadrature Rules Involving Some Nonclassical Weight Functions*, NASA Technical Note D-5785, Ames

Research Center, National Aeronautics and Space Administration, Moffett Field, California, May 1970, iv + 36 pp., 27 cm. Available from National Technical Information Service, Operations Division, Springfield, Virginia 22151. Price \$3.00.

Nodes, t_{iN} , and weight coefficients, W_{iN} , are herein tabulated (Tables 1–12) to 25S (in floating-point form) for twelve Gauss quadrature formulas

$$\int_0^1 w(a, \beta, \gamma; t) f(t) dt = \sum_{i=1}^N W_{iN} f(t_{iN}) + E_N,$$

where the weight function, w , is of the form $t^\alpha(1 - t^\alpha)^\beta$ and $N = 2(2)8(4)16, 24$. The corresponding values of the parameters α, β, γ are, respectively, (3, $\pm 1/2, 0$), (4, $\pm 1/2, 0$), (6, $\pm 1/2, 0$), (8, $\pm 1/2, 0$), (3, $\pm 1/2, \pm 1/2$), (2, $-3/4, 0$), and (2, $-2/3, 0$). The coefficient k_N in the error term

$$E_n = k_N [(2N)!]^{-1} f^{(2N)}(\tau), \quad 0 < \tau < 1,$$

is tabulated (in Table 13) to 5S for the same values of N .

The zeroth moment $M_0 = \int_0^1 w(t) dt$ and the coefficients b_i, g_i of the three-term recurrence relation for the monic orthogonal polynomials associated with the enumerated weight function are given to 25S in Tables 14–25, for $j = 1(1)26$.

An introduction of 11 pages contains a detailed discussion of the numerical difficulties overcome in the construction of these unique tables and of the checks that were applied to test their accuracy. Appended is a listing of a double-precision version of the algorithm used in calculating these tables. As the authors note, this computer program can be used to find additional quadrature rules of the type considered in this very useful report, which also includes a list of 16 valuable references.

J. W. W.

44 [2.10].—R. PIESSENS, *Gaussian Quadrature Formulae for Integrals Involving Bessel Functions*, 30 pages of tables and 3 pages of explanatory text, reproduced on the microfiche card attached to this issue.

These tables consist of 14D values of the abscissas, x_k , and weights, A_k , in the Gaussian quadrature formula

$$\int_{i_{n,s-1}}^{i_{n,s}} J_n(x) f(x) dx = (-1)^{s+1} \sum_{k=1}^N A_k f(x_k),$$

for $n = 0, 1, 2, s = 1(1)20$, and $N = 2(2)8$. The limits of integration are pairs of successive zeros of $J_n(x)$, for the stated ranges of n and s .

The calculation of these tables was performed on an IBM 1620 system at the Computing Centre of the University of Leuven, using algorithms described by Golub & Welsch [1].

The authors refer to similar tables of Longman [2], which consist of 10D entries corresponding to $n = 0, 1, s = 1(1)20$, and $N = 16$. The present tables constitute a valuable and unique supplement to these earlier ones.

J. W. W.

1. GENE H. GOLUB & JOHN H. WELSCH, "Calculation of Gauss quadrature rules," *Math. Comp.*, v. 23, 1969, pp. 221-230.

2. I. M. LONGMAN, "Tables for the rapid and accurate numerical evaluation of certain infinite integrals involving Bessel functions," *MTAC*, v. 11, 1957, pp. 166-180.

45[2.10,7].—É. N. GLONTI, *Tablitsy Kornei i Kvadraturnykh Koeffitsientov Polinomov Iakobi* (Tables of the Roots and Quadrature Coefficients of Jacobi Polynomials), Computing Center, Acad. Sci. USSR, Moscow, 1971, xiv + 236 pp., 27 cm. Price 2.14 rubles.

This is an extensive tabulation of the zeros $x_k^{(n)}$ and Christoffel numbers $A_k^{(n)}$ of the Jacobi polynomials $P_n(p, q, z)$ orthogonal on the interval $(0, 1)$ with weight function $x^{q-1}(1-x)^{p-q}$. The parameters of p, q range through the values $q = 0.1(0.1)1.0, p = (2q-1)(0.1)(q+1)$, while $n = 2(1)15$. The precision is 15S in the zeros and 15D in the coefficients. The only published table that is comparable in scope is that of Krylov et al. [1], which covers a somewhat larger region of the parameters, but is restricted to $n \leq 8$ and a precision of only 8S.

An additional table of quadrature errors

$$e_j^{(n)} = \left| \int_0^1 x^{q-1}(1-x)^{p-q} x^{2n+j} dx - \sum_{k=1}^n A_k^{(n)} [x_k^{(n)}]^{2n+j} \right|,$$

for $0 \leq j \leq 16$ and $2 \leq n \leq 15$, appears in the introduction. These errors grow slightly as q increases for fixed p and sharply increase with p for fixed q . Consequently, only the errors for $q = 1, p = 2$ and $q = 0.1, p = -0.8$ are tabulated, representing the largest and smallest errors, respectively, in the tabular range.

The introduction also includes a collection of formal relationships satisfied by Jacobi polynomials, comments on the computation and use of the tables, and information facilitating interpolation.

W. G.

1. V. I. KRYLOV, V. V. LUGIN & L. A. IANOVICH, *Tablitsy dlia Chislennogo Integrirovaniia Funktsii so Stepennymi Osobennostiami*, Izdat. Akad. Nauk BSSR, Minsk, 1963.

46[2.35, 6, 13.35].—JAMES W. DANIEL, *The Approximate Minimization of Functionals*, Prentice-Hall, Inc., Englewood Cliffs, N. J., 1971, xi + 228 pp., 24 cm. Price \$9.50 (cloth).

This book presents the basic theory relevant to problems of approximately minimizing functionals and an analysis of some of the numerical methods available for their solution. It is an informative, useful, and very readable book. The author's exposition is clear, his prose is smooth, and the text is attractively printed. Exercises, almost all of a theoretical nature, are interspersed throughout the text, and references to nearly 200 books and papers, most of them quite recent, are well documented. Although practical methods are discussed, theoretical considerations dominate. There is little discussion of the comparative merits of these methods, although some very rough guidelines are given in an epilogue.

In Chapter 1, "Variational Problems in an Abstract Setting", basic functional analysis relevant to minimization problems is reviewed, various notions of convexity

are introduced, and some results on minimizing sequences are proved. Most of this material is presented in the context of Banach spaces.

Chapter 2, "Theory of Discretization", presents theory applicable to the convergence of approximate solutions of discretized variational problems.

Chapter 3, "Examples of Discretization", presents specific examples of discretization schemes that can be analyzed by this theory. Particular examples are taken from regularization, optimal control, Chebyshev solution of differential equations, calculus of variations, two-point boundary value problems and the Ritz method.

After introducing criticizing sequences and presenting some general convergence results, Chapter 4, "General Banach-Space Methods of Gradient Type", analyzes various steplength algorithms in great detail. There is also a very brief discussion of direction algorithms and penalty function methods for constrained problems. As the author states in his preface, "... this is not a text on mathematical programming."

Chapter 5, "Conjugate-Gradient and Similar Methods in Hilbert Space", is devoted to a concise but thorough survey of the theory of the conjugate gradient method in Hilbert space. Much of the original work in this area was done by the author.

The remaining four chapters treat minimization in real finite-dimensional spaces. In Chapter 6, "Gradient Methods in R^l ", the results of Chapters 4 and 5 are strengthened. In particular, the nature of the limit sets of criticizing sequences are explored, and improved convergence results are given for variants of the conjugate gradient method. There is an oversight in the proof of one of these results, i.e., Theorem 6.4.1, which makes it false as stated. We must have $\bar{p}_0 = r_0$ as well as $z_0 = x_n$ for $z_l = h_n$. Therefore, the theorem is true only for the restart algorithm.

Variable metric methods are described in Chapter 7, "Variable-Metric Gradient Methods in R^l ". The emphasis is on proofs of finite termination for quadratics and convergence. In the discussion preceding Theorem 7.4.2, the following statement appears regarding the one-parameter family of variable metric methods first considered by Broyden: "... "If $\langle \nabla f(x_{n+1}), p_n \rangle = 0$ for all n , then the directions $p_n / \|p_n\|$ are independent of the parameters $\{\beta_i\}$." This remarkable result was just recently proved by Dixon. However, although the above statement is true, it does not follow directly from Theorem 7.4.2 or from its proof.

Newton's methods and variants of it, generalized versions of iterative linear methods such as the Jacobi, Gauss-Seidel, SOR and ADI methods, and methods for solving nonlinear least squares problems are the topics covered in Chapter 8, "Operator-Equation Methods". "Only the barest outline", to quote the author, of this important area is presented in this very brief chapter.

"Avoiding Calculation of Derivatives" is the subject and title of Chapter 9. Modifications of the Davidon-Fletcher-Powell, Newton, Gauss-Newton, and Levenberg-Marquardt methods that employ difference approximations to derivatives are described. There is also a section on methods which use only function values, but which are not based upon difference approximations to derivatives. The focus is primarily on Zangwill's version of Powell's method. Here, as elsewhere in the book, the outcome of an isolated computational experiment with a particular algorithm is presented. However, precise details of the algorithm are not given. Numerical results presented in this manner serve no useful purpose in this reviewer's opinion.

Although one could argue with the emphasis of this book, it is a valuable addition to the expanding literature on optimization. It includes some new results not previously published and expounds its subject in a rigorous, yet readable manner.

D. G.

47 [2.35,13.35].—PETER L. FALB & JAN L. DE JONG, *Some Successive Approximation Methods in Control and Oscillation Theory*, Academic Press, New York, 1969, viii + 240 pp., 24 cm. Price \$13.50.

It is with considerable pleasure that I recommend this fine book by Falb and de Jong. It should prove useful to a wide class of readers with interests in applied mathematics, computational techniques, and control and optimization of nonlinear dynamic systems.

The importance of the book lies mainly in its approach to the subject. The significant problems associated with approximating solutions of nonlinear systems and with the efficient application of digital computers to this class of problems are discussed from a central point of view in the rich mathematical setting of functional analysis. This makes possible on the one hand a rigorous treatment of convergence for a number of algorithms that have been found useful in the computer solution of problems in optimal control, and, on the other hand, provides a framework that makes clear the essential features of the various algorithms. It also provides a useful way of comparing the computational efficiency of various methods after first applying them to the solution of a "standard" problem in optimal control. The results obtained by this approach have been much needed since convergence proofs in the past have either been quite restrictive or nonexistent even for algorithms that have proved quite useful in practice.

Such a unification is obtained by observing that the application of the necessary conditions for optimal control generally gives a two-point boundary value problem, and that the latter may be viewed as a nonlinear operator equation on a suitable Banach space. The results obtained by the authors then follow mainly by application of the contraction mapping principle. The work of the distinguished Soviet mathematician Kantorovich was obviously crucial at this point, for he was among the first to realize the power of functional analysis in the unification of the theory of iterative methods and provided the basic theorems that now support much of the results in this area.

The techniques discussed in the book are all indirect methods since they proceed by first finding a solution to the two-point boundary value problem given by the necessary conditions and then establishing that under suitable conditions this solution provides an optimal value for the functional under consideration. The specific algorithms considered in detail are Newton's method and variations of it, and Picard's method. These are applied to several numerical examples and the results are discussed in terms of the convergence theory.

If there is a lack in this book, it is one of scope. The powerful techniques of functional analysis, so aptly applied by the authors, could have been applied in a wider context that would have allowed discussion of a number of the direct methods. These are basically gradient, or hill climbing, procedures, and conjugate direction techniques and have proved of considerable practical value. It would also have

provided a point of departure for the development of new methods, topological in nature, that take advantage of the mixed topology inherent in the optimal control problem: the fact that the control space has a weak topology and the state space a strong topology. Apparently, this remains to be exploited: the yield in terms of generality and computational efficiency may be considerable. But this is a small disappointment for one reader who, being stimulated, naturally wishes for more.

This book is an important contribution to the literature on mathematics of computation and has gone a good distance toward filling a serious gap in the field.

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48[3].—J. B. ROSEN, O. L. MANGASARIAN & K. RITTER, Editors, *Nonlinear Programming*, Academic Press, New York, 1970, xii + 490 pp., 24 cm. Price \$10.50.

This book presents the proceedings of a Symposium in Nonlinear Programming held at the University of Wisconsin at Madison, on May 4–6, 1970. According to the preface, “. . . one of the main purposes of this Symposium was to further strengthen the existing relationship between theory and computational aspects of this subject.” In this, the editors have succeeded admirably.

Of the 17 papers listed below, the first nine are devoted primarily to computational algorithms. The emphasis in this group of papers is nicely balanced between the practical and algorithmic and the theoretical. This is especially true of the papers by Powell, McCormick, and Ritter. Particularly noteworthy is the paper by Bartels, Golub and Saunders. It applies numerically stable and efficient LU and QR matrix decomposition techniques to linearly constrained problems. Until recently, such techniques have gone practically unnoticed by mathematical programmers. The paper by Zoutendijk is a very clear and concise summary of his feasible directions approach applied to several algorithms. Daniel's paper extends to constrained problems the results on steplength algorithms contained in Chapter 4 of his book “The Approximate Minimization of Functionals”, and Polak's paper presents the basic ideas for implementing algorithms that he expounds in particular cases in his book, “Computational Methods in Optimization”.

The next four papers deal with theoretical aspects of nonlinear programming. Lemke's paper is a good summary of recent results relating to the complementarity problem.

The last four papers consider the application of nonlinear programming to areas such as mathematical analysis and the physical sciences, statistics and probability, and L_p approximation. These papers do not give nonlinear programming solutions to specific problems, rather they indicate how some of the problems that arise in these basic areas can be viewed in terms of nonlinear programming.

A Method of Centers by Upper-Bounding Functions with Applications

P. HUARD

A New Algorithm for Unconstrained Optimization M. J. D. POWELL

A Class of Methods for Nonlinear Programming: II Computational Experience

R. FLETCHER, SHIRLEY A. LILL

- Some Algorithms Based on the Principle of Feasible Directions. .G. ZOUTENDIJK
 Numerical Techniques in Mathematical Programming.
 R. H. BARTELS, G. H. GOLUB, M. A. SAUNDERS
- A Superlinearly Convergent Method for Unconstrained Minimization.
 K. RITTER
- A Second Order Method for the Linearly Constrained Nonlinear Programming
 Problem.GARTH P. MCCORMICK
- Convergent Step-Sizes for Gradient-Like Feasible Direction Algorithms for
 Constrained OptimizationJAMES W. DANIEL
- On the Implementation of Conceptual Algorithms.E. POLAK
- Some Convex Programs Whose Duals Are Linearly Constrained.
 R. TYRREL ROCKAFELLAR
- Sufficiency Conditions and a Duality Theory for Mathematical Programming
 Problems in Arbitrary Linear Spaces.LUCIEN W. NEUSTADT
- Recent Results on Complementarity Problems.C. E. LEMKE
- Nonlinear Nondifferentiable Programming in Complex Space.BERTRAM MOND
- Duality Inequalities of Mathematics and Science.R. J. DUFFIN
- Programming Methods in Statistics and Probability Theory.OLAF KRAFFT
- Applications of Mathematical Programming to l_p Approximation.
 I. BARRODALE, F. D. K. ROBERTS
- Theoretical and Computational Aspects of Nonlinear Regression.R. R. MEYER
 D.G.

49[3.25, 13.40].—W. ALLEN SPIVEY & ROBERT M. THRALL, *Linear Optimization*, Holt, Rinehart & Winston, Inc., New York, 1970, xii + 530 pp., 24 cm. Price \$15.75.

This is a well written and leisurely paced book that should appeal to application-oriented management and computer scientists who still insist on a mathematically sound presentation. A pleasant feature of the book is the extended set of appendices, approximately 25 percent of the whole book, which cover all the needed background material such as sets, functions, foundation and linear algebra. Another useful feature is the abundance of problems at the end of each chapter. The core of the book is the simplex algorithm for solving linear programs. It is presented in this book at three levels: a geometrical and application-oriented level in the first chapters, a second level consisting of the conventional constructive simplex algorithm itself and, finally, in the condensed Tucker tableau form. The book also includes some special topics and applications such as the assignment problem, the capacitated transportation problem, game theory, decomposition and upper-bound constraints. The book is highly recommended as a textbook for a first course in linear programming in operations research and industrial engineering departments and, especially, for students with limited mathematics background.

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- 50 [7].—C. MOSIER & D. D. SHILLADY, *A Fast, Accurate Approximation for $F_0(z)$ Occurring in Gaussian Lobe Basis Electronic Calculations*, Chemistry Department, Virginia Commonwealth University, Richmond, Virginia. Ms. of 3 typewritten pp. and 2 computer sheets deposited in the UMT file.

The mathematical function referred to in the title is defined by the definite integral $F_0(z) = \int_0^1 \exp(-zu^2) du$, which is expressible in terms of the error function by the relation $F_0(z) = \frac{1}{2}(\pi/z)^{1/2} \text{Erf}(z^{1/2})$, as the authors note.

Specifically, the function $F_0(z)$ is herein approximated for positive z not exceeding 22.5 by a quartic polynomial in $z - s_i$, where the interval i and the corresponding shift s_i are calculated from a given value of z by simple formulas presented in the explanatory text. An accompanying table consists of 16S decimal values (in floating-point form) of the coefficients of this approximating polynomial for $i = 1(1)119$.

The authors claim that the error in their approximation is everywhere less than $4 \cdot 10^{-12}$. Moreover, tests performed by the authors on an IBM 360/50 system have revealed their algorithm to be faster than those based on comparable approximations cited in the literature.

J. W. W.

- 51 [7].—ROBERT PIESSENS & MARIA BRANDERS, *Chebyshev Polynomial Expansions of the Riemann Zeta Function*, 3 pages of tables and 2 pages of explanatory text, reproduced on the microfiche card attached to this issue.

Herein are six 23D tables of the coefficients of the respective expansions of $x\zeta(x+1)$ and $\zeta(x+k)$ for $k = 2(1)5, 8$ in terms of the shifted Chebyshev polynomials $T_n^*(x)$, for $0 \leq x \leq 1$.

These tables were calculated on an IBM 1620 at the Computing Centre of the University of Leuven, and each table was checked by calculating $\zeta(x)$ therefrom for several values of x and then comparing the results with corresponding entries in the tables of McLellan [1].

Coefficients of the Chebyshev expansion of $x(\zeta + 1)$ have been published to 20D by Luke [2]; however, several entries are incorrect beyond 16D, as noted by the present authors. As a further check on Table 1 in the set under review, this reviewer has successfully compared it with a similar, unpublished 40D table of Thacher [3].

J. W. W.

1. ALDEN MCLELLAN IV, *Tables of the Riemann Zeta Function and Related Functions*, Desert Research Institute, University of Nevada, Reno, Nevada, 1968. (See *Math. Comp.*, v. 22, 1968, pp. 687–688, RMT 69.)

2. YUDELL L. LUKE, *The Special Functions and Their Approximations*, v. II, Academic Press, New York and London, 1969.

3. H. C. THACHER, JR., *On Expansions of the Riemann Zeta Function*.

- 52 [7].—GORO SHIMURA, *Introduction to the Arithmetic Theory of Automorphic Functions*, Princeton Univ. Press, Princeton, N.J., 1971, xiv + 267 pp., 24 cm. Price \$10.00.

The present book is an advanced work on some of the most fascinating chapters

in number theory. The author's two main topics are the theory of complex multiplication of elliptic and elliptic modular functions and applications of the theory of Hecke operators to zeta functions of algebraic curves and abelian varieties. Since the prerequisites for studying these topics are quite formidable, probably only the most sophisticated readers will be able to comprehend the book in its entirety.

Professor Shimura has divided his book into three parts which make successively increasing demands upon the reader's mathematical background. The first of these consists of chapters on Fuchsian groups of the first kind, automorphic forms and functions, and the zeta functions associated with modular forms. This material is accessible to those who have mastered the usual introductory graduate courses. However, the Riemann-Roch Theorem and a theorem of Wedderburn about an algebra with radical are stated and used without proof. The next section encompasses elliptic curves, Abelian extensions of imaginary quadratic fields, complex multiplication of elliptic curves, and modular functions of higher level. The classical results of Kronecker, Webber, Takagi and Hasse concerning the construction of maximal Abelian extensions of imaginary quadratic fields by adjoining special values of elliptic and elliptic modular functions are derived by modern methods. Specifically, the adèle-theoretic formulation of class field theory as presented in Weil's *Basic Number Theory* and some concepts from algebraic geometry are used freely. The final section treats zeta functions of algebraic curves and Abelian varieties and arithmetic Fuchsian groups. The Hasse-Weil Conjecture, the construction of class fields over real quadratic fields and Fuchsian groups obtained from quaternion algebras are among the topics discussed.

Professor Shimura has made his book even more useful by providing an appendix summarizing the required algebraic geometry and an extensive bibliography.

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53[8].—G. W. HILL, *Reference Table: "Student's" t-Distribution Quantiles to 20D*, Technical Paper No. 35, Division of Mathematical Statistics, Commonwealth Scientific and Industrial Research Organization, Melbourne, Australia, 1972, 24 pp., 25 cm. Copy deposited in the UMT file.

Quantiles of Student's t -distribution corresponding to the two-tail probability levels $P(t | n) = 0.9(-0.1)0.1, \{5, 2, 1\} \times 10^{-r}$ for $r = 2(1)5, 10^{-s}$ for $s = 6(1)10(5)30$, and for $n = 1(1)30(2)50(5)100(10)150, 200, \{24, 30, 40, 60, 120\} \times 10^r$ where $r = 1(1)3$, and also for $n = \infty$, are herein tabulated to 20D for $t < 10^3$, otherwise to 20S. These numbers were originally calculated to about 25S on a CDC 6400 system prior to rounding to the tabular precision, and elaborate checks applied at successive stages of the calculations and to the final reproduction inspire acceptance of the author's claim of accuracy of the tabular entries to within half a unit in the least significant digit.

We are informed in the introduction that this table is not intended for daily use, but rather has been designed to provide reference values for resolving discrepancies in previous tables and for determining errors of various approximations.

The explanatory text also includes outlines of the various methods available for the multiple-precision computation of the tabulated quantiles, the probability integral and frequency function for the t -distribution, and the normal probability integral and its inverse.

The exceptionally high precision of this definitive table, as well as its accuracy, should make it a basic reference table for statisticians, as implied in the title.

J. W. W.

54[9].—J. C. P. MILLER, *Primitive Root Counts*, University Mathematical Laboratory, Cambridge, England. Ms. of 15 pp. deposited in the UMT file.

This tabulation of counts of primes with specified primitive roots was started in connection with the preparation of a set of tables of indices and primitive roots compiled by the present author in collaboration with A. E. Western [1]. The counts listed therein (Table 8) have been considerably extended in the present tables, which are based on calculations completed in 1966 on EDSAC 2, using programs prepared by M. J. Ecclestone.

The main listing of counts herein includes all those primes less than 250,000 for which the integer a is a primitive root, where $\pm a = 2(1)60$. These counts are given for such primes occurring in successive intervals of 10^4 integers, with subtotals for successive intervals of $5 \cdot 10^4$ and 10^5 integers, as well as a grand total for each a . Corresponding to $\pm a = 3, 5, 7, 11, 13$, and 17 , these counts are extended in a supplementary table to all such primes less than 10^6 , appearing in successive intervals of $5 \cdot 10^4$ integers, with subtotals at every fifth interval, and the corresponding grand totals. The corresponding counts of all primes in these intervals are also given, and a numerical comparison is made between the cumulative tabular counts and the corresponding counts predicted from Artin's conjecture as elaborated upon in [1].

The large amount of new material in this manuscript certainly provides a valuable supplement to the corresponding data in [1], which will be of particular interest to number theorists.

J. W. W.

I. A. E. WESTERN & J. C. P. MILLER, *Indices and Primitive Roots*, Royal Society Mathematical Tables, Vol. 9, University Press, Cambridge, 1968. (See *Math. Comp.*, v. 23, 1969, pp. 683–685, RMT 51.)

55[9].—SAMUEL YATES, *Partial List of Primes with Decimal Periods Less than 3000*, Moorestown, N. J. Ms. of 30 computer sheets (undated) deposited in the UMT file.

Known primitive prime factors of integers of the form $10^n - 1$ are here tabulated for 1564 positive integers n less than 3000. Complete factorizations are listed for the first 30 values of n and for 15 higher values, not exceeding $n = 100$. All admissible primes under $4 \cdot 10^7$ have been tested as factors throughout the range of the table.

Brillhart and Selfridge [1] have proved that $(10^n - 1)/9$ is prime only for $n = 2, 19$, and 23 if $n < 359$. The present table permits this limit to be raised to $n < 379$.

This compilation updates and supplements an earlier one [2], which was limited to prime values of n in the same range.

J. W. W.

1. J. BRILLHART & J. L. SELFRIDGE, "Some factorizations of $2^n \pm 1$ and related results," *Math. Comp.*, v. 21, 1967, pp. 87-96.
2. SAMUEL YATES, "Factors of repunits," *J. Rec. Math.*, v. 3, 1970, pp. 114-119.

56[10].—RUDOLF KOCHENDÖRFFER, *Group Theory*, McGraw-Hill, London, 1970, vii + 297 pp., 24 cm. Price £5.—

This book, an English translation of the original 1966 German edition, provides an excellent introduction to group theory, with the emphasis placed on finite groups. Besides the standard topics, some fairly recent theorems (e.g., on Carter subgroups of solvable groups) are included.

The book is written very straightforwardly, with a minimum of notation and a maximum of clarity; in this respect, it compares favorably with other texts, such as *Group Theory* by W. R. Scott, which cover somewhat more ground at the expense of readability. There are 127 exercises, many of them being examples for the general theory (of these, however, 49 are concentrated in the first two chapters).

The author's choice of topics shows (in the reviewer's opinion) excellent taste. There are 13 chapters, as follows: groups and subgroups, homomorphisms, Sylow subgroups of finite groups, direct products, abelian groups, extensions of groups, permutation groups, monomial groups and the transfer, nilpotent and supersoluble groups, finite p -groups, finite soluble groups, miscellaneous topics (e.g., the Burnside Problem), representations (including proofs of theorems of Burnside and Frobenius). There is a useful bibliography of books and articles.

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57[12].—R. DUBUC, G. LAMBERT-CAREZ, M. GRATTON, L. ROY & A. SHAPIRO, *Dictionnaire Anglais-Français, Français-Anglais de l'Informatique*, Dunod, Quebec, 1971, xi + 214 pp., 22 cm. Price 24F, paperbound.

This book contains some six thousand words and phrases from the area of computer science, both in English and in French. Contrary to the usual concepts of lexicology, the authors have not attempted to record the current usage of the French scientific language in the field of computers. Rather, they have tried to invent a new language, since they feel that the current usage involves too much borrowing from the English vocabulary. While nobody can dispute this fact, it appears doubtful that their endeavor will have any measure of success. There seem to be two reasons for this: The first reason is that it is far from certain that French speaking computer specialists actually feel the need to purify the professional jargon they have been using for many years. The second reason for the probable failure of this enterprise is the proposed vocabulary itself. Many of the French words or phrases have been coined by the authors themselves, using some standard Latin or Greek roots, prefixes and suffixes. Besides the fact that one would expect the book to indicate clearly which phrase is extracted from the existing literature and which one is a creation of the authors, it turns out that many of the new words are unbelievably pedantic. Although most of these neologisms are, in fact, logically derived from the appropriate

roots, a large number of them are either so unfamiliar as to be difficult to remember or simply awfully discordant. Examples: 'programmoïde' for 'software', 'discotrope' for 'disk-oriented', 'ordinolingue' for 'in machine language', 'disque-programmo-thèque' for 'library disk'.

Besides these matters of principle, there are a number of lesser shortcomings. The translation sometimes lacks consistency: 'control card' becomes 'carte de controle' while 'control character' is translated as 'caractère de commande'. Even though the authors claim to have consulted a large body of existing literature, nowhere do they specifically reference their sources. They do not explain precisely what domain is covered by their dictionary: A number of terms related to electronics are included ('adjustable voltage divider'), but one finds almost no terms from the mathematical aspects of computer science or from the metatheory (how does one translate 'Algol-like' or 'context-free'?). Finally, some of the translations are inaccurate: e.g., 'actual size' translated by 'grandeur réelle' which in fact means 'real quantity' or 'real value'. In conclusion, even though French-speaking professionals may indeed feel the need for an accurate dictionary of computer science terms, it seems far from certain that this book will answer their needs.

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58[12].—T. F. FRY, *Computer Appreciation*, Philosophical Library, New York, 1971, viii + 237 pp., 23 cm. Price \$15.00.

In the 237 pages of his book "Computer Appreciation", Mr. T. F. Fry has collected a comprehensive body of essential computer science material and presented it in an easy-to-read, easy-to-understand manner. This carefully worded book represents a worthwhile contribution to a field which, unfortunately, is all too often cluttered with so-called elementary books written in a language which only the sophisticated computer programmer can comprehend. Not so with Mr. Fry's book. He provides excellent reading for students of almost any basic course in computer science, and, more particularly, for students of business.

I believe the questions which are to be found at the end of each chapter are not only appropriate but also sufficiently stimulating to provide ample material for discussion in the class room.

If Mr. Fry is guilty of over-simplification, this is intentional rather than accidental and he is to be complimented rather than criticized. This book will not teach the reader *any* computer language, but it was not designed to do so. It succeeds very well in providing the necessary background for computer appreciation—which indeed, is the book's title.

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59[12].—T. E. HULL & D. D. F. DAY, *Computers and Problem Solving*, Addison-Wesley (Canada) Ltd., Don Mills, Ontario, 1970, xi + 276 pp., 25 cm. Price \$6.50.

The authors have put two books under one cover. The first would aptly be titled "Computer Fundamentals" and the second "Collected Problems without Solutions". The first part of this volume provides the student with a clear introduction to the concept of an algorithm through flow charting. The flow charting procedures are then related to computers by the introduction of both Fortran and a sample machine language. The sections on Fortran are clear but their order of presentation may be questioned as indicated by the authors in the introduction. The Fortran discussion is interspersed with references to machine code and, in fact, the section titled Fortran Arithmetic has its examples done in machine code. Early sections rely on material that is not explained in depth until later sections, as input/output. Three quarters of the way through the Fortran explanation, a problem section appears, followed by more Fortran language material, subroutines and more I/O. The solutions to the problems themselves are not presented within the text, although details are given in the answers to exercises. The otherwise extreme clarity of Part One is obscured by its organizational format. A clever instructor, however, may be able to reorganize the material to advantage.

Part Two of the book is composed entirely of problems touching all aspects of computing. It will broaden the horizon of any student with regard to possible uses of programming knowledge. The authors have taken an "open ended" approach to problem solving. They present the problem and give the student no direction as to its solution. The problem areas range from numerical analysis, statistics, string processing, computer graphics, and business applications to compilers and interpreters. Part Two on its own can provide any instructor with a wealth of well-defined projects to motivate the programming student and would go well with some other text on programming fundamentals.

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60[12].—KENNETH P. SEIDEL, *Cobol*, Goodyear Publishing Co., Pacific Palisades, California, 1971, viii + 122 pp., 23 cm. Price \$3.95.

This is an easily read and moderately complete book about COBOL as used on the IBM 360, RCA Spectra 70, and XDS-Sigma Systems. It is definitely not a text for a beginning course in data processing, as it provides no introduction to basic computer concepts. From the start, the text plunges into COBOL fundamentals and takes the reader through the language in the order in which COBOL programs are written. The identification division, environment division, data division, working storage, and procedure are explained in an easily usable form, along with one or two examples carried through to solutions. The section on the data division is preceded by a well conceived chapter on data file structure, necessary to the COBOL programmer. Only one complete program is presented as illustration. This is a major failing of the book. Unless the reader is well acquainted with the art of programming,

he may be hard pressed to apply the limited examples in the book to other problems. Advanced topics in COBOL are covered as far as inter-module linkage, which again assumes more foreknowledge on the reader's part. Before concluding with the COBOL sort feature, a well thought-out guide to COBOL language debugging is presented in a form that quickly points the beginning user to his sources of error. Any programming language book would do well to use this section as a model. In summation, this is a fine book for the experienced programmer wishing to add another language to his list.

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