REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS

The numbers in brackets are assigned according to the indexing system printed in Volume 22, Number 101, January 1968, page 212.

1 [1].—Petr Beckmann, *A History of π (Pi)*, Second Edition, The Golem Press, Boulder, Colorado, 1971, 196 pp., 24 cm. Price $6.30.

Bernard Baruch once remarked that "Every man has a right to his own opinion, but no man has a right to be wrong in his facts." It is with this in mind that I shall review the present book. The author admits he is neither an historian nor a mathematician and uses this to excuse his avoidance of mathematical rigor and lack of hesitation in giving vent to (controversial) opinions. Pi is looked upon as "a quaint little mirror of the history of man." We shall not discuss the opinions expressed by the author concerning the Roman Empire or his scathing remarks all through the book concerning the Soviet Union (pp. 52, 54, 55, 76, 80, 125, 147, etc.) except to forewarn the reader that this book is more than a technical history of the calculation of π. The author, however, hopes "that this little book might stimulate nonmathematical readers to become interested in mathematics." It is inevitable, then, that we must look at errors in history and mathematics, errors in which this book abounds.

This is the second edition of the work, appearing not quite a year after the first. The most astonishing thing about the first edition was the absence of any mention of the calculation of π after 1946; not one word was said about electronic computers and the calculation of π to 100,000 decimals by Daniel Shanks and John W. Wrench, Jr. (*Math. Comp.*, v. 16, 1962, pp. 76–99). It appears that the present edition was rushed into print to remedy that omission while sales were brisk. The book has a new chapter (18) on the computer age, and the end sheets of the book have been imprinted with the first 10,000 decimals of π as found by Shanks and Wrench. Unfortunately, even here on the end sheets, it is erroneously stated that the computation was programmed for, and performed on, an IBM 704, instead of an IBM 7090 (as correctly stated on page 180).

What is more, Chapter 18 is marred by several errors of statement regarding the computing time that was required in the evaluation of π to 100,265 decimal places on the IBM 7090 system. Page 181 is quite garbled and incorrect. What it should report is that the computation of 24 arctan(1/8) required only 2 hours and 34 minutes (because of using shifting on the binary machine instead of regular division, which would have required 6 hours and 7 minutes), the computation of 8 arctan(1/57) required 3 hours and 7 minutes, and the computation of 4 arctan(1/239) required 2 hours and 20 minutes. This information is contained in the paper by Shanks and Wrench, a copy of which was sent to the author by Dr. Wrench soon after the first edition of the book appeared, so that this reviewer wonders how the author dreamed up the incorrect story that only 34 minutes was needed to compute 4 arctan(1/239) and that this was possible because the third term "converges fastest because of the small argument."

Also on page 181, there is an incorrect account of the later calculations of π.

Again, this information was sent to the author by Dr. Wrench (letter of 10 December 1970), so that we still must wonder at the careless manner in which facts are reported. What we should read in paragraph 3 is that the calculation to 250,000 decimals was done by Jean Gilloud and Jean Filliatre; whereas the calculation a year later to 500,000 places was done by Jean Gilloud and Michele Dichampt.

On page 99 it is erroneously stated that Abraham Sharp used an arcsine series to compute π to 72 decimal digits. The series he did use was the arctangent series displayed at the top of page 140.

On page 100 the author fails to mention that Levi B. Smith and John W. Wrench, Jr. jointly calculated π to 808 decimal places and their final, corrected result was published in a joint paper with D. F. Ferguson in *Mathematical Tables and Other Aids to Computation* (now *Math. Comp.*), v. 3, 1948–1949, pp. 18–19. This fact, as well as the account of the unpublished extension of this approximation to 1157 decimal places by Smith and Wrench, appears in the paper of Wrench cited in the bibliography on page 191.

On page 37 there is an incorrect argument to show the existence of infinitely many primes. Assuming there is a largest prime p, the author would have us believe that $p! + 1$, which is not divisible by $2, 3, \cdots, p$, "is therefore a prime" when the simple counterexample $5! + 1 = 121 \neq$ prime reveals the fallacy here. This is a common mistake in textbooks.

On page 109 we are told that "The Goldbach conjecture has been proved for numbers greater than 10^{10}." No reference is cited and this reviewer would like to see such a proof, as he believes no such proof has been given.

The account on page 47 does not explain why certain constructions using compass and straightedge are inadmissible (sliding measurements used to make two segments of equal length, as in certain trisections).

The index is virtually unchanged from the first edition, so that the new names are not possible to retrieve save by a page-by-page perusal.

Among the interesting misprints and other errors in this book, we offer finally the following sampling: on page 92, line 19, for "enginnering" read "engineering"; on page 95, next to the last line, for "and" read "an"; on page 101, line 14 from bottom, for "exhibite" read "exhibited"; on page 102, line 13 from bottom, Dase was born in 1824, not 1840, but the correct date is given on page 100; on page 108, line 11 from bottom, we should read "1920's"; on page 152, equation (18) is wrong, for "tan" read "tanh"; on page 165, line 10 from bottom, for "ansered" read "answered"; on pages 180–181 and the end sheets, the author is uncertain whether to use one or two "*m*'s" in spelling various derivatives of the word "program". The Index should be carefully checked for incorrect page numbers. The statement on page 146, lines 9 and 10 from bottom, seems in rather poor taste.

Besides the insertion of the new chapter and end sheets, the main way that changes have been made is by the addition of six new footnotes on page 188 and some additions to the bibliography and chronological table.

We have then a hastily contrived second edition that continues to garble important history and present some incorrect facts. This could have been a very lively and factually correct reference work if the author had been more careful. As it is, the best advice is that this is an interesting and useful book but one must not believe something merely because it is so stated in this book; there are so many errors that one must check with other sources. The book should be rewritten entirely and the manuscript should be examined by qualified mathematicians and historians before being committed to the printed page; for despite the author's protestations at being neither mathematician nor historian, many readers will undoubtedly quote from the book as gospel truth. The history of π may be embellished but it must first of all be correct.

Subject to the limitations we have discussed above, the book under review gives an interesting account of the calculations of π from Biblical and Greek times down to the present day. The book is nicely illustrated and printed in a pleasing format.

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2 **[2, 3, 4, 5, 7, 8]**.—A. M. COHEN, J. F. CUTTS, R. FIELDER, D. E. JONES, J. RIBBANS & E. STUART, *Numerical Analysis*, McGraw-Hill Book Co. (UK) Ltd., Maidenhead, Berkshire, England, xiv + 341 pp., 23 cm. Price £3.50.

"This book contains the material which now forms the basis of undergraduate honours courses in the College (Wales Institute of Science and Technology) and only assumes an elementary knowledge of real and complex variable theory. It aims to form a foundation for an appreciation of numerical work and to pave the way to understanding more advanced treatises and research papers."

The authors achieve their goal. They present careful descriptions of numerical methods and apply them to illustrative examples. Many of the mathematical analyses of the methods are found among the numerous problems at the end of each chapter. But, the level of some of these exercises may require a more than elementary facility with complex analysis. For example, in Chapter 2, on the roots of polynomials, one of the exercises asks for a proof of Rouché's theorem, with the hint to use the result of the previous problem in which the integral of $1/(z - a)$ is evaluated over the unit circle. To avoid this concise presentation of the wealth of material would have made the book impossibly long. Nevertheless, this softly (but sturdily) covered book will prove to be a valuable supplementary text for advanced undergraduate and beginning graduate students, who are learning about numerical methods for the first time. Much practical analysis is given in detail. The authors have a sense of humor and of reality as exemplified by the following quotation taken from the solution to a numerical exercise of Chapter 3: "One should not assume that questions are always correctly posed." Indeed, a student who was not aware of this principle might have wasted some effort in trying to establish the text's purposely incorrectly set problem. The book concludes with a large set of miscellaneous exercises and selected solutions for all of the text problems. The chapter headings are: Introduction; Polynomials and their zeros; Interpolation and differentiation; Orthogonal polynomials; Numerical integration (quadrature); Series summation; Function approximation; Direct methods for the solution of simultaneous linear equations;

The algebraic eigenvalue problem; Iterative methods for the solution of simultaneous linear equations; Ordinary differential equations; Partial differential equations; The solution of simultaneous non-linear equations and optimization; Monte Carlo methods.

E. I.

3 [2.05, 2.35, 3, 4].—H. R. SCHWARZ, H. RUTISHAUSER & E. STIEFEL, translated by P. Hertelendy, *Numerical Analysis of Symmetric Matrices*, Prentice-Hall, Inc., Englewood Cliffs, N. J., 1973, xi + 276 pp., 24 cm.

Here we have a translation of a book that grew out of course notes by Professor Schwarz who drew from notes by the late Professor Rutishauser of Switzerland. Very good courses they must have been, with the derivation of the mathematical problems given as much emphasis as the numerical methods to solve them. This book would be a good text for a senior-level course in the United States.

After a first chapter on basics, including Choleski's decomposition tailored to band matrices, the four remaining chapters are entitled: Relaxation Methods, Data Fitting, Eigenvalue Problems, and Boundary Value Problems. Sixteen Algol procedures are developed in the text and Fortran IV versions are given in an appendix. It is not claimed that these programs are the last word in sophistication, clarity was the watchword. It is worth pointing out that the translation of the procedures was fairly easy because programs of this sort do not depend on the Algol features, such as recursion and block structure, which have no counterpart in Fortran. The authors are experienced at exposition and the book seems to be well written, well translated, and well laid out. Somehow the clarity of thinking, so characteristic of the authors, comes through in their written words.

It is not incumbent upon a textbook to be up to date, but in the field of numerical methods it is highly desirable. In this respect, there is a most puzzling defect in part of the book. I can only conclude that the text was actually written in 1964, although the German version did not come out until 1968. How else can one explain the fact that the eigenvalue chapter is a beautiful presentation of methods, some of which are ten years out of date? The QR algorithm is not mentioned, but LR and QD receive close attention. For positive definite tridiagonal matrices, there appears to be some advantage in using the QD formulation, but in practice it is a nuisance to have to maintain positive definiteness when using shifts.

Another little clue to the time lag in publication is the fact that the Sturm sequence algorithm is used in its standard form instead of the simpler, more convenient version in which the successive quotients of the polynomials are evaluated, rather than the polynomials themselves. These quotients are just the diagonal elements of U in the LU decomposition.

These are comparatively minor criticisms of a work which takes a very nice, coherent set of topics and presents them so well.

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4 [2.20].—CHIH-BING LING, *On Values of Roots of Monomial-Transcendental Equations*, Department of Mathematics, Virginia Polytechnic Institute and State University, Blacksburg, Virginia. Ms. of 21 typewritten pp. deposited in the UMT file.

Herein are tabulated to 11S, in floating-point form, the first 50 real (or complex) roots of equations of the type $f(z) \pm z = 0$ where $f(z)$ represents one of the trigonometric functions $\tan z$, $\cot z$, $\sec z$, $\csc z$ (or the corresponding hyperbolic functions) and the exponential function $e^{\pm z}$.

The introduction includes details of the underlying calculations and a list of references to various applications of the tables.

These tables supersede similar ones published jointly by the author and Yeung [1], which have been found to be generally unreliable, except for the tabulated roots of $\tan z \pm z = 0$.

The author has informed this reviewer that the present tabular values have been thoroughly checked by substitution in the appropriate equations. As a further check, the reviewer has successfully compared the roots of $\tan z = z$ as herein tabulated with the corresponding 40D values calculated by Robinson [2]. Also, the accuracy of the tables corresponding to $f(z) = e^{\pm z}$ has been confirmed by independent calculations to 13S by Fettis [3].

This set of tables may be considered a sequel to a table [4] by the same author, consisting of roots of similar equations where $f(z)$ is, respectively, $\sin z$, $\cos z$, and the corresponding hyperbolic functions.

J. W. W.

1. S.-F. YEUNG & C.-B. LING, "On values of roots of monomial-transcendental equations," *Hung-Ching Chow Sixty-fifth Anniversary Volume*, National Taiwan University, Taipei, Taiwan, December 1967, pp. 196–204.

2. H. P. ROBINSON, *Roots of* $\tan x = x$, Lawrence Berkeley Laboratory, University of California, Berkeley, California, December 1972, ms. deposited in the UMT file. (See *Math. Comp.*, v. 27, 1973, p. 999, RMT **44**.)

3. H. E. FETTIS, Private communication.

4. C.-B. LING, *Values of the Roots of Eight Equations of Algebraic-Transcendental Type*, Virginia Polytechnic Institute, Blacksburg, Virginia, June 1965, ms. deposited in the UMT file. (See *Math. Comp.*, v. 20, 1966, p. 175, RMT **16**.)

5 [3, 9, 10].—MORRIS NEWMAN, *Integral Matrices*, Academic Press, New York, 1972, xvii + 224 pp., 24 cm. Price $14.00.

This book is a gem. It definitely belongs in the library of anyone interested in rings, matrices, number theory, or group theory.

The first five chapters cover the basic material on equivalence, similarity, and congruence of matrices. By page 15, the Hermite normal for a matrix over a principal ideal domain (p.i.d.) appears. But typical of the rest of the book, Dr. Newman gives the reader something new and interesting even in dealing with such a classical result as this: in Theorem II.4, he discusses the number of classes with respect to left equivalence which have a fixed determinant. By page 22, we have the proof that every left ideal in the matrix ring over a p.i.d. is also principal. It is short, easy, and devoid of the usual excess baggage found in most presentations of this theorem. On page 33 there is a very nice argument showing that the Smith normal form is multiplicative, and again on page 35 we are treated to something new with a count of the number of two-sided equivalence classes of matrices with fixed determinant. An interesting application of the Smith form, virtually never found in linear algebra texts, is made on pages 37 and 38 to the solutions of linear diophantine equations.

By page 49, the standard similarity theory over a field is complete and the author goes on to discuss similarity over Z. Here he obtains the Latimer-MacDuffee theorem relating similarity classes over Z for which $f(A) = 0$ with ideal classes in $Z[\theta]$ where θ is a root of the monic integral irreducible polynomial $f(\lambda)$.

Chapter IV begins with the study of congruence of matrices over fields, including the characteristic-2 case. Witt's theorem and a statement of the Hasse-Minkowski theorem appear. A very nice argument is presented to prove that there are only finitely many congruence classes of symmetric integral matrices of a given determinant whose quadratic forms do not properly represent 0. Chapter V is entitled, "Combined similarity and equivalence", and contains a proof of the theorem of M. Hall, Jr. and H. J. Ryser that asserts that if two n-tuples over a field of characteristic not 2 have the same Euclidean square length, then they are orthogonal transforms of one another. Chapter VI provides a quick and self-contained introduction to the Minkowski geometry of numbers including a derivation of sharp bounds for the largest value for the arithmetic minimum of a positive definite form.

Chapter IX contains in about ten pages a mini course in group representation theory which is then used to study automorphs of positive definite quadratic forms and the finite subgroups of the two- and three-dimensional general linear groups over the integers. A circulant is a polynomial in the full cycle permutation matrix, and Chapter X contains a number of interesting results of O. Taussky, R. C. Thompson and the author on this fascinating subject. The last chapter resumes the study of quadratic forms and contains among other items Mordell's inequality for the Hermite constant and Minkowski's proof of the finiteness of the class number.

Chapters VII and VIII on general matrix groups and the classical modular group comprise about 60 pages in this 224-page book. The author includes much of his own fundamental work in this field here. Perhaps a list of the items he covers will convey some idea of the content: the general and special linear groups, the symplectic group, congruence groups over principal ideal rings, generators of the modular group, ranks of subgroups, an original proof of Wohlfahrt's theorem on congruence groups, parabolic class numbers, genus, and discrete subgroups of the special linear group over **R**.

Each of the eleven chapters ends with a problem set. Almost none of these are easy or routine, but some of the more excruciating ones include hints for solutions.

The theory of integral matrices is a large and difficult field. As the author states in his introduction, it "is a huge subject which extends into many different areas of mathematics." Despite this, Dr. Newman has written an accessible text which leads the reader up to the boundaries of current research in the subject. He has a friendly attitude towards a newcomer to the subject, both in his style of writing and his selection of explicit methods whenever possible. The book can be used for an advanced undergraduate or graduate course assuming only that the students have completed basic courses in modern algebra, matrix theory, and number theory.

Chapters I through V, IX and X could be the core of an advanced, one-semester course in matrix theory including elementary group representation theory. Selected topics from the remaining chapters could more than easily complete a one-year sequence.

This reviewer believes that *Integral Matrices* will certainly take its place among the very best in mathematical expositions: it deals with interesting material; it is packed with information; and it is intelligible.

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6 [7, 9].—ROBERT SPIRA, *Table of* $e^{\pi\sqrt{n}}$, Michigan State University, East Lansing, Michigan. Ms. of 9 typewritten pp. deposited in the UMT file.

This unpublished table consists of 15D values of $e^{\pi\sqrt{n}}$ for $n = 1(1)200$. Because of the increasing size of the integer parts of these numbers, the corresponding number of significant figures in the tabular entries ranges from 17 to 35. In the introduction we are informed that this table was calculated in order to test the author's general multiple-precision Fortran subroutines for the elementary functions. Each entry was computed in about four seconds on a CDC 3600 system, using 117S decimal arithmetic.

The author refers to a listing of decimal approximations to six of these numbers in the FMRC *Index* [1], and he notes his confirmation of terminal-digit errors in two of them, originally announced by Larsen [2].

This table should be of particular interest to number-theorists because of the known relation between the fractional part of $e^{\pi\sqrt{n}}$ and the number of classes of binary quadratic forms of determinant equal to $-n$, as mentioned by D. H. Lehmer [3].

J. W. W.

1. A. FLETCHER, J. C. P. MILLER, L. ROSENHEAD & L. J. COMRIE, *An Index of Mathematical Tables*, 2nd ed., Addison-Wesley Publishing Co., Reading, Massachusetts, 1962.
2. *Math Comp.*, v. 25, 1971, p. 200, MTE **474.**
3. *MTAC*, v. 1, 1943, pp. 30–31, QR **1.**

7 [9].—R. P. BRENT, *The Distribution of Prime Gaps in Intervals up to* 10^{16}, Australian National University, 1973, iv + 62 pp. deposited in the UMT file.

These tables are analogous to the Table 2 of Brent's paper [1]. For all primes p such that $N < p < N'$, the number of gaps

$$p_{i+1} - p_i = g$$

are tabulated for each $g = 2, 4, 6, \cdots$ that occurs in (N, N'). The *estimated* total number of gaps is

$$P = \int_N^{N'} dx/\log x,$$

while the number for $g = 2$ or for $g = 4$ is the well-known

$$E_2 = E_4 = 1.3203236317 \int_N^{N'} dx/\log^2 x.$$

For larger g, Brent uses his formulae developed in [1].

The first 21 tables are for the intervals

$$(10^j, 10^j + 10^6), \qquad j = 6(1)15;$$
$$(10^j, 10^j + 10^7), \qquad j = 7(1)14;$$
$$(10^6, 10^j), \qquad j = 7, 8, 9.$$

For each interval there is listed the first and last prime; the observed population for each g: O_g; the expected number E_g for $g = 2(2)80$ according to the aforementioned formulas; the expected number for $g > 80 = P - \sum_2^{80} E_g$; the normalized differences $(O_g - E_g)/(E_g)^{1/2}$; and a χ^2 computed for these 41 degrees of freedom. The χ^2 vary from 20 to 73 and seem to suggest that, if anything, the distribution agrees "too well" with the expected distribution.

For the remaining four intervals

$$(10^j, 10^j + 2\cdot 10^7), \qquad j = 15, 16,$$
$$(10^j, 10^j + 10^8), \qquad j = 14, 16,$$

only the empirical data are given, not the expected values or χ^2.

There is included a 13-page Fortran and 360 Assembly Language program. One sees that the estimating integrals were computed with a 16-point Gauss integration. There also is a 3-page text.

The empirical counts in the interval $(10^{14}, 10^{14} + 10^8)$ were tabulated earlier by Weintraub [2]. The data agree.

D. S.

1. R. P. Brent, "The distribution of small gaps between successive primes," *Math. Comp.*, v. 28, 1974, pp. 315–324.
2. S. Weintraub, UMT 27, *Math. Comp.*, v. 26, 1972, p. 596.

8 [9].—Edgar Karst, *The Third* 2500 *Reciprocals and their Partial Sums of all Twin Primes* $(p, p + 2)$ *between* (239429, 239431) *and* (393077, 393079), University Computer Center, The University of Arizona, Tucson, Arizona, February 1973. Ms. of 207 computer sheets deposited in the UMT file.

9 [9].—Daniel Shanks & Carol Neild, *Brun's Constant*, Computation and Mathematics Department, Naval Ship Research and Development Center, Bethesda, Maryland, April 1973. Ms. of 67 computer sheets deposited in the UMT file. For a detailed review of these unpublished tables, see pp. 295–296 of this issue.

10 [9].—KARL C. RUBIN, *Table of* $A^{(k)}(n)$, Woodrow Wilson High School, Washington, D. C., 1973, ms. of 4 pages deposited in the UMT file.

This is an extension of Wagstaff's table [1] of $A^{(k)}(n)$. This is the cardinality of the largest subset of the natural numbers 1 to n wherein no k numbers are in arithmetic progression. Wagstaff computed these for $k = 3(1)8$ and for all $n = 1, 2, \cdots$ up to

$$A^{(3)}(53) = 17, \quad A^{(4)}(52) = 26, \quad A^{(5)}(74) = 48,$$

$$A^{(6)}(52) = 38, \quad A^{(7)}(53) = 42, \quad A^{(8)}(57) = 46.$$

Here, $k = 6(1)8$ are extended up to

$$A^{(6)}(80) = 55, \quad A^{(7)}(94) = 72, \quad A^{(8)}(80) = 64$$

using Wagstaff's method [2] on a SPC-16 minicomputer. The ratios

$$A^{(k)}(n)/n$$

for these three k have therefore been only reduced slightly. The conjecture is that they $\to 0$ as $n \to \infty$.

The author suggests that a further extension is "somewhat impractical" since "$A^{(6)}(80) = 55$ ran for several nights."

D. S.

1. SAMUEL S. WAGSTAFF, JR., *Math. Comp.*, v. 26, 1972, pp. 767–771.
2. S. S. WAGSTAFF, JR., *Math. Comp.*, v. 21, 1967, pp. 695–699.

11 [9].—HUGH WILLIAMS, LARRY HENDERSON & KEN WRIGHT, *Two Related Quadratic Surds Having Continued Fractions with Exceptionally Long Periods*, University of Manitoba, 1973, 177 computer sheets deposited in the UMT file.

It was known [1], [2] that the prime

$$p = 26437680473689$$

has two properties. (A) All numbers < 151 are quadratic residues of p. (B) The class number $h(p)$ of $Q(\sqrt{p})$ equals 1. It follows that the periodic continued fractions

$$\tfrac{1}{2}(\sqrt{p} - 5141757) = \frac{1}{1} + \frac{1}{3} + \frac{1}{940} + \frac{1}{3} + \cdots, \tag{1}$$

and

$$\sqrt{p} - 5141758 = \frac{1}{1} + \frac{1}{1} + \frac{1}{1880} + \frac{1}{1} + \cdots \tag{2}$$

(which are related by Hurwitz's transformation) have periods that are exceptionally long. It turns out here that these periods are 18334815 and 18331889, respectively.

In the period for (1), 7609286 or 41.5018% of the partial quotients equal 1, 3117706 or 17.0042% equal 2, 1706864 or 9.3094% equal 3, and so on, until, finally, 1 partial quotient—the largest—equals 5141757. The first table deposited—73 pages long—lists these frequencies and percentages for each of the 4759 different partial quotients a that occur in (1).

The empirical percentages may be compared with the Gauss-Kuzmin law of almost all continued fractions wherein a occurs with the percentage

$$100 \log[1 + 1/(a^2 + 2a)]/\log 2.$$

For $a = 1, 2, 3$, this gives the values 41.5037, 16.9925, 9.3109, respectively, in close agreement with the empirical data.

In (1), each $a = 1, 2, 3, \cdots$ up to 1471 occurs. The first missing values are 1472, 1525, 1538, 1648, etc. The largest a in (1) are 5141757, 2570878, 1713918, 1285439, etc. These large a have an interesting distribution that we now explain.

All a are given by

$$a = [(\sqrt{p} + B_n)/2A_{n+1}]$$

where $(\pm A_n, B_n, \mp A_{n+1})$ is a reduced binary quadratic form of discriminant

$$p = B_n^2 + 4A_nA_{n+1}.$$

By the special properties of p mentioned above, each $A_{n+1} = k$ ($k = 1$ to 150) will occur in the period 2^r times, where r is the number of distinct primes dividing k. Thus, the first missing k is 151 since this is the first quadratic nonresidue of p, while $a = 13183$ occurs 16 times since $k = 2 \cdot 3 \cdot 5 \cdot 13$ is divisible by four primes.

Since all $k < 151$ occur here, one might guess that large a occur here with a greater frequency than is predicted by the Gauss-Kuzmin law. Not so. The latter predicts $18334815 \log(1 + 1/100001)/\log 2 \approx 265$ partial quotients $a > 10^5$, while, in fact, there are only 163. The reason, of course, is that (1) is nonetheless a quadratic surd, and therefore cannot have rational approximations that are as good as a transcendental number has. Thus, no $a > 5141757 \approx \sqrt{p}$ can occur; none can occur between 5141757 and $2570878 \approx \frac{1}{2}\sqrt{p}$; etc.

The second table—76 pages long—gives the same data for (2). Here there are 4957 different values of a; the first missing a are 1262, 1388, 1612, 1621, etc.; the largest a are $10283516 \approx 2\sqrt{p}$, $3427838 \approx 2\sqrt{p}/3$, $2056703 \approx 2\sqrt{p}/5$, etc.; and Gauss-Kuzmin again holds well for the small a.

Also deposited is the 14 page computer program (for a 360/65) that was used. Apparently, each run took 6 and a fraction minutes central processor time for the $18.33 \cdot 10^6$ partial quotients.

D. S.

1. DANIEL SHANKS, "The infrastructure of a real quadratic field and its applications," *Proceedings of the* 1972 *Number Theory Conference*, Boulder, Colorado, 1972, pp. 217–224.
2. DANIEL SHANKS, "Five number-theoretic algorithms," *Proceedings of the Second Manitoba Conference on Numerical Mathematics*, 1972, Winnipeg, Manitoba, Canada, pp. 51–70.

12 [9].—HORST G. ZIMMER, *Computational Problems, Methods, and Results in Algebraic Number Theory*, Lecture Notes in Math., No. 262, Springer-Verlag, Berlin, Heidelberg, New York, 1972, 103 pp., 25 cm. Price D.M. 16,—.

This is an excellent survey of problems, methods of computation, and tabular matter in the general area of algebraic number theory. There are 12 chapters, headed: Finite Fields, Factorization of Polynomials, Galois Groups, Continued Fractions, Field Extensions, Modules and Orders, Products of Linear Forms, Units in Algebraic Number Fields, Class Numbers of Algebraic Number Fields, Class Groups and Class Fields of Algebraic Number Fields, Diophantine Equations, and the Hasse Principle for Cubic Surfaces. Each chapter contains a concise theoretical discussion of the relevant subject matter, a description of the main computational algorithms required, a statement of the significant problems in the area, and a wealth of information on tabular material and results obtained by computation. There is an extensive bibliography containing 408 items, some of which are manuscripts yet unpublished. Although no claim to completeness is made by the author (indeed, the subject matter grows too fast to allow any such claim) the material presented is comprehensive, important, and arranged in an attractive and easily assimilated manner. The author has performed a valuable service to the mathematical community in producing this compilation, and it should retain its interest for a long time to come.

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13 [12].—JONATHAN L. GROSS & WALTER S. BRAINERD, *Fundamental Programming Concepts*, Harper & Row, Publishers, New York, 1972, x + 304 pp., 24 cm. Price $8.95.

This book is intended for use as a text in an introductory computer programming course, probably for those outside the field of computer science. It uses the BASIC programming language. About the first fifth of the book is concerned with teaching BASIC itself, and the remainder is concerned with various applications.

The most appealing aspect of this text is the wide range of interesting examples that the authors have chosen. There are chapters devoted to information processing, character manipulation, simple numerical methods, simulation using Monte Carlo methods, solution of puzzles, and artificial intelligence. Each of these chapters contains sample programs for several different problems, as well as a good collection of exercises. The variety and imaginativeness of these problems is illustrated by the chapter on simulation, which contains the following programs: simulation of pollution level in a lake; results of two different betting policies at roulette; waiting times for tellers at a bank; political polling; and a search problem in archaeology. Typical programs for these problems consist of about 25 BASIC statements, which should indicate how difficult these problems are.

The book is rather fast-paced, and I suspect that the typical student in a programming course for non-majors would find it difficult. The explanation of BASIC, in particular, has very little redundancy in it, and would not be suitable for self-study. At the same time, the range of topics covered, the lack of depth of the programming section, and the choice of BASIC as a language would make this book unsuitable for computer science majors.

I noticed that one of the sample programs includes the use of a computed GO TO statement, though this statement is never described in the book. This creates the suspicion that there are other such difficulties. There is a chapter at the end on machine language, compilers, and similar topics. This chapter seems rather badly written and hard to follow; the explanation of compilers and operating systems is so abbreviated as to be virtually worthless.

This book would be appropriate for a class of interested students and an instructor inclined to provide a great deal of extra explanation. For a less interested class (for instance, a required course) or for an instructor who wanted to use a text more intensively, I would not recommend the book.

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14 [12].—RALPH E. GRISWOLD, *The Macro Implementation of SNOBOL4*, W. H. Freeman and Co., San Francisco, Calif., 1972, xii + 310 pp., 26 cm. Price $14.95.

A case study of machine-independent software development.

SNOBOL is a computer programming language designed for the manipulation of symbolic, as contrasted to numeric, data, and character strings in particular. It is by far the best and most popular language of its type, and has been implemented on many different machines. SNOBOL originated in 1963, and SNOBOL4 is the present version of it. For a long time there has been interest in the approach used in implementing SNOBOL4, and this book explicates that approach.

As the author points out, there is a scarcity of books describing implementations of programming languages in depth. This book is a fine example of how such a description ought to be written down. It is a hazardous subject, for a writer can easily be bogged down in the details without elucidating the larger themes; or, conversely, a writer may avoid the details only to have the reader forever asking, "How did he *really* do it?" Implementations have a way of becoming less and less coherent as they become encrusted with extensions, and as the implementors seek to compensate for early faults of judgment. Thus the object being described is far more complex and less rational than one would like it to be.

This book is both clear and well-organized, and contains a variety of material. The coherence of the presentation is remarkable; the logical development of concepts and techniques is worthy of a mathematics text. The book is divided into four parts: the SNOBOL4 language, the organization of the system, the language in which the implementation was done, and an overview of the results. The most interesting and challenging section is the one on the system organization; the section on the implementation language is more detailed and of less general interest, though I would not quarrel with the decision to include this material.

The opening section contains some background material about SNOBOL4, a condensed exposition of the language itself, and a brief discussion of those language features that influence the implementation. The exposition of the language manages to be concise without being obscure; someone not familiar with SNOBOL4 but with a good background in programming should be able to learn SNOBOL4 from this exposition without much trouble.

The section on system organization opens with a brief overview and then proceeds to a discussion of the elementary data structures used throughout the system. A discussion of the interpretive mechanism follows, and an understanding of these two issues, data structure and interpretation, is the foundation on which the rest of the book is based. The remainder of the section contains no major surprises, but does cover rather thoroughly the various features of the language and the way they are realized. The discussion of pattern matching is particularly interesting, as the SNOBOL4 pattern is the single most novel and significant concept in the language. For someone with no experience in compiler construction, this section provides not only a presentation of the solution to various problems, but an understanding of how language leads to an implementation design.

The third section discusses the implementation language, which is actually a collection of (more or less) machine-independent macros. The system is written in terms of these macros, and the macros are then defined for each machine on which SNOBOL4 is implemented. There is a discussion of the implementation on two machines: the IBM 360 and the CDC 6000 series, with some comparisons between them. Sample macro definitions are given in machine language. This section probably will not be read closely by anyone other than a prospective SNOBOL4 implementor, though a casual reading is worthwhile even for the nonspecialist.

The concluding section is entitled "Retrospect and Prospect", and includes both a history of the project and an appraisal of the present status of the language. The problems that have been encountered with the implementation are discussed frankly, though the reader may not be made aware of the seriousness of some of these problems. The inefficiencies of SNOBOL4 are severe enough so as to have discouraged many users, and indeed, there is a case reported in the literature where a compiler bootstrap, done with SNOBOL4, had to be abandoned because of the severity of these problems. My personal experience also verifies this difficulty. The section ends with a discussion of the SPITBOL compiler and some thoughts about a SNOBOL machine.

The book is generally free of errors and has been well laid out. A distinct typeface, without proportional spacing, is used for all fragments of computer programs, a practice that ought to be followed more widely than it is. There are interesting exercises scattered throughout the book.

I heartily recommend this book both to the experienced language implementor, for whom it will be a valuable reference work, and to the student of compilers and programming languages, for whom it will provide a clear and palatable introduction to the area.

PAUL ABRAHAMS

15 [12].—A. P. YERSHOV, Editor, *The ALPHA Automatic Programming System*, J. McWilliam, translator, Academic Press, London and New York, 1971, xi + 247 pp., 23 cm. Price $15.00 (A.P.I.C. Studies in Data Processing No. 7).

This anthology is a detailed description of a 24-pass compiler for ALPHA, the Russian dialect of Algol 60. It was written about ten years ago and translated only recently. A previous review in another journal [1] describes the limitations and operating characteristics of the machine, the compiler, and the book. The author of that review admits that he did not check all sections. Over an embarrassingly long period, I have read the entire book. Some sections are extremely difficult reading and some are just plain hard. The authors sometimes gave too much detail (like bit maps and octal constants), but the translator was more often too literal in his interpretation. Several sections refer to a "scale of $\cdots$" and it took a long time for me to guess that a scale is a vector. The most mysterious sentence in the whole book is, "It is perfectly obvious that if the PP [compiler] is always started according to a single plan, dealing with many problems will be like breaking a butterfly on the wheel." That sentence must have been in the kernel of the book (that which vanishes on translation). A smaller problem for the western reader is that he is assumed to be familiar with the ALPHA language. I believe only one other book about ALPHA has been translated into English [2]. I have not read it.

I disagree with the opening statement of the previous review. This book does not describe the development of a compiler. I wish it had. How does one successfully manage a team of eleven programmers (35 man-years) developing a large system (45,000 instructions) on a small machine (4096 words)? This aspect is mentioned briefly in the planning documents which were included as appendices—Yershov even laments the scarcity of such papers. This subject is not discussed in the post-project reports which form the main part of the book. These reports are detailed descriptions of various parts of the final ALPHA compiler.

The a priori and ad hoc limitations of the 24-pass ALPHA compiler are in several ways different from those of a one-pass compiler for a superset of Algol 60 [3] from my own experience. We felt obliged to implement recursive procedures because our additional data types were recursively defined. Ad hoc limitations arose from trying to squeeze everything into one pass. We ignored dynamic own arrays. The limitations of ALPHA, on the other hand, seem to arise from the decision to optimize as much as possible. Recursive procedures are not permitted, side effects are ignored, the value of the expression $E - E$ is not necessarily zero (even in the absence of side effects), and so on. Dynamic own arrays were implemented but never used in the first year of experimental operation of the translator. Some very fancy optimization methods were developed, like application of a heuristic solution of the generalized map coloring problem to memory optimization. (Variables are colored by the memory locations allocated for them.) No comparison of fancy methods with more ordinary methods was reported. For each method the interesting measurement would be the number of production runs required to "pay" for the incremental cost in compilation using the fancy method versus the ordinary.

Several efforts in this country are related to concepts developed in the ALPHA translator. The relationship might have been ancestral if we had paid more attention to the Russian efforts. Let us travel back in time to Novosibirsk of ten years ago and imagine what advice we would give as consultants to the ALPHA group.

Several improvements can be made on the hash addressing scheme used in the ALPHA translator. Perhaps the most surprising is that discovered by Brent [5]. A survey is given by Knuth in [6].

Abrams shows how many of the APL operations on arrays can be carried out on descriptions of the arrays [7]. This relates to the treatment of subscript expressions in ALPHA. Wagner takes the notion of loop fusion for componentwise array options somewhat further and describes several other algorithms for matrix multiplication, optimizing on the choice of algorithm for each product [8].

Cocke and Schwartz also develop a method for factoring common subexpressions by using hash addressing [9]. Lipovski finds largest combinable classes by representing the constraints by a boolean formula and converting it to disjunctive normal form [10]. This method might apply to memory economy.

Wegbreit's treatment of procedures allows for progressive refinement of the code as additional information is supplied by the programmer [11]. ALPHA looks at all the calls on a procedure and extracts all the information it can. I have examined the notion of call by result in terms of operator definition [12]. I give some examples which have a nonprocedural, declarative flavor.

In conclusion, the Russians of ten years ago had some interesting ideas about compilation of programs on and for small machines. The worth of the book is approximately equal to the patience it takes to read it.

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