

Mathematics of Computation

Coden: MCMPAF

Volume 28, Number 125

Pages 1—347

January 1974

Published by the American Mathematical Society

PROVIDENCE, RHODE ISLAND

Editorial Committee

- EUGENE ISAACSON, Chairman, New York University, Courant Institute of Mathematical Sciences, 251 Mercer Street, New York, New York 10012
JAMES H. BRAMBLE, Department of Mathematics, Cornell University, Ithaca, New York 14850
ALSTON S. HOUSEHOLDER, Department of Mathematics, Ayres Hall, The University of Tennessee, Knoxville, Tennessee 37916
JOHN W. WRENCH, JR., Naval Ship Research and Development Center, Bethesda, Maryland 20034

Technical Editor

- CHARLOTTE W. JOHN, New York University, Courant Institute of Mathematical Sciences, 251 Mercer Street, New York, New York 10012

Board of Associate Editors

- JAMES W. DANIEL, Department of Mathematics, University of Texas at Austin, Austin, Texas 78712
WALTER GAUTSCHI, Computer Sciences Department, Purdue University, Lafayette, Indiana 47907
DONALD GOLDFARB, Department of Computer Sciences, School of Engineering, The City College of the City University of New York, 139th Street & Convent Avenue, New York, New York 10031.
HEINZ-OTTO KREISS, Computer Science Department, University of Uppsala, Uppsala, Sturegaten 4, Sweden
YUDELL L. LUKE, Department of Mathematics, University of Missouri at Kansas City, Kansas City, Missouri 64110
JAMES N. LYNES, Argonne National Laboratory, 9700 South Cass Avenue, Argonne, Illinois 60439
BERESFORD PARLETT, Department of Computer Science, University of California, Berkeley, California 94720
PHILIP RABINOWITZ, Department of Applied Mathematics, The Weizmann Institute of Science, Rehovot, Israel
JOHN R. RICE, Division of Mathematical Sciences, Purdue University, Lafayette, Indiana 47907
DANIEL SHANKS, Naval Ship Research and Development Center, Bethesda, Maryland 20034
HANS J. STETTER, Institut für Numerische Mathematik, Technische Hochschule Wien, Karlsplatz 13, A-1040 Wien, Austria

Information for Subscribers

The journal is published quarterly in one volume per year, with issues numbered serially since Volume 1, Number 1. The subscription price is \$24.00. All back volumes are available. For Volumes 1–19 (1943–1965), prices are \$20.00 per volume; for Volumes 20–23 (1966–1969), \$24.00 per volume; for Volumes 24–26 (1970–1972), \$30.00 per volume; for Volume 27 (1973), \$36.00.

Unpublished Mathematical Tables

The editorial office of the journal maintains a repository of Unpublished Mathematical Tables (UMT). When a table is deposited in the UMT repository a brief summary of its contents is published in the section *Reviews and Descriptions of Tables and Books*. Upon request, the chairman of the editorial committee will supply copies of any table for a nominal cost per page.

Subscriptions, address changes, business communications and payments should be sent to:

AMERICAN MATHEMATICAL SOCIETY
P. O. Box 6248
Providence, Rhode Island 02904

Copyright © 1974, American Mathematical Society
Second-class postage paid at Providence, Rhode Island, and at additional mailing offices

Mathematics of Computation

A Quarterly Journal

Edited by

EUGENE ISAACSON, Chairman

JAMES H. BRAMBLE	JAMES W. DANIEL	WALTER GAUTSCHI
DONALD GOLDFARB	A. S. HOUSEHOLDER	HEINZ-OTTO KREISS
YUDELL L. LUKE	JAMES N. LYNESS	BERESFORD PARLETT
PHILIP RABINOWITZ	JOHN R. RICE	DANIEL SHANKS
HANS J. STETTER		JOHN W. WRENCH, JR.

CHARLOTTE W. JOHN, Technical Editor

XXVIII

NOS. 125–128

1974

Mathematics of Computation

TABLE OF CONTENTS

JANUARY 1974

Convergence Rates of Parabolic Difference Schemes for Non-Smooth Data VIDAR THOMÉE & LARS WAHLBIN	1
Fourth Order Difference Methods for the Initial Boundary-Value Problem for Hyperbolic Equations JOSEPH OLIGER	15
An Implicit Fourth Order Difference Method for Viscous Flows DANIEL S. WATANABE & J. RICHARD FLOOD	27
Numerical Solution of Lundquist Equations of Magnetohydrodynamics R. L. JOHNSTON & S. K. PAL	33
Numerical Solution of Plateau's Problem by a Finite Element Method MASAHIRO HINATA, MASAOKI SHIMASAKI & TAKESHI KIYONO	45
Global Solution of the Generalized Abel Integral Equation by Implicit Interpolation H. BRUNNER	61
Convergence Rates of Approximate Least Squares Solutions of Linear Integral and Operator Equations of the First Kind M. Z. NASHED & GRACE WAHBA	69
Computational Techniques Based on the Lanczos Representation J. N. LYNESS	81
On Chebyshev-Type Quadratures WALTER GAUTSCHI & HIROKI YANAGIWARA	125
A Note on the Optimal Addition of Abscissas to Quadrature Formulas of Gauss and Lobatto Type ROBERT PIESSENS & MARIA BRANDERS	135
Limiting Precision in Differential Equation Solvers L. F. SHAMPINE	141
On the Stability and Accuracy of One-Step Methods for Solving Stiff Systems of Ordinary Differential Equations A. PROTHERO & A. ROBINSON	145
A Class of A -Stable Advanced Multistep Methods JACK WILLIAMS & FRANK DE HOOG	163
Vector-Valued Approximation and Its Application to Fitting Exponential Decay Curves GENEVA G. BELFORD	179
Differences of Fractional Order J. B. DIAZ & T. J. OSLER	185
A New Algorithm for the Chebyshev Solution of Overdetermined Linear Systems PAUL T. BOGGS	203
On the Number of Nonzeros Added when Gaussian Elimination is Performed on Sparse Random Matrices I. S. DUFF	219
Triangular Factorization and Inversion by Fast Matrix Multiplication JAMES R. BUNCH & JOHN E. HOPCROFT	231
Reduction of the Eigenproblem for Hermitian Persymmetric Matrices MARVIN J. GOLDSTEIN	237
Computation of Modified Bessel Functions and Their Ratios . . D. E. AMOS	239
Inequalities for Modified Bessel Functions INGEMAR NÄSELL	253

Some Integrals Involving Associated Legendre Functions . .	S. N. SAMADDAR	257
Reduction Formulas for Multiple Series	M. L. GLASSER	265
Applications of a Continued Fraction Algorithm to Some Class Number Problems	M. D. HENDY	267
On the 3-Rank of Quadratic Fields and the Euler Product	CAROL NEILD & DANIEL SHANKS	279
Brun's Constant	DANIEL SHANKS & JOHN W. WRENCH, JR.	293
Iterates of the Unitary Totient Function	MOHAN LAL	301
A Note on Congruent Numbers	RONALD ALTER & THADDEUS B. CURTZ	303
Some Factorizations of $10^n \pm 1$	I. O. ANGELL & H. J. GODWIN	307
Four Large Amicable Pairs	H. J. J. TE RIELE	309
12,758	ROBERT E. DRESSLER & THOMAS PARKER	313
The Distribution of Small Gaps Between Successive Primes	RICHARD P. BRENT	315
REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS		325
BECKMANN 1, BRENT 7, COHEN, CUTTS, FIELDER, JONES, RIBBANS & STUART 2, GRISWOLD 14, GROSS & BRAINERD 13, KARST 8, LING 4, NEWMAN 5, RUBIN 10, SCHWARZ, RUTISHAUSER & STIEFEL 3, SHANKS & NIELD 9, SPIRA 6, WILLIAMS, HENDERSON & WRIGHT 11, ZIMMER 12, YERSHOV, EDITOR 15.		
TABLE ERRATA		
DE MORGAN 508, YEUNG & LING 509.		
CORRIGENDUM		343
KARST		
SUPPLEMENT TO		
A Note on the Optimal Addition of Abscissas to Quadrature Formulas of Gauss and Lobatto Type . . ROBERT PIESSENS & MARIA BRANDERS pp. 135–139, this issue		

Information for Contributors

Manuscripts should be typewritten double-spaced in the format used by the journal. For journal abbreviations, see the latest *Mathematical Reviews* volume index. An author should submit the original and one copy of the manuscript and retain one copy. The author may suggest an appropriate editor for his paper. It is recommended that the author acquaint himself with the pertinent material contained in "Information for Contributors to Mathematics of Computation" and "Manual for Authors," both of which are available upon request from the American Mathematical Society. All contributions intended for publication and all books for review should be addressed to Eugene Isaacson, Chairman, Editorial Committee, Mathematics of Computation, New York University, Courant Institute of Mathematical Sciences, 251 Mercer Street, New York, New York 10012. Institutions sponsoring research reported in the journal are assessed page and microfiche charges.

Each article submitted for publication must be accompanied by a brief and reasonably self-contained abstract, and by AMS (MOS) subject classification numbers. If a list of key words and phrases is included, it will be printed as a footnote on the first page. A list of the classification numbers may be found in the Index to Mathematical Reviews, Volume 39 (June 1970).

Microcard Edition

Volumes 1–14 (1943–1960) are available on Microcards at \$39.00 for the complete set and may be purchased from Microcard Editions, Inc., 901 26th Street, N. W., Washington, D.C. 20037.

Cumulative Index to Mathematics of Computation

The CUMULATIVE INDEX TO MATHEMATICS OF COMPUTATION is a compilation, by author and subject, of all material which has appeared in *Mathematics of Computation* and its predecessor, *Mathematical Tables and Other Aids to Computation*, during the years 1943 through 1969 (twenty-three published volumes). The INDEX contains over 7,000 entries.

This is an unusual publication since the character of the journal is unique, not only publishing research papers, but also reviews of material on mathematics of computation and a table errata section which covers a number of other publications. In addition an unpublished mathematical tables' file is maintained.

The new classification system developed in 1969 is used in the SUBJECT INDEX to classify all research papers, tables, reviews, table errata, queries and replies, and microfiche.

The AUTHOR INDEX gives bibliographical information on all of the items published in the journal.

462 pages; list price \$19.95; institutional member price \$14.96;
individual member price \$9.98;

ISBN 0-8218-4000-2; to order, please specify MCOMIN/1

Please send orders to
AMERICAN MATHEMATICAL SOCIETY
P. O. Box 6248
Providence, Rhode Island 02904

Some Integrals Involving Associated Legendre Functions . .	S. N. SAMADDAR	257
Reduction Formulas for Multiple Series	M. L. GLASSER	265
Applications of a Continued Fraction Algorithm to Some Class Number Problems	M. D. HENDY	267
On the 3-Rank of Quadratic Fields and the Euler Product	CAROL NEILD & DANIEL SHANKS	279
Brun's Constant	DANIEL SHANKS & JOHN W. WRENCH, JR.	293
Iterates of the Unitary Totient Function	MOHAN LAL	301
A Note on Congruent Numbers	RONALD ALTER & THADDEUS B. CURTZ	303
Some Factorizations of $10^n \pm 1$	I. O. ANGELL & H. J. GODWIN	307
Four Large Amicable Pairs	H. J. J. TE RIELE	309
12,758	ROBERT E. DRESSLER & THOMAS PARKER	313
The Distribution of Small Gaps Between Successive Primes	RICHARD P. BRENT	315
REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS		325
BECKMANN 1 , BRENT 7 , COHEN, CUTTS, FIELDER, JONES, RIBBANS & STUART 2 , GRISWOLD 14 , GROSS & BRAINERD 13 , KARST 8 , LING 4 , NEWMAN 5 , RUBIN 10 , SCHWARZ, RUTISHAUSER & STIEFEL 3 , SHANKS & NIELD 9 , SPIRA 6 , WILLIAMS, HENDERSON & WRIGHT 11 , ZIMMER 12 , YERSHOV, EDITOR 15 .		
TABLE ERRATA		
DE MORGAN 508 , YEUNG & LING 509 .		
CORRIGENDUM		343
KARST		
SUPPLEMENT TO		
A Note on the Optimal Addition of Abscissas to Quadrature Formulas of Gauss and Lobatto Type . .	ROBERT PIESSENS & MARIA BRANDERS	
pp. 135–139, this issue		344

Mathematics of Computation

TABLE OF CONTENTS

JANUARY 1974

Convergence Rates of Parabolic Difference Schemes for Non-Smooth Data VIDAR THOMÉE & LARS WAHLBIN	1
Fourth Order Difference Methods for the Initial Boundary-Value Problem for Hyperbolic Equations JOSEPH OLIGER	15
An Implicit Fourth Order Difference Method for Viscous Flows DANIEL S. WATANABE & J. RICHARD FLOOD	27
Numerical Solution of Lundquist Equations of Magnetohydrodynamics R. L. JOHNSTON & S. K. PAL	33
Numerical Solution of Plateau's Problem by a Finite Element Method MASAHIRO HINATA, MASAACKI SHIMASAKI & TAKESHI KIYONO	45
Global Solution of the Generalized Abel Integral Equation by Implicit Interpolation H. BRUNNER	61
Convergence Rates of Approximate Least Squares Solutions of Linear Integral and Operator Equations of the First Kind M. Z. NASHED & GRACE WAHBA	69
Computational Techniques Based on the Lanczos Representation J. N. LYNESS	81
On Chebyshev-Type Quadratures WALTER GAUTSCHI & HIROKI YANAGIWARA	125
A Note on the Optimal Addition of Abscissas to Quadrature Formulas of Gauss and Lobatto Type ROBERT PIESSENS & MARIA BRANDERS	135
Limiting Precision in Differential Equation Solvers L. F. SHAMPINE	141
On the Stability and Accuracy of One-Step Methods for Solving Stiff Systems of Ordinary Differential Equations A. PROTHERO & A. ROBINSON	145
A Class of A -Stable Advanced Multistep Methods JACK WILLIAMS & FRANK DE HOOG	163
Vector-Valued Approximation and Its Application to Fitting Exponential Decay Curves GENEVA G. BELFORD	179
Differences of Fractional Order J. B. DIAZ & T. J. OSLER	185
A New Algorithm for the Chebyshev Solution of Overdetermined Linear Systems PAUL T. BOGGS	203
On the Number of Nonzeros Added when Gaussian Elimination is Performed on Sparse Random Matrices I. S. DUFF	219
Triangular Factorization and Inversion by Fast Matrix Multiplication JAMES R. BUNCH & JOHN E. HOPCROFT	231
Reduction of the Eigenproblem for Hermitian Persymmetric Matrices MARVIN J. GOLDSTEIN	237
Computation of Modified Bessel Functions and Their Ratios . . D. E. AMOS	239
Inequalities for Modified Bessel Functions INGEMAR NÅSELL	253