

# Mathematics of Computation

**Coden: MCMPAF**

**Pages 349—677**

**Volume 28, Number 126**

**April 1974**

*Published by the American Mathematical Society*

PROVIDENCE, RHODE ISLAND



### Editorial Committee

- EUGENE ISAACSON, Chairman, New York University, Courant Institute of Mathematical Sciences, 251 Mercer Street, New York, New York 10012  
JAMES H. BRAMBLE, Department of Mathematics, Cornell University, Ithaca, New York 14850  
ALSTON S. HOUSEHOLDER, Department of Mathematics, Ayres Hall, The University of Tennessee, Knoxville, Tennessee 37916  
JOHN W. WRENCH, JR., Naval Ship Research and Development Center, Bethesda, Maryland 20034

### Technical Editor

- CHARLOTTE W. JOHN, New York University, Courant Institute of Mathematical Sciences, 251 Mercer Street, New York, New York 10012

### Board of Associate Editors

- JAMES W. DANIEL, Department of Mathematics, University of Texas at Austin, Austin, Texas 78712  
WALTER GAUTSCHI, Computer Sciences Department, Purdue University, Lafayette, Indiana 47907  
DONALD GOLDFARB, Department of Computer Sciences, School of Engineering, The City College of the City University of New York, 139th Street & Convent Avenue, New York, New York 10031  
HEINZ-OTTO KREISS, Computer Science Department, University of Uppsala, Uppsala, Sturegaten 4, Sweden  
YUDELL L. LUKE, Department of Mathematics, University of Missouri at Kansas City, Kansas City, Missouri 64110  
JAMES N. LYNES, Argonne National Laboratory, 9700 South Cass Avenue, Argonne, Illinois 60439  
BERESFORD PARLETT, Department of Computer Science, University of California, Berkeley, California 94720  
PHILIP RABINOWITZ, Department of Applied Mathematics, The Weizmann Institute of Science, Rehovot, Israel  
JOHN R. RICE, Division of Mathematical Sciences, Purdue University, Lafayette, Indiana 47907  
DANIEL SHANKS, Naval Ship Research and Development Center, Bethesda, Maryland 20034  
HANS J. STETTER, Institut für Numerische Mathematik, Technische Hochschule Wien, Karlsplatz 13, A-1040 Wien, Austria

### Information for Subscribers

The journal is published quarterly in one volume per year, with issues numbered serially since Volume 1, Number 1. The subscription price is \$36.00. All back volumes are available. For Volumes 1–19 (1943–1965), prices are \$20.00 per volume; for Volumes 20–23 (1966–1969), \$24.00 per volume and Volumes 24–26 (1970–1972), \$30.00 per volume; for Volume 27 (1973), \$36.00.

### Unpublished Mathematical Tables

The editorial office of the journal maintains a repository of Unpublished Mathematical Tables (UMT). When a table is deposited in the UMT repository a brief summary of its contents is published in the section *Reviews and Descriptions of Tables and Books*. Upon request, the chairman of the editorial committee will supply copies of any table for a nominal cost per page.

---

Subscriptions, address changes, business communications and payments should be sent to:

AMERICAN MATHEMATICAL SOCIETY  
P. O. Box 6248  
Providence, Rhode Island 02940

Copyright © 1974, American Mathematical Society  
Second-class postage paid at Providence, Rhode Island, and at additional mailing offices



# Mathematics of Computation

## TABLE OF CONTENTS

APRIL 1974

On Fourier-Toeplitz Methods for Separable Elliptic Problems D. FISCHER, G. GOLUB, O. HALD, C. LEIVA & O. WIDLUND	349
High-Order Finite-Difference Methods for Poisson's Equation H. J. VAN LINDE	369
Finite Element Methods for Parabolic Equations . . . . MILOŠ ZLÁMAL	393
Semidiscrete Least-Squares Methods for Second Order Parabolic Problems with Nonhomogeneous Data . . . . . J. THOMAS KING	405
Stable Approximations for Hyperbolic Systems with Moving Internal Boundary Conditions . . . . . M. GOLDBERG & S. ABARBANEL	413
The Application of Implicit Runge-Kutta and Collocation Methods to Boundary-Value Problems . . . . . RICHARD WEISS	449
Approximation by Aliasing with Application to "Certaine" Stiff Differential Equations . . . . . ARTHUR DAVID SNIDER & GARY CHARLES FLEMING	465
Recursive Collocation for the Numerical Solution of Stiff Ordinary Dif- ferential Equations . . . . . H. BRUNNER	475
On Semicardinal Quadrature Formulae I. J. SCHOENBERG & S. D. SILLIMAN	483
Quadrature Formulas for Semi-Infinite Integrals RAVINDRA KUMAR & M. K. JAIN	499
Methods for Modifying Matrix Factorizations P. E. GILL, G. H. GOLUB, W. MURRAY & M. A. SAUNDERS	505
Modifying Pivot Elements in Gaussian Elimination . . . . . G. W. STEWART	537
The Evaluation of Determinants by Expansion by Minors and the General Problem of Substitution . . . . . W. M. GENTLEMAN & S. C. JOHNSON	543
A Characterization of Superlinear Convergence and Its Application to Quasi-Newton Methods . . . . . J. E. DENNIS, JR. & JORGE J. MORÉ	549
Evaluation of a Constant Associated with a Parking Problem M. LAL & P. GILLARD	561
Splines with Nonnegative $B$ -Spline Coefficients C. DE BOOR & JAMES W. DANIEL	565
On the Conditional Equivalence of Two Starting Methods for the Second Algorithm of Remez . . . . . R. E. HUDDLESTON	569
A Note on Chambers' Method . . . . . J. A. BLACKBURN & Y. BEAUDOIN	573
Numerical Computation of a Generalized Exponential Integral Function W. F. BREIG & A. L. CROSBIE	575
Rational Chebyshev Approximations for the Modified Bessel Functions $I_0(x)$ and $I_1(x)$ . . . . . J. M. BLAIR	581
A Stable Algorithm for Computing the Inverse Error Function in the "Tail-End" Region . . . . . HENRY E. FETTIS	585
The Minimum Root Separation of a Polynomial GEORGE E. COLLINS & ELLIS HOROWITZ	589
Error Analysis of a Computation of Euler's Constant W. A. BEYER & M. S. WATERMAN	599
Confluent Expansions for Functions of Two Variables . . V. L. DESHPANDE	605
Some Definite Integrals of the Product of Two Bessel Functions of the Second Kind: (Order Zero) . . . . . M. L. GLASSER	613
On Weird and Pseudoperfect Numbers . . . . . S. J. BENKOSKI & P. ERDÖS	617



A New Factorization Technique Using Quadratic Forms	D. H. LEHMER & EMMA LEHMER	625
Factoring Large Integers	R. SHERMAN LEHMAN	637
A New Function Associated with the Prime Factors of $\binom{n}{k}$	E. F. ECKLUND, JR., P. ERDÖS & J. L. SELFRIDGE	647
Sums of Distinct Primes from Congruence Classes Modulo 12	ROBERT E. DRESSLER, ANDRZEJ MAKOWSKI & THOMAS PARKER	651
Irregular Prime Divisors of the Bernoulli Numbers	WELLS JOHNSON	653
The Character Table of an Eight-Dimensional Orthogonal Group	DAVID C. HUNT	659
The Character Table of Fischer's Simple Group, $M(23)$	DAVID C. HUNT	660
REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS		663
BAUER, GARABEDIAN & KORN 22, BEYER & WATERMAN 19, COLOMBO & LAVOINE 26, JOHN 27, KORN 28, MILLER & THATCHER, Editors 20, MITRINOVIC 23, MITRINOVIC 24, PHILLIPS & HANSON 18, REID, Editor 21, ROSE & WILLOUGHBY, Editors 21, SCHULTZ 16, STOER 17, TRANTER 25.		
TABLE ERRATA		677
LEHMER 510.		
MICROFICHE SUPPLEMENT		
Gauss Quadrature Rules with $B$ -Spline Weight Functions	JAMES L. PHILLIPS & RICHARD J. HANSON	
Some Integrals Involving the Modified Bessel Function $K_0$	M. L. GLASSER	
Tables for Numerical Computation of a Generalized Exponential Integral Function	W. F. BREIG & A. L. CROSBIE	
The Character Table of an Eight-Dimensional Orthogonal Group	DAVID C. HUNT	
The Character Table of Fischer's Simple Group, $M(23)$	DAVID C. HUNT	
Tables for Rational Chebyshev Approximations for the Modified Bessel Functions $I_0(x)$ and $I_1(x)$	J. M. BLAIR	

### Information for Contributors

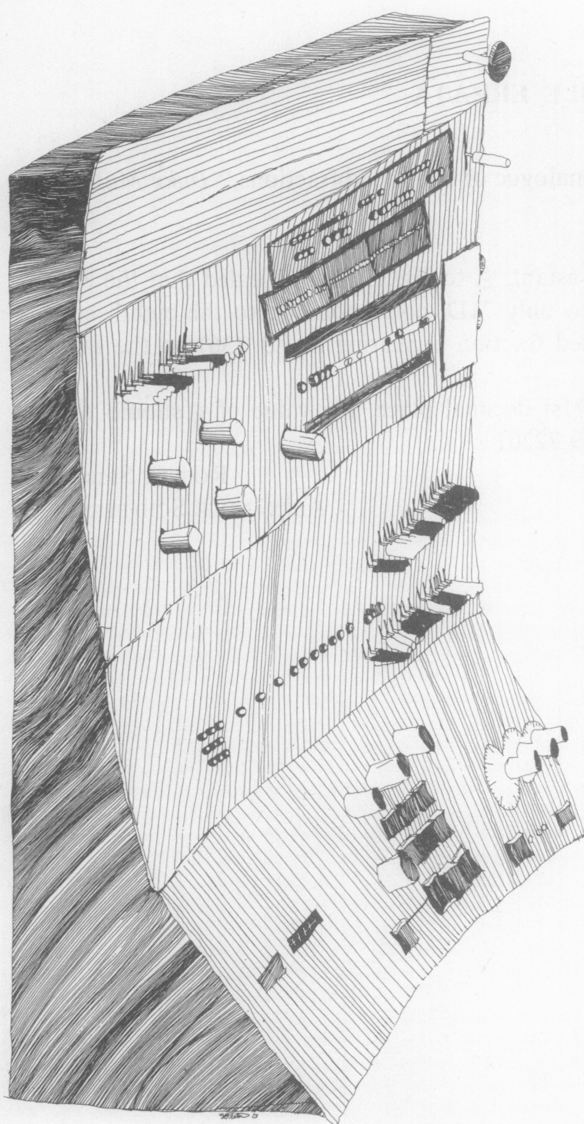
Manuscripts should be typewritten double-spaced in the format used by the journal. For journal abbreviations, see the latest *Mathematical Reviews* volume index. An author should submit the original and one copy of the manuscript and retain one copy. The author may suggest an appropriate editor for his paper. It is recommended that the author acquaint himself with the pertinent material contained in "Information for Contributors to Mathematics of Computation" and "Manual for Authors," both of which are available upon request from the American Mathematical Society. All contributions intended for publication and all books for review should be addressed to Eugene Isaacson, Chairman, Editorial Committee, Mathematics of Computation, New York University, Courant Institute of Mathematical Sciences, 251 Mercer Street, New York, New York 10012. Institutions sponsoring research reported in the journal are assessed page and microfiche charges.

Each article submitted for publication must be accompanied by a brief and reasonably self-contained abstract, and by AMS (MOS) subject classification numbers. If a list of key words and phrases is included, it will be printed as a footnote on the first page. A list of the classification numbers may be found in the Index to Mathematical Reviews, Volume 39 (June 1970).

### Microcard Edition

Volumes 1–14 (1943–1960) are available on Microcards at \$39.00 for the complete set and may be purchased from Microcard Editions, Inc., 901 26th Street, N. W., Washington, D.C. 20037.





Prentice-Hall  
offers 6  
distinguished,  
new texts on  
computer math...

*Spline Analysis*

Martin M. Schultz, Yale University

An introduction to the theory, applications and computational aspects of finite elements. 1973, 156 pp. \$10.50

*Floating-Point Computation*

Pat H. Sterbenz, INM Systems Research Institute

A detailed discussion of floating-point computation. 1974 approx. 352 pp. \$15.00

*Numerical Methods*

Germund Dahlquist, Univ. of Stockholm, Ake Bjorck, Linkoping Univ., Ned Anderson, Massachusetts Inst. of Technology

Contains an introduction to the ideas and concepts of numerical analysis. 1974 approx. 576 pp. \$14.50

*Graph Theory with Applications to Engineering Science*

Narsingh Deo, Indian Inst. of Technology, Kanpur, India

Offers complete treatment of the fundamentals of graph theory, emphasizing graph-theoretic algorithms. 1974 approx. 700 pp. \$14.50

*An Analysis of the Finite Element Method*

Gilbert Strang, Massachusetts Institute of Technology and George Fix, University of Maryland

Details the finite element method—developed by civil and aerospace engineers for the numerical solution of structural problems. 1973 approx. 320 pp. \$16.00

*Computer Approaches to Mathematical Problems*

Jurg Nievergelt, Univ. of Illinois, Urbana-Champaign, J. Craig Farrar Ohio Univ. and Edward M. Reingold, Univ. of Illinois, Urbana-Champaign

Focuses on problems that require mathematical concepts for their formulation, and computer techniques for their practical solution. 1974 approx. 272 pp. \$8.95

For further information write, Robert Jordan, Dept. J-916, Prentice-Hall, Englewood Cliffs, N.J. 07632.

Art by Stephen Frimm.



A New Factorization Technique Using Quadratic Forms	D. H. LEHMER & EMMA LEHMER	625
Factoring Large Integers	R. SHERMAN LEHMAN	637
A New Function Associated with the Prime Factors of $\binom{n}{k}$	E. F. ECKLUND, JR., P. ERDÖS & J. L. SELFRIDGE	647
Sums of Distinct Primes from Congruence Classes Modulo 12	ROBERT E. DRESSLER, ANDRZEJ MAKOWSKI & THOMAS PARKER	651
Irregular Prime Divisors of the Bernoulli Numbers	WELLS JOHNSON	653
The Character Table of an Eight-Dimensional Orthogonal Group	DAVID C. HUNT	659
The Character Table of Fischer's Simple Group, $M(23)$	DAVID C. HUNT	660
REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS		663
BAUER, GARABEDIAN & KORN 22, BEYER & WATERMAN 19, COLOMBO & LAVOINE 26, JOHN 27, KORN 28, MILLER & THATCHER, Editors 20, MITRINOVIC 23, MITRINOVIC 24, PHILLIPS & HANSON 18, REID, Editor 21, ROSE & WILLOUGHBY, Editors 21, SCHULTZ 16, STOER 17, TRANTER 25.		
TABLE ERRATA		677
LEHMER 510.		
MICROFICHE SUPPLEMENT		
Gauss Quadrature Rules with $B$ -Spline Weight Functions	JAMES L. PHILLIPS & RICHARD J. HANSON	
Some Integrals Involving the Modified Bessel Function $K_0$	M. L. GLASSER	
Tables for Numerical Computation of a Generalized Exponential Integral Function	W. F. BREIG & A. L. CROSBIE	
The Character Table of an Eight-Dimensional Orthogonal Group	DAVID C. HUNT	
The Character Table of Fischer's Simple Group, $M(23)$	DAVID C. HUNT	
Tables for Rational Chebyshev Approximations for the Modified Bessel Functions $I_0(x)$ and $I_1(x)$	J. M. BLAIR	

---

The editorial committee would welcome readers' comments about this microfiche feature. Please send comments to Professor Eugene Isaacson, MATHEMATICS OF COMPUTATION, Courant Institute of Mathematical Sciences, New York University, 251 Mercer Street, New York, New York 10012.



# Mathematics of Computation

## TABLE OF CONTENTS

APRIL 1974

On Fourier-Toeplitz Methods for Separable Elliptic Problems D. FISCHER, G. GOLUB, O. HALD, C. LEIVA & O. WIDLUND	349
High-Order Finite-Difference Methods for Poisson's Equation H. J. VAN LINDE	369
Finite Element Methods for Parabolic Equations . . . . MILOŠ ZLÁMAL	393
Semidiscrete Least-Squares Methods for Second Order Parabolic Problems with Nonhomogeneous Data . . . . . J. THOMAS KING	405
Stable Approximations for Hyperbolic Systems with Moving Internal Boundary Conditions . . . . . M. GOLDBERG & S. ABARBANEL	413
The Application of Implicit Runge-Kutta and Collocation Methods to Boundary-Value Problems . . . . . RICHARD WEISS	449
Approximation by Aliasing with Application to "Certaine" Stiff Differential Equations . . . . . ARTHUR DAVID SNIDER & GARY CHARLES FLEMING	465
Recursive Collocation for the Numerical Solution of Stiff Ordinary Dif- ferential Equations . . . . . H. BRUNNER	475
On Semicardinal Quadrature Formulae I. J. SCHOENBERG & S. D. SILLIMAN	483
Quadrature Formulas for Semi-Infinite Integrals RAVINDRA KUMAR & M. K. JAIN	499
Methods for Modifying Matrix Factorizations P. E. GILL, G. H. GOLUB, W. MURRAY & M. A. SAUNDERS	505
Modifying Pivot Elements in Gaussian Elimination . . . . . G. W. STEWART	537
The Evaluation of Determinants by Expansion by Minors and the General Problem of Substitution . . . . . W. M. GENTLEMAN & S. C. JOHNSON	543
A Characterization of Superlinear Convergence and Its Application to Quasi-Newton Methods . . . . . J. E. DENNIS, JR. & JORGE J. MORÉ	549
Evaluation of a Constant Associated with a Parking Problem M. LAL & P. GILLARD	561
Splines with Nonnegative $B$ -Spline Coefficients C. DE BOOR & JAMES W. DANIEL	565
On the Conditional Equivalence of Two Starting Methods for the Second Algorithm of Remez . . . . . R. E. HUDDLESTON	569
A Note on Chambers' Method . . . . . J. A. BLACKBURN & Y. BEAUDOIN	573
Numerical Computation of a Generalized Exponential Integral Function W. F. BREIG & A. L. CROSBIE	575
Rational Chebyshev Approximations for the Modified Bessel Functions $I_0(x)$ and $I_1(x)$ . . . . . J. M. BLAIR	581
A Stable Algorithm for Computing the Inverse Error Function in the "Tail-End" Region . . . . . HENRY E. FETTIS	585
The Minimum Root Separation of a Polynomial GEORGE E. COLLINS & ELLIS HOROWITZ	589
Error Analysis of a Computation of Euler's Constant W. A. BEYER & M. S. WATERMAN	599
Confluent Expansions for Functions of Two Variables . . V. L. DESHPANDE	605
Some Definite Integrals of the Product of Two Bessel Functions of the Second Kind: (Order Zero) . . . . . M. L. GLASSER	613
On Weird and Pseudoperfect Numbers . . . . . S. J. BENKOSKI & P. ERDÖS	617