REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS

The numbers in brackets are assigned according to the indexing system printed in Volume 22, Number 101, January 1968, page 212.

29 [**2.05, 2.05.6, 6**].—P. L. BUTZER, J.-P. KAHANE & B. SZ.-NAGY, Editors, *Linear Operators and Approximation,* ISNM vol. 20, Birkhäuser Verlag, Basel, Switzerland, 1972, 506 pp., 25 cm. Price sfr.84.—.

This book contains the lectures presented at the conference on *Linear Operators and Approximation* held at the Oberwolfach Mathematical Research Institute, August 14–22, 1971. Four papers are included in addition to the 38 lectures presented. The fly leaf states that the book's goal is to "elucidate the actual state of research in the vast field of approximation and related topics", but the book's title better indicates the main thrust of the book. Thus, one does obtain a good sampling of current (in 1971) research in the area described by the key words: approximation theory, functional analysis and operator theory.

The number of papers and their length preclude individual mention of the papers. The average is at the high quality that one expects from an Oberwolfach conference. The papers are divided into five sections as follows:

1. Operator Theory (9 papers)
2. Topics in Functional Analysis (8 papers)
3. Approximation in Abstract Spaces (9 papers)
4. Harmonic Analysis and Approximation (7 papers)
5. Spline and Algebraic Approximation (9 papers).

There is a large and interesting set of new and unsolved problems which were collected during the conference.

The quality of production is high with few misprints although there is some puzzling use of small type on pages 475–477.

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30 [**2.05, 2.10, 2.15, 2.25, 2.30, 2.35, 4, 13.05, 13.15, 13.35**].—P. R. GRAVES-MORRIS, Editor, *Padé Approximants and their Applications,* Proceedings of a Conference held at the University of Kent, Canterbury, England, July 17–21, 1972, Academic Press, London and New York, 1973, x + 354 pp., 23 cm. Price £5.50.

Padé approximants are a two-dimensional array of rational functions which are associated with a given power series by requiring that their Maclaurin expansions agree with the given series to as many terms as possible. Approximations of this type are encountered in many branches of classical analysis: the theory of meromorphic functions, continued fractions, the moment problem, operator theory, etc. The subject, for a long time exclusive hunting-ground for analysts, has recently attracted the attention not only of numerical analysts, but also of research workers in applied mathematics and the physical sciences. To numerical analysts the theory of Padé approximation has become of interest primarily as a source of useful approximation techniques. Beyond this, the theory has important bearings on nonlinear convergence acceleration procedures, e.g., the epsilon algorithm, and on stability theory for discretization methods in differential equations (a topic not covered in these proceedings). For theoretical physicists and chemists, Padé methods provide a means of extracting maximum information from the series expansions which they encounter. In statistical mechanics, e.g., they are used to predict the singular behavior of thermodynamic functions. Given this new activity in an old field, inevitably, attempts are being made at generalizing the basic idea of Padé approximation. Some of these consider series expansions other than power series (e.g., series in Legendre polynomials), others go beyond the boundaries of rational approximation, seeking approximants in the form of solutions of quadratic (or higher-degree) equations with polynomial coefficients. Still others use the orthogonality concept as their point of departure.

In the summer of 1972, an international symposium on Padé approximants was held at the University of Kent in England, with the purpose of bringing together people representing all these diverse points of view. The symposium was divided into a "School" and a "Conference". The "School" involved a number of introductory tutorial lectures, which are to be published separately by the Institute of Physics. The "Conference", on the other hand, consisted of a series of technical papers describing recent and current progress in the field. The volume under review is the proceedings of this conference. The contributions have largely the character of brief expository reviews. They are grouped into five sections, the first two containing papers on mathematical and numerical aspects, the remaining three on applications to theoretical physics and simulation and control theory. Unfortunately, no index of any kind is provided.

The individual authors and their titles are as follows.

I. Mathematical Properties of Padé Approximants

J. L. Gammel, Review of two recent generalizations of the Padé approximant

J. S. R. Chisholm, Convergence properties of Padé approximants

W. J. Thron, Recent approaches to convergence theory of continued fractions

J. Nuttall, Variational principles and Padé approximants

D. Masson, Padé approximants and Hilbert spaces

R. C. Johnson, Alternative approach to Padé approximants
J. Fleischer, Nonlinear Padé approximants for Legendre series
F. V. Atkinson, Orthogonal polynomials and lacunary approximants

II. Numerical Analysis and Numerical Methods
George A. Baker, Jr., Recursive calculation of Padé approximants
P. J. S. Watson, Algorithms for differentiation and integration
Jacek Gilewicz, Numerical detection of the best Padé approximant and determination of the Fourier coefficients of the insufficiently sampled functions
A. C. Genz, Applications of the ϵ-algorithm to quadrature problems
W. B. Gragg, On Hadamard's theory of polar singularities
William B. Jones, Truncation error bounds for continued fractions and Padé approximants
I. M. Longman, Use of Padé table for approximate Laplace transform inversion
A. Ronveaux, Padé approximant and homographic transformation of Riccati's phase equations
V. Zakian, Properties of I_{MN} approximants

III. Critical Phenomena and Padé Approximants
George A. Baker, Jr., Generalised Padé approximant bounds for critical phenomena
Michael E. Fisher, Critical phenomena—series expansions and their analysis
G. S. Joyce and A. J. Guttmann, A new method of series analysis
C. Isenberg, A comparison of the vibrational properties of H.C.P. and F.C.C. crystals
F. Harbus and H. E. Stanley, Ising model antiferromagnets with tricritical points
F. Harbus, R. Krasnow, D. Lambeth, L. Liu and H. E. Stanley, Ising, planar and Heisenberg models with directional anisotropy
Sava Milosevic, Calculation of the equation of state near the critical point for the Heisenberg model using Padé approximants

IV. Atomic, Nuclear and Particle Physics and Padé Approximants
A. K. Common, Applications of the moment problem
C. López and F. J. Ynduráin, The moment problem and stable extrapolations with an application to forward Kp dispersion relations
J. A. Tjon, Application of Padé approximants in the three-body problem
C. R. Garibotti, Padé approximants in potential scattering
P. R. Graves-Morris and J. F. Rennison, Padé approximants and the Lippmann Schwinger equation
D. Bessis, Padé approximants in quantum field theory
M. Pusterla, Model field theories and Padé approximants
G. Turchetti, Padé approximants in nucleon-nucleon dynamics

V. SIMULATION AND CONTROL

M. I. Sobhy, Applications of Padé approximants in electrical network problems

J. B. Knowles, A. B. Keats and D. W. Leggett, The simulation of a continuously variable transport delay

Y. Shamash, Approximation of linear time-invariant systems

S. C. Chuang, Frequency domain approximation technique for optimal control

H. P. Debart, A Padé Chebyshev approximation in network theory

The reader interested in the applications of Padé approximation to theoretical physics may also wish to consult [1], which contains another recent cross-section of work in this field.

W. G.

1. G. A. BAKER, JR. & J. L. GAMMEL, Editors, *The Padé Approximant in Theoretical Physics*, Academic Press, New York and London, 1970.

31 [2.05, 2.35, 3.25, 13.35]. —L. COLLATZ & W. WETTERLING, Editors, *Numerische Methoden bei Optimierungsaufgaben* (in German and English), International Series of Numerical Mathematics, vol. 17, Birkhäuser Verlag, Basel, 1973, 136 pp., 25 cm. Price: approximately \$10.—.

Proceedings of a conference on Numerical Methods for Optimization Problems which was held November 14—20, 1971, in Oberwolfach, Germany. In addition to the papers appearing in the book, the following are given in the foreword as the most important points raised in discussions among the participants:

1. Many of the familiar methods used in optimization problems, being frequently the developments of those not oriented toward numerical analysis or computation, must be more closely examined than they have been in the past for their numerical fitness, and improved, if necessary.

2. In iterative methods, the determination of an initial approximation is frequently more difficult than the execution of the method itself. This should be taken into consideration when numerical algorithms are developed.

3. Several new (asymptotic) methods for integer programming have appeared. Still, the standard difficulty remains that their computational complexity is not constrained by a bound depending only upon the dimension of the problem.

4. New numerical methods which have been developed at universities are frequently unusable for the practitioner because the originators have not tested their methods sufficiently on applications problems and, consequently, cannot give adequate directions for their employment. This deficiency has to be remedied.

Papers presented were:

1. Bereanu, B.: "The Cartesian Integration Method in Stochastic Linear Programming."

A computational method for finding the expected value and the probability-distribution function for the optima of stochastic linear programs is proposed when the constraint matrix, right-hand side, and cost vector are affine functions of a random vector sampled from an r-dimensional compact interval and when certain other assumptions hold.

2. Collatz, L.: "Anwendungen der Dualitaet der Optimierungstheorie auf nichtlineare Approximationsaufgaben [Applications of Duality in Optimization Theory to Nonlinear Approximation Problems]."

Given the space $C(B)$ of continuous functions over $B \subseteq R^n$ (compact), some $f \in C(B)$, and a subspace of functions $w(x, \alpha)$ in $C(B)$ which depend upon parameters $\alpha = (\alpha_1, \cdots, \alpha_p)$, consider the primal problem: α, δ_1, δ_2 so that $-\delta_1 \leqq w(x, \alpha) \leqq \delta_2$ for all x in B and $\delta_1 + \delta_2$ is minimal. This problem generalizes that of Chebyshev approximation. Its dual is: find linear, bounded, nonnegative functionals l_1, l_2 on $C(B)$ so that $l_2(w) - l_1(w)$ is nonnegative for all $w(x, \alpha)$, so that $l_2(1) = l_1(1) = 1$, and so that $l_1(f) - l_2(f)$ is maximal. The application of a weak duality theorem to obtain bounds on $\delta_1 + \delta_2$ is described via some simple examples.

3. Eckhardt, U.: "Iterative Loesung linearer Ungleichungssysteme [Iterative Solution of Linear Inequality Systems]."

For M a given subset of a Hilbert space H, K the convex hull of M, and b a given point in H, an algorithm is sketched which attempts to generate a sequence in K converging to b. Should b not lie in the closure of K, a point $\hat{x}$ is produced so that the inner product $\langle \hat{x}, y - b \rangle$ is positive for all y in M. This algorithm is applied to get derivative-free truncation-error expressions for numerical quadrature formulas with positive weights. The algorithm is imprecisely defined in the paper, but reference is given to a technical report which contains further details.

4. Fleischmann, B.: "Eine primale Version des Benders'schen Dekompositionsverfahrens und seine Anwendung in der gemischt-ganzzahligen Optimierung [A Primal Version of Benders' Decomposition Algorithm and Its Application in Mixed Integer Programming]."

The problem in question is

$$C^T x + f(y) = \max, \qquad Ax + F(y) \leqq b, \qquad y \in S.$$

Benders splits this problem into a sequence of alternately linear programming problems involving x and optimization problems over S. These "S-problems" are solved by a dual method. Difficulties with this approach are pointed out, and a modification is given defining the S-problems so that a primal method can reasonably be applied. Some numerical results are presented.

5. Glashoff, K.: "Schwache Stetigkeit bei nichtlinearen Kontrollproblemen [Weak Continuity in Nonlinear Control Problems]."

A class of nonlinear control problems is presented

$$\underset{u \in Q}{\text{minimize}}\, C(u)$$

where Q (the set of permissible control functions) is a subset of a certain Hilbert space and c is a certain type of control functional. The theorem that a functional which is weakly semicontinuous from below on a Hilbert space takes on its minimum over weakly compact subsets of the space is used to give existence results. Conditions on control problems of the given class which guarantee the appropriate compactness and continuity are discussed.

6. Gustafson, S. A.: "Die Berechnung von verallgemeinerten Quadraturformeln vom Gaussschen Typus, eine Optimierungsaufgabe [The Calculation of Generalized Quadrature Formulas of Gaussian Type, an Optimization Problem]."

A number of examples involving the approximation of functionals over a class of absolutely monotone functions are presented. Nonlinear systems are encountered, the solution of which motivates the discussion of an extremal property connected with the moment cone of Chebyshev systems. Some numerical results are presented.

7. Krabs, W.: "Stetigkeitsfragen bei der Diskretisierung konvexer Optimierungsprobleme [Questions of Continuity Connected with the Discretization of Convex Optimization Problems]."

For X a convex subset of a normed linear space E and for Z a normed vector space partially ordered by the cone Y, the problem considered is: find $\inf f(x)$ subject to $x \in X$ and $g(x) \in Y$. Here $f: E \to \mathbf{R}$ is convex and $g: E \to Z$ is concave. General sequences of approximating problems involving sets X_m and functions g_m and f_m are considered, and some results concerning the convergence of the solutions for these problems are presented. A linear control example is investigated in some detail.

8. Kubik, K.: "Das Problem Slalom oder optimale Linienfuehrung innerhalb eines Korridors—ein nichtlineares Optimierungsproblem [The Slalom Problem, or Optimal Curve Fitting Within a Corridor—A Nonlinear Optimization Problem]."

A generalization of interpolation is discussed: the problem of constructing a smooth, low-curvature, minimum-length curve between two points so that it intersects each of the discs in one given collection while avoiding each of those in a second collection. The pragmatics of an optimization-scheme are presented. No theory. Delightfully illustrated.

9. Lempio, F.: "Dualitaet und optimale Steuerungen [Duality and Optimal Control]."

For the most general formulation of an optimization problem, a dual problem is specified and a theorem of weak inclusion stated, all without assumptions about linearity, convexity, or differentiability. An example control problem is posed, its dual is studied, and an upper bound on the solution of the primal problem is obtained from the weak inclusion theorem. Then a strong inclusion theorem is derived in the case of linear optimization.

10. Locher, F.: "Optimale definite Polynome und Quadraturformeln [Optimal Definite Polynomials and Quadrature Formulas]."

A definite polynomial is defined as one which does not change sign on $[-1, +1]$. The problem considered is that of finding the extreme value for $\int_{-1}^{+1} p_n(x)w(x)dx$ with p_n taken from among all definite monic polynomials of degree $\leqq n$ and $w(x)$ a given nonnegative weight function. Application of the solution to this problem is made to determine some quadrature formulas and to solve a problem posed by P. Kirchberger.

11. Sibony, M.: "Some Numerical Techniques for Optimal Control Governed by Partial Differential Equation" (sic).

Considered as problem (1) is: find $u \in X$ so that $F(u) \leqq F(v)$ for all $v \in X$, where X is a closed convex subset of a Banach space and F is a given functional. A reformulation of this with greater detail in the case of control problems governed by partial differential equations is given as problem (2). Several illustrations of obtaining numerical solutions for both problem types are presented.

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32[2.05.3, 2.20, 2.40].—RICHARD P. BRENT, *Algorithms for Minimization Without Derivatives*, Prentice-Hall, Englewood Cliffs, N. J., 1973, xii + 195 pp., 24 cm. Price \$12.—.

This well written very readable book should be of particular interest to numerical analysts working on methods for finding zeros and extrema of functions. The book is concerned primarily with functions of a single variable and exclusively with methods which use only function values; no derivative evaluations are required by any of the algorithms presented. It also emphasizes algorithmic details that are extremely important for developing reliable computer codes.

In the first chapter, a very useful summary is given of the material covered in subsequent chapters. As these chapters are relatively self-contained, this enables the reader to easily determine which sections of the book to read according to his interests.

Fundamental results on Taylor series, polynomial interpolation, and divided differences are found in Chapter 2. These results are proved under slightly weaker assumptions than one usually finds in the literature. Chapter 3 presents a unified treatment of successive polynomial interpolation for finding zeros of a function and its derivatives; i.e., zeros, stationary points, inflexion points, etc.

The next four chapters are devoted, respectively, to algorithms for:

1. Finding a zero of a function, given an interval in which it changes sign.
2. Finding a local minimum of a function defined on a given interval.
3. Finding the global minimum of a function of one or several variables, given upper bounds on the second derivatives. (This is practical for at most two or three variables.)
4. Finding a local minimum of a function of several variables.

In each case, convergence, the rate of convergence and the effect of rounding errors are analyzed. ALGOL programs that are claimed to be fast and reliable in the presence of roundoff are given along with a substantial amount of numerical results which supports these claims. FORTRAN implementations of the first two algorithms are also given in the Appendix. The book also contains an extensive and up-to-date bibliography relevant to nonlinear optimization.

The reviewer's only complaint about the book is its title which implies that it is about the minimization of nonlinear functions of several variables. Actually, less than one-third of the book deals with this subject. The title, "Algorithms for finding zeros and extrema of functions without calculating derivatives," of an earlier version of the book that appeared as a Stanford University report, gives a more accurate description of the book's contents.

In any case, this book is an excellent reference work for optimizers and root finders who should find the programs in it of great value.

D. G.

33 [2.45, 10].—W. W. Boone, F. B. Cannonito & C. R. Lyndon, Editors, *Word Problems. Decision Problems and the Burnside Problem in Group Theory,* North-Holland Publishing Co., Amsterdam and London, 1973, viii + 646 pp., 23 cm. Price $42.—.

This work is a collection of papers which were presented at a conference at the University of California, Irvine, in September, 1969. A detailed review of all of the 21 papers would require an excessive amount of space; the potential reader will have to consult the "Mathematical Reviews" for an item by item review. However, the book has a unifying theme, namely, the use of algorithmic methods, and this fact is well presented in a very good introduction by the editors who also give proper credit to the pioneering work of Thue and Dehn.

Of the 646 pages of the book, 282 are occupied by a paper on "The existence of infinite Burnside groups" by J. J. Britton. It contains a new proof of the Novikov-Adjan Theorem that, for all sufficiently large odd numbers e, the Burnside groups with exponent e and at least two generators are infinite. The paper is practically self-contained, using little more than the concepts of free group and free product (according to a claim made by the author). Nevertheless, it is probably one of the most difficult mathematical papers ever to appear in print. It is impossible to skim it, and it is very important to know that it has been checked and commented on in detail by J. Mennicke.

Novikov and Adjan had shown that $e \geqq 4381$ is a lower bound (since improved by them) for the Burnside exponent. No such lower bound appears in Britton's paper which, however, establishes a connection between Burnside groups and a generalization of groups investigated by Tartakovskiĭ.

A paper by S. I. Adjan on "Burnside groups of odd exponent and irreducible systems of group identities" in the present volume sketches the author's proof that in a 2-generator group defined by the identities (i.e., relations valid for any pair x, y of group elements)

$$(x^{ke}y^{ke}x^{-ke}y^{-ke})^e = 1 \qquad (e \text{ odd}, e \geqq 4381, k \text{ ranges over all prime numbers})$$

none of the identities can be derived from the others in the system. This implies that there exists a two-generator group with a recursively enumerable set of identities (i.e., a free group of a variety) and with an unsolvable word problem.

The third important paper connected with the Burnside problem is the construction of a nonsolvable group of exponent 5 by S. Bachmuth, H. Y. Mochizuki and D. W. Walkup. The paper is based on the construction of a particular nonnilpotent associative ring and provides also the construction of a Lie ring of characteristic 5 which satisfies the third Engel condition and is not nilpotent. An appendix of the paper contains a FORTRAN program which the authors used to verify some algebraic relations.

These are the contributions to the Burnside problem which fill about half of the volume. The other half contains excellent survey articles and original papers. All of the survey articles concern theories to which the author has contributed heavily himself, but, in some of them, a very large part consists of previously unpublished material. They are surveys only because of the careful outlay of background information and of the references to ramifications. This is true, in particular, of the papers by D. J. Collins: "The word, power, and order problems in finitely presented groups", by C. F. Miller III: "Some connections between Hilbert's 10-th problem and the theory of groups" (written prior to the publication of Matejasevich's famous paper which, however, is briefly considered in an "Added in proof" note) and by D. Tamari: "The associativity problem for monoids and the word problem for semigroups and groups".

The surveys in the usual sense are those by W. Haken: "Connections between topological and group theoretical decision problems", by C. F. Miller III: "Decision problems in algebraic classes of groups" and by P. E. Schupp: "A survey of small cancellation theory". The third one will probably be superseded soon by a forthcoming book by the author and R. C. Lyndon.

The research papers are listed below in alphabetical order:

S. Aanderau: A proof of Higman's imbedding theorem, using Britton extension of groups (18 pp.).

F. S. Cannonito: The algebraic invariance of the word problem in groups (16 pp.).

R. W. Gatterdam: The Higman theorem for primitive-recursive groups—A preliminary report (6 pp.).

S. Lipschutz: On the word problem and T-fourth-groups (10 pp.).

J. McCool and A. Pietrowski: On a conjecture of W. Magnus (4 pp.).

R. McKenzie and R. J. Thompson: An elementary construction of unsolvable word problems in group theory (18 pp.).

T. G. McLaughlin: A non-enumerability theorem for infinite classes of finite structures (4 pp.).

A. W. Mostowski: Uniform algorithms for deciding group theoretic problems (28 pp.).

B. H. Neumann: The isomorphism problem for algebraically closed groups (10 pp.).

H. Schiek: Equations over groups (6 pp.).

L. Wos and G. Robinson: Maximal modules and refutation completeness: semidecision procedures in automatic theorem proving.

It is not possible to comment on these papers in a general review, but it should at least be mentioned that the paper by B. H. Neumann is preparing the ground for the recent Boone-Higman theorem which characterises the intersection of the algebraically closed groups.

An appendix contains fifty research problems. With one exception, the name of the person raising the question is not given. In several cases, it would be very hard to guess since the problem is "in the air".

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34[4].—HANS J. STETTER, *Analysis of Discretization Methods for Ordinary Differential Equations,* Springer Verlag, New York, 1973, xvi + 338 pp., 24 cm. Price $44.40.

In the eleven years since Henrici's now classical book "Discrete Variable Methods in Ordinary Differential Equations" (J. Wiley & Sons, 1962) appeared, there have been a plethora of new and variants of existing methods in the literature, particularly for the initial value problem which is the major concern of the exhaustive analysis in this excellent book. Professor Stetter places these methods in a solid mathematical framework, and in doing so, extends the existing theory and presents many new results. To quote from the preface: "This text is *not* an introduction to the use of finite-difference methods; rather, it assumes that the reader has a knowledge of the field, preferably including practical experience in the computational solution of differential equations," and from the end of Chapter 1: "In the remainder of this treatise the many practical aspects of the numerical solution of ordinary differential equations by discretization methods will *not* be considered,...." My sole objection to the first statement is the word "text". This treatise provides a fundamental and complete background for those concerned with the theoretical development of the field. However, in addition to assuming a good intuitive grasp of the practical aspects necessary to put the results in perspective, it assumes a moderate mathematical sophistication. This is a book neither for an engineer wanting to know "How to solve it", nor for a beginning student. (Further, its price makes me unwilling to require it as a text, even for an advanced seminar following a course based on, for example, Henrici's text (op. cit.), although it puts the current state of the field into shape such that a course of this nature would be well-structured and exciting.)

The book starts with an abstract discussion of discretization methods and their stability, consistency and convergence. Particular emphasis is placed on the asymptotic expansion of the local and global discretization error, a topic in which Stetter has made many new contributions, both in previous papers and in the pages of this book. The second chapter specializes the ideas to discretization methods for initial value problems and introduces some important ideas, in particular "strong exponential stability". This refers to the exponential reducing property of a method for a class of exponentially stable differential equations, and is as close as the book comes to discussing stiff problems. However, it is possible that the approach will provide the tools for an analysis of this currently open area. The four remaining chapters cover virtually all classes of methods. In addition to explicit R-K and linear multistep methods, the analysis is shown to be applicable to implicit R-K, cyclic multistep, predictor-corrector, hybrid (off step points), power series, Groebner-Knapp-Wanner, R-K-Felhberg, Nordsieck and extrapolation methods. Stetter's asymptotic error theory sheds new light on Butcher's concept of "effective order". (Further important results on global asymptotic error estimation can be found in Stetter's recent paper "Economical Global Error Estimation", in Proceedings of the International Symposium on Stiff Differential Equations, edited by R. A. Willoughby, expected in 1974, Plenum Press.)

Generally, the analysis is performed assuming a "coherent grid". This is one in which the grid is specified by $t_\nu = t_{\nu-1} + h\theta(t_\nu)$ where θ is a piecewise constant strictly positive function. The lack of an analysis for more general grid points leaves out an important open area of research, as the vast majority of practical algorithms not only vary the step size frequently, but also vary the method from step to step.

Although criticism of this first class work is presumptuous, permit me to make two observations. Firstly, this book starts with abstractions and then gives examples (but there are an ample number). I feel that it would have been easier to read if the examples had preceded and motivated each definition. Since the reader is assumed to be familiar with discretization methods, they could have provided good introductory material for each concept. Secondly, it is necessary to read this book from the beginning in order to read any later section. Although there is a lot of cross referencing, there is no table of notations, and by the midpoint there are so many notational conventions (e.g. which type of letter represents a member of which space, which letter represents the local error, etc.) that a quick run through the book is not possible. However, the time spent in carefully reading the first two chapters is amply rewarded by the density of material presented in relatively few pages of the last four chapters.

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35 [5]. —GILBERT STRANG & GEORGE J. FIX, *An Analysis of the Finite Element Method,* Prentice-Hall, Englewood Cliffs, N. J., 1973, xiv + 306 pp., 24 cm. Price $16.—.

Few are able to present sophisticated analytical developments in a simple, practical, and easily digestible form. Strang and Fix have succeeded! Their book shows the mark of men who have contributed to the development of the field. They get right to the heart of the subject matter, without introducing extraneous mathematical embellishments. Their prose is a pleasure to read, their mathematical taste is elegant, and their numerical techniques are simply described and evaluated for efficiency. This book will be a delightful graduate text and reference work for a broad audience. Its spirit can best be described by quoting from the Preface:

"Its purpose is to explain the effect of each of the approximations that are essential for the finite element technique to be computationally efficient. We list here some of these approximations:

(1) interpolation of the original physical data
(2) choice of a finite number of polynomial trial functions
(3) simplification of the geometry of the domain
(4) modification of the boundary conditions
(5) numerical integration of the underlying functional in the variational principle
(6) roundoff error in the solution of the discrete system.

"These questions are fundamentally mathematical, and so are the authors. Nevertheless, this book is absolutely not intended for the exclusive use of specialists in numerical analysis. On the contrary, we hope it may help establish closer communication between the mathematical engineer and the mathematical analyst. It seems to us that the finite element method provides a special opportunity for this communication: the theory is attractive, the applications are growing, and best of all, the method is so new that the gap between theory and application ought not yet to be insurmountable.

"Of course, we recognize that there are obstacles which cannot be made to disappear. One of them is the language itself; we have kept the mathematical notations to a minimum, and indexed them (with definitions) at the end of the book. We also know that, even after a norm has been interpreted as a natural measure of strain energy, and a Hilbert space identified with the class of admissible functions in a physically derived variational principle, there still remains the hardest problem: to become comfortable with these ideas, and to make them one's own. This requires genuine patience and tolerance on both sides, as well as effort. Perhaps this book at least exhibits the kind of problems which a mathematician is trained to solve, and those for which he is useless."

E. I.

36 [**5, 6**]. —I. I. Gihman & A. V. Skorohod, *Stochastic Differential Equations*, translated from the Russian, Springer Verlag, New York, 1972, viii + 354 pp., 24 cm. Price $27.90.

This book is a treatise on stochastic equations by two well-known authors in probability theory. It is a theoretical work and much attention is devoted to developing the foundations: existence, uniqueness, regularity, etc. The book presents many other results; for example it treats asymptotic behavior, which should be of direct interest to readers with applied interests. The authors do not treat specific applied problems in detail. Let us give a brief review of the contents of the book which is arranged in two parts.

Part I deals with one-dimensional stochastic differential equations of first order exclusively. Chapter 1 gives a succinct treatment of K. Itô's theory of stochastic integrals. Chapter 2 deals with existence and uniqueness, again following K. Itô's methods, i.e., a stochastic Picard iteration method. The authors are careful here, as well as in the rest of the book, to single out the necessary hypotheses and avoid unnatural restrictions. In Chapter 2, they give a thorough analysis of the dependence of solutions on parameters. This is important in establishing the connection of stochastic differential equations and partial differential equations. Chapter 3 analyzes the connection just mentioned and the Markov character of the solutions of stochastic equations. One finds that probabilistic methods yield very strong results on the existence, uniqueness, and regularity of parabolic partial differential equations. Moreover, these results do not depend upon uniform ellipticity assumptions. Chapter 4 deals with the asymptotic behavior of the solutions of stochastic equations. The results of this chapter are very sharp and many are not available elsewhere. Chapter 5 treats problems on a finite interval and gives a thorough analysis of boundary behavior as well as asymptotic behavior.

Part II examines systems of stochastic differential equations. Chapter 1 deals with vector stochastic equations in general and not only with stochastic equations that lead to Markov processes. Many of the concepts here are due to the authors. Chapter 2 specializes in stochastic equations without after-effect (i.e., Markov) but includes jump processes as well as diffusions. The existence, uniqueness, and regularity theory is not given in detail again since it is similar to that of Part I. The results are stated in full, however. The latter sections of Chapter 2 contain a wealth of information on the connections between functionals of solutions of stochastic equations and partial differential equations. Chapter 3 is perhaps the most important chapter in the book from the point of view of applications. A number of very strong results on asymptotic behavior for large time and as a parameter tends to a limiting value are obtained. Bogoliubov's averaging method is extended to stochastic equations.

This book, along with H. P. McKean's "Stochastic Integrals", Academic Press, New York, 1968, provide excellent foundations and up to date information on stochastic differential equations.

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37 [7].—H. P. Robinson, *Tables of the Derivative of the Psi Function to* 58 *Decimals,* University of California, Lawrence Berkeley Laboratory, Berkeley, California, October 1973. Ms. of 11 pp. deposited in the UMT file.

This unpublished set of tables consists of 58D values of the trigamma function, $\Psi'(x)$, for $x = n + a$, where $n = 0(1)50$ and $a = 0$, 1/2, 1/3, 2/3, 1/4, 3/4, 1/5, 2/5, 3/5, 4/5.

The tabular values were calculated on a Wang 720C electronic calculator by means of the (stable) backward recursion formula $\Psi'(x-1) = \Psi'(x) + (x-1)^{-2}$, starting with values corresponding to $x = 1000 + a$, which were calculated by the appropriate asymptotic series. The terminal values in this recurrence were checked by the reflection formula $\Psi'(x) + \Psi'(1-x) = \pi^2\csc^2 x$.

It may be appropriate to remark here that these excellent tables possess much higher precision than published tables [1], [2] of this function, which extend to at most 19D.

J. W. W.

1. British Association for the Advancement of Science, *Mathematical Tables,* v. 1, third edition, Cambridge University Press, Cambridge, England, 1951.

2. H. T. Davis, *Tables of the Mathematical Functions,* second edition, v. 2, The Principia Press of Trinity University, San Antonio, Texas, 1963. (For references to additional tables, with lower precision, see A. Fletcher, J. C. P. Miller, L. Rosenhead, and L. J. Comrie, *An Index of Mathematical Tables,* second edition, v. 1, Addison-Wesley, Reading, Massachusetts, 1962, p. 298.)

38 [**7, 9, 10**].—N. J. A. SLOANE, *A Handbook of Integer Sequences,* Academic Press, New York, 1973, xiii + 206 pp., 24 cm. Price $10.—.

This unique book contains a listing of 2372 numbered integer sequences (some interrelated) of special interest to researchers in combinatorial theory (including graph theory) and number theory. It represents the culmination of the author's efforts in accumulating such information over a period of seven years, beginning in 1965.

The main tabulation of sequences is prefaced by three explanatory chapters entitled, respectively, Description of the Book, How to Handle a Strange Sequence, and Illustrated Description of Some Important Sequences.

The four criteria used by the author in selecting a sequence for inclusion in the main table are: the sequence must consist of nonnegative integers; it must be infinite; the first two terms must be, respectively, 1 and n, where n is between 2 and 999, inclusive; enough terms must be known to separate the sequence from its tabulated neighbors; and the sequence should have appeared in the scientific literature and be considered well-defined and interesting. The author concedes that his somewhat arbitrary selection of sequences has been necessarily subjective, and in recognition of the limitations of the present list he states that he plans to issue supplements from time to time that will include any necessary corrections, new sequences, and extensions to the original sequences. In the opinion of this reviewer, certain artificial sequences such as the mixed decimal digits of π and e might be appropriately omitted in future editions.

It should be mentioned that some sequences composed of the absolute value of selected alternating sequences have been included. Also, to meet the second criterion some sequences have been subjected to Procrustean manipulation so that an initial integer 1 has been inserted before the first term if this exceeds 1, and extra 1's and 0's have been suppressed. Whenever possible, enough terms of a sequence are given to fill two lines.

The great majority of the tabulated sequences have been taken from sources (including this journal) in the fields of combinatorial theory and number theory. Included are sequences relating to combinations and permutations, graphs and trees, geometries, dissections, polyominoes, Boolean functions, partitions and other number-theoretic problems. A brief discussion of such sequences constitutes Chapter 3.

A few of the entries in the main table consist of about the first 60 digits in the decimal expansions of certain well-known mathematical constants such as the square roots of 2, 3, and 5; the cube roots of 2 and 3; the natural logarithms of 2, 3, and 10; π, γ, ϕ (the golden ratio) and their natural logarithms; e and its common logarithm; Khintchine's constant; π^2 and $\pi^{1/2}$. Sequences relating to the continued-fraction expansions of some of these constants are also included, as well as sequences consisting of the first 12 or more powers of the integers 2 through 19 (excluding 10 as trivial).

A bibliography of 319 entries lists the sources of the sequences in the main table. This is followed by a convenient index, which gives the listed numbers of sequences relating to a specific topic, the principal sequence of its type being identified therein by an asterisk.

This unusual book may be considered as a companion to the report of Robinson and Potter [1], which deals with the identification of noninteger numbers having a prescribed decimal expansion. Because of its wealth of material and extensive bibliography, the book should be of considerable educational value to many readers.

J. W. W.

1. H. P. ROBINSON & ELINOR POTTER, *Mathematical Constants,* Report UCRL-20418, Lawrence Radiation Laboratory, University of California, Berkeley, California, March 1971. (See *Math. Comp.*, v. 26, 1972, pp. 300-301, RMT 12.)

39 [7, 13.15].—PETER BECKMANN, *Orthogonal Polynomials for Engineers and Physicists,* The Golem Press, Boulder, Colorado, 1973, viii + 280 pp., 24 cm. Price $15.—.

The preface gives several reasons for writing the book and states that, as the title implies, it is for the "application minded reader, and I have tried to be convincing without splitting hairs. The reader looking for local limit lemmas and epsilonics has the wrong book." There is little I find to recommend in this volume. The author's style and apparent sense of humor seem out of place to the reviewer. One of the reasons given for writing the book is that, though orthogonal polynomials were once a "bewildering and confusing collection of many systems...", they "can now be derived from a general weighting function...." The author's statement is not clear. Further, he means the classical orthogonal polynomials, but he does not say so. On this point, one of the appendices is a collection of integrals and other relations involving the classical orthogonal polynomials lifted from [1]. The economy noted by the author does not seem to apply here for he is apparently unaware of the fact that many properties for the Laguerre and Hermite polynomials (the latter is a special case of the former) follow from those of Jacobi by invoking a certain confluent limit process; see [2, Vol. 1., Chapter 8].

Epsilons and deltas aside, to be convincing one should expect complete and correct statements. The author states in the preface, "I have endeavored to give applications to some of the problems where today's action is—numerical analysis, random processes, wave scattering and the like. I did include a brief chapter on partial differential equations, but I suspect that today's engineers will be more appreciative of things like the telescoping trick for orthogonal expansions, which is given here in somewhat more general form than..." in a recent book which shall pass unnamed. The author is referring to the scheme for evaluating a sum involving orthogonal polynomials by use of the recursion formula for such polynomials employed in the backward direction. On p. 120, he says that the scheme is "unbeknownst to many wizards of numerical analysis." The author seems unaware that the principle is far more general than the one he gives. He presents no references to original papers, and the round-off error problem of computing with recursion formulas is not even mentioned.

The volume is divided into two parts. Part I—Theory, is composed of three chapters—introduction, orthogonal functions and orthogonal polynomials. Part II—Applications—is also composed of three chapters—numerical analysis, partial differential equations, and probability theory and random processes.

If some one must seek information on the topics covered in this book, I believe he has the wrong book.

Y. L. L.

1. I. S. Gradshteyn & I. M. Ryzhik, *Tables of Integrals, Sums, Series and Products* (in Russian), 4th ed., Moscow, 1963. (See *Math. Comp.*, v. 20, 1966, pp. 616-617 and references given there.)

2. Y. L. Luke, *The Special Functions and Their Approximations*, Vols. 1 and 2, Academic Press, 1969. (See *Math. Comp.*, v. 26, 1972, pp. 297-299.)

40[7].—Harry E. Rauch & Aaron Lebowitz, *Elliptic Functions, Theta Functions, and Riemann Surfaces*, The Williams and Wilkins Co., Baltimore, Maryland, 1973, xii + 292 pp., 24 cm. Price $20.—.

According to the preface, this volume is designed as a text for advanced undergraduate students and graduate students who have had an introductory course in complex variable theory. The authors also have in mind readers interested in self-study, applied workers and researchers in pure mathematics.

The subject matter of the text is aptly described by its title. The material is quite classical and is divided into four chapters—I. Riemann Surfaces and Elliptic Functions, II. Theta Functions and Elliptic Functions, III. Elliptic Integrals of Second and Third Kinds and the Representation of Elliptic Functions, IV. Transformation Theory. There are also seven appendices spread throughout the text.

There is no dearth of material from which mathematical courses can be developed. Fundamental courses aside, it is subject matter of current research interest which for the most part dictates the contents of a course. The topics covered by the volume at hand are classical and have been thoroughly worked over. As far as I know, except for numerous mathematical tables of functions, there is little research done on the subjects. The authors claim in the preface that there is a revival of interest in theta functions of several variables, and as a consequence believe treatment of the single variable case useful. However, there are no references to back this statement. Indeed, except for a few references to books (and these are mostly classical and not easily inaccessible to most readers), there are but two references to original papers and these are dated 1957 and 1965. References to mathematical tables and related data are absent. It is unfortunate that a number of references to books and papers dating from about 1956 which are accessible to most readers are not noted.

There are no exercises and this coupled with our above comments suggests that the volume is not likely to be used as a text. On the other side of the ledger, the tome is quite readable and contains much information of value to applied and pure research workers.

Y. L. L.

41[7].—L. S. Srinath, K. R. Sarma & S. V. Patankar, Editors, *Basic Engineering and Mathematical Tables*, Tata McGraw-Hill Publishing Co., Bombay and New Delhi, 1971, x + 136 pp., 24 cm. Price $9.95.

This compact set of mathematical and engineering tables has been designed primarily to meet the needs of students of engineering and science for such information.

The first part, subtitled Mathematical Tables, contains a total of 14 numerical tables consisting, respectively, of 4S or 5S values of the standard elementary functions, zeros of the Bessel function of the first kind, and the normal distribution function. In addition, this part includes sets of formulas from algebra, geometry, trigonometry, analytic geometry, calculus (including tables of 174 indefinite and 80 definite integrals), complex variables, vector analysis, operational calculus, and statistics. Also included is a table of 35 physical quantities (with dimensions and units), as well as related tables of the principal physical atomic constants, and a useful list of conversion factors.

The latter part of the book, subtitled Engineering Tables, consists of a total of 74 engineering tables, arranged under the broad headings of analytic mechanics; mechanics of solids; and physical, mechanical, electrical, and thermal properties of substances.

Careful examination has revealed that several of the tables contain erroneous entries. For example, on p. 22 last-place errors occur in the values of e^{-x} for $x = .04$ and $.06$, in e^x for $x = .47$ (where only a 4D approximation appears), in $\sinh x$ for $x = 5.2$ and 5.3, and in $\cosh x$ for $x = 5.2$. More serious errors occur on p. 60 in the 3D table of the first nine positive zeros of $J_p(x)$ for $p = 0(1)5$, where half the entries require correction. The source of this table can be traced ultimately to the table of Bourget, whose errors are discussed and corrected in the FMRC *Index* [1].

Furthermore, the table of the normal distribution function (pp. 68–69) contains a total of 16 errors, all in the last decimal place except for the entry corresponding to $z = 1.82$, where one should read .9656 in place of .9556. On p. 102 this reviewer discovered through recalculation a total of 15 errors in the table of $e^{\mu\theta}$.

In addition to these tabular errata a significant number of serious typographical errors (in addition to obvious ones) appear in the formulas and text. For example, on p. 23, in the expression for the second root of the cubic equation, read $A - B$ instead of $A - 3$, and in the following line $d^2/27$ should be replaced by $d^3/27$. On p. 34 the constant e is inadvertently represented as the sum of a terminating series. On p. 51, in formula 194 an extraneous minus sign obtrudes in the integrand, and in the last line on p. 61 it is clear that $E(n \pm \phi)$ should be replaced by $E(n\pi \pm \phi)$. In the table of torsional deflection constants (p. 93) the symbol k should be replaced by K, and in formula 4 we should read $q = a_i/a_0 = b_i/b_0$. Also, the reader may be confused by formula 7 because the quantities a and b therein actually represent the lengths of the half-sides of the rectangular cross-section, which differs from the notation adopted in related formula 5.

Finally, it should be mentioned that although the authors state in the preface that they consulted several source books, reference books, and handbooks while selecting the present tables, they do not identify these sources nor do they include a bibliography to assist those users who may desire information regarding more extensive tables.

A careful emendation of these tables is clearly required before their overall reliability can match their evident utility.

J. W. W.

1. A. Fletcher, J. C. P. Miller, L. Rosenhead & L. J. Comrie, *An Index of Mathematical Tables*, Second edition, Addison-Wesley, Reading, Mass., 1962. (See v. II, p. 791.)

42 [8].—Charles E. Land, *Tables of Critical Values for Testing Hypotheses about Linear Functions of the Normal Mean and Variance*. II, Department of Statistics, Atomic Bomb Casualty Commission, Hiroshima, Japan 730. Ms. of 69 computer sheets (reduced) deposited in the UMT file.

The calculation of these unpublished tables was carried out by the author while he was a member of the Department of Statistics at Oregon State University.

They represent expanded versions of similar 3D tables [1] previously deposited by the author in the UMT file. The tabular precision and the tabulated significance levels α in [1] remain unchanged, but the number of degrees of freedoms listed in Table 1 (one-sided tests) is now

$$\nu = 3(1)30(5)50(10)100(20)200(50)500(100)1000,$$

while in Table 2 (two-sided tests) $\nu = 2(2)20$. Furthermore, the range of the parameter ξ has now been extended beyond $\xi = 100$ for $\nu \leqq 14$ in Table 1, so that the upper limit of ξ progressively increases (at intervals varying from 10 to 500) with decreasing ν until it attains the value 5000 when $\nu = 3$. Similarly, in Table 2 the upper limit of ξ ranges from 100 for $\nu \geqq 16$ to 4000 when $\nu = 2$. Also, the interval $0 \leqq \xi \leqq 0.1$ has now been divided into 10 subintervals. However, as in the earlier version of Table 2, the critical values corresponding to ξ between 0.01 and 0.2 have been progressively omitted as ν increases to 14, because of corresponding loss of precision in the calculations.

Expansion of these tables was necessary in order to obtain the author's new tables of standard confidence limits, which will be described in a subsequent review.

J. W. W.

1. Charles E. Land, *Tables of Critical Values for Testing Hypotheses about Linear Functions of the Normal Mean and Variance*, Department of Statistics, Oregon State University, Corvallis, Oregon, ms. deposited in the UMT file. (See *Math. Comp.*, v. 25, 1971, p. 941, RMT **44**.)

43 [**9**].—Jack Alanen, *Empirical Study of Aliquot Series*, Report MR 133, Stichting Math. Centrum, Amsterdam, x + 121 pp., July 1972.

This is essentially a reprint of Alanen's Yale thesis. The introduction discusses the use of the computer in attacking number-theoretic problems. He is concerned with the construction and proof of algorithms, and quotes Dijkstra: "Testing shows the presence, not the absence, of bugs." An aliquot sequence ("series") is $s^0(n) = n$, $s^k(n) = s(s^{k-1}(n))$, where $s(n) = \sigma(n) - n$ is the sum of the aliquot parts of n, the divisors of n apart from n itself. He classifies aliquot sequences as purely periodic ("sociable numbers of index k"), ultimately periodic, and unbounded. He lists the 13 purely periodic sequences then known with period ("index") greater than 2. These are due to Poulet ($k = 5, 28$), Borho, Cohen and David ($k = 4$). Further 4-cycles have since been found by David and by Root. He defines $s(0) = 0$ and so includes among the ultimately periodic sequences those called terminating by the reviewer and Selfridge [1]. He lists the six known candidates (276, 552, 564, 660, 840, 966) for "main" n sequences with $n < 1000$ which may remain unbounded; the reviewer, D. H. Lehmer, Selfridge and Wunderlich [2] have now pursued these to terms 433, 181, 265, 168, 195, and 184, which contain 36, 35, 31, 33, 31, and 32 digits.

He gives properties of $s(n)$ and points out that it induces a digraph on the nonnegative integers as vertices. There is part of the drawing of the diagraph containing the perfect number $P = 8128$. He gives the members of the monotonic sequence $s^k(3P)$, $0 \leqq k \leqq 48$. (The sequence $s^k(9336)$ leads into this $s^k(3P)$ at $s^4(9336) = s^3(3P) = 9P$, and, in [2], $s^k(3P)$ was thereby continued until

$$s^{95}(3P) = 22\ 14196\ 97766\ 28194\ 23647\ P,$$

where it is still monotonic. The theory of $s^k(n\,P)$ with n odd and P perfect has been given by te Riele [3]. For large P and $n = 27$ the sequence has been continued to $k = 136$ [3], [4].)

If the invalence of an integer is zero, Alanen calls it "untouchable". He conjectures that no odd number other than 5 is untouchable, and refers to this as a strengthened Goldbach conjecture (or "weakened", since if $n = p+q+1$, where p, q are distinct odd primes, then pq is a value of $s^{-1}(n)$, but there are in general other than "Goldbach" solutions to the equation $s(x) = n$). He gives the value of the invalence of n and a list of all solutions of $s(x) = n$ for $0 \leqq n \leqq 100$, the non-Goldbach solutions for $101 \leqq n \leqq 500$, and a list of the 570 untouchable numbers $2, 5, 52, 88, \cdots$ less than 5000.

Two sections develop algorithms for determining all untouchable numbers, and all cycles, below a given bound. Two others give the results of computer calculations (see next review) and a specification of the algorithms used. There are 33 references.

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1. RICHARD K. GUY & J. L. SELFRIDGE, "Interim report on aliquot series," *Proc. Winnipeg Conf. on Numerical Math.*, October 1971, pp. 557-580.

2. RICHARD K. GUY, D. H. LEHMER, J. L. SELFRIDGE & M. WUNDERLICH, "Second report on aliquot sequences," *Proc. Winnipeg Conf. on Numerical Math.*, October 1973.

3. H. J. J. TE RIELE, "A note on the Catalan-Dickson conjecture," *Math. Comp.*, v. 27, 1973, pp. 189-192.

4. Corrigendum, *ibid.*, p. 1011.

44[9].—JACK ALANEN, *Tables of Aliquot Sequences*, 7 volumes, each of approximately 600 pages of computer output, filed in stiff covers and presented to the reviewer.

This is the output produced in connection with the author's thesis (see previous review). One volume investigates, by various algorithms, (certain subclasses of) partitions of n, and also carries out other algorithms designed to find all solutions of $s(x) = n$, where $s(x)$ is the sum of the divisors of x, other than x itself. A second volume continues the previous work, lists all n sequences with $n \leqq 48303$ for which $s^k(n) = 6$ for some k (the largest k in this range is 33) and lists the members of the aliquot sequence $s^k(138)$ for $0 \leqq k \leqq 112$ (it was earlier shown by D. H. Lehmer that $s^{177}(138) = 1$, the maximum term being $s^{117}(138) = 179931895322$). The other five volumes give all terms of all n sequences for $1 \leqq n \leqq 10000$, $10001 \leqq n \leqq 20000$, $20001 \leqq n \leqq 30000$, $30001 \leqq n \leqq 40000$, $40001 \leqq n \leqq 48303$ and the rank of the bounding term, where the bounding term is either 1, or a member of a cycle, or the first term of the sequence which exceeds 10^{10}. Of the sequences associated with the first 40,000 integers, 33450 terminate at 1, 5676 exceed Alanen's bound of 10^{10}, 325 become periodic at a perfect number (6, 496 or

8128), 495 become periodic with period 2 (amicable pair), and 54 lead into one of the two Poulet cycles.

For a review of Paxson's related tables see *Math. Comp.*, v. 26, 1972, UMT **38**, pp. 807-809.

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45 [10].—P. A. MORRIS, *Characteristic Polynomials of Trees on up to* 14 *Nodes*, University of the West Indies, St. Augustine, Trinidad, West Indies, December 1973. Ms of 8 pp. + 57 computer sheets deposited in the UMT file.

Herein are listed the coefficients of the characteristic polynomials of the adjacency matrices of all trees with 13 or fewer nodes.

The list was generated in two ways to provide a check on the calculations, which were performed on a 1 CL 1902A and an IBM 1620, respectively. The first method employs a theorem of Collatz and Singowitz [1], which asserts that if $\phi(T) = \sum_{k=0}^{n} (-1)^k a_k \lambda^{n-k}$ is the characteristic polynomial of a tree T on n nodes, then $a_{2k+1} = 0$ and a_{2k} equals the number of ways of finding k mutually nonadjacent edges in T. The second method uses a known decomposition theorem [2], which states that if u is a node of valency 1 connected to a node v, $T - uv$ is the tree (together with the isolated node u) formed by deleting the edge uv, and $T - u - v$ is the forest formed by deleting nodes u and v and their incident edges, then $\phi(T) = \phi(T - uv) - \phi(T - u - v)$.

A further check of the accuracy of the list was made by comparison with the corresponding data in the table of Mowshowitz [3], which includes all trees on 10 or fewer nodes.

AUTHOR'S SUMMARY

1. L. COLLATZ & U. SINGOWITZ, "Spektren endlichen Graphen," *Abh. Math. Sem. Univ. Hamburg*, v. 21, 1957, pp. 63-77.

2. F. HARARY, C. KING, A. MOWSHOWITZ & R. C. READ, "Cospectral graphs and digraphs," *Bull. London Math. Soc.*, v. 3, 1971, pp. 321-328.

3. A. MOWSHOWITZ, "The characteristic polynomial of a graph," *J. Combinatorial Theory Ser. B*, v. 12, 1972, pp. 177-193.

46 [12].—LOUIS D. GREY, *A Course in* APL/360 *with Applications*, Addison-Wesley Publishing Co., Inc., Reading, Mass., 1973, xviii + 332 pp., 24 cm. Price $7.50 (paperbound).

If this paper-back book were used merely as a reference manual for APL programmers, it would serve a useful function since it is well organized, comprehensive and well documented. But the text is far more than a work of reference. It is an excellent vehicle for teaching this most elegant and succinct language, one which is considered by some to be a serious competitor with the other two scientific languages—Fortran and PL/I—for prominence in the world of programming languages.

Divided into three major parts, the book leaves nothing of importance unsaid. The reader is introduced to the language at a steady pace, one designed for rapid learning, and provides as pleasant a learning experience as one could hope for. One is carried from the mechanics of sitting at an APL terminal through the APL character set, to all the many varieties of functions available to the APL user.

Despite the richness of the language, one is not left in the frustrating position of not being able to see the forest for the trees. Each topic is handled carefully and each description is more than adequate.

In addition to the numerous examples given, each chapter is followed by a series of exercises making the text admirably suitable for classroom use. Chapter 19 contains an interesting discussion of what lies ahead for APL. Though it is perhaps too early to say with any certainty to what degree the APL language will be accepted by the science and engineering community, this text can only help to pave the way for its acceptance.

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47 [12].—NIKLAUS WIRTH, *Systematic Programming: An Introduction*, Prentice-Hall, Inc., Englewood Cliffs, N. J., 1973, 167 pp., 24 cm. Price $10.50.

During the past year or so, the notion of "structured programming" has become quite popular in the computer field. Although the term is still rather ill-defined, it seems to include concepts such as top-down programming, goto-less programming, program verification, and the chief programmer team method of project organization. Wirth's introductory textbook "Systematic Programming" is an attempt to teach some of these ideas to the student when he is first learning about computers. Program verification is presented as being part of programming itself; the *goto* statement is barely mentioned; and an excellent chapter is devoted to the process by which an outline of a program is fleshed out in top-down style until it becomes functional.

This is definitely not a book about "how to code". It requires considerable mathematical aptitude and knowledge on the part of the student (though Wirth disclaims this requirement). Program structure is emphasized at the same time that linguistic details about PASCAL, the language used for the sample programs, are kept to a minimum. A student who intends to continue in the computer field would need to study a programming language in far greater detail, though a mathematician trying to learn about the nature of programming would be quite satisfied with Wirth's presentation.

The first four chapters are a sketchy introduction to the nature of algorithms, computers, and programming systems. Unfortunately, these chapters are too abstract, and a student is unlikely to learn much from them. Chapter 5 introduces flowcharts and the rules of analytic program verification; with each flowchart component, the corresponding verification method is given. The reader may conclude, however, that the emphasis on program verification is a red herring, for in the later chapters when the programs become more complicated, these techniques are hardly used at all. Chapter 6 discusses, very briefly, how one can know that a program will terminate. Chapter 7 (about a quarter of the way through the book) first introduces some details of PASCAL, and again relates each type of statement and each construction to the corresponding flowchart and verification rule. Chapter 8 is devoted to data types.

In Chapter 9 some programs based on recurrence relations are shown; the first two of these are programs for computing inverses and square roots by approximation. The invariant given in the program for computing inverses is incorrect; it should be

$$(1-\epsilon)/x < A < 2/x.$$

Furthermore, the example is confusing in that the range of the variable x should be restricted to $0 < x < 1$ rather than $0 < x < 2$ if the approximation is to be reasonable. (The algorithm given will yield an inverse of 1 for all numbers greater than 1.) The algorithms also have an unpleasant out-of-the-hat quality; a student with any curiosity will be utterly puzzled as to how he could have found them for himself.

Chapters 10 through 14 present a variety of topics: textfiles (a primitive model of input-output), arrays, subroutines and functions, transformation of number representations, and simple text manipulation. The presentation on functions is unnecessarily obscure; the first example of a value-returning function is an integration procedure that itself has a functional parameter. A simpler example of a value-returning function should have been given prior to this complicated one.

The last chapter is devoted to stepwise program development, and presents three case studies: solving a set of linear equations, finding the first n prime numbers, and generating sequences of N characters from an alphabet of 3 characters such that no two immediately adjacent subsequences are identical. Each program is developed as a sketch in which informal statements are expanded to more detailed statements until the program has been fully formalized; the complete programs are then modified to incorporate various improvements. This part of the presentation is very well done.

Overall, "Systematic Programming" is a difficult, frequently obscure, but challenging book. Its approach is heavily mathematical, both in style and in the choice of examples. For students whose primary interest is numerical mathematics, it would be a good choice. For the general computer science student, it would not be, for its view of programming is too abstract and specialized.

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48[12, 13.40].—E. S. PAGE & L. B. WILSON, *Information Representation and Manipulation in a Computer,* Cambridge Univ. Press, New York, N. Y., 1973, v + 244 pp., 23 cm. Price $5.95.

I was quite pleased to receive this book for review, since I have been searching for a long time for a satisfactory text for a course in data structures; this is the first text to fill the need. The classic text on the subject is Knuth's "Art of Computer Programming", but the relevant material is spread over two volumes. Furthermore, the explanations are on too high a level for an undergraduate course. Page and Wilson's book might well be entitled, "A Simplified Version of Knuth". Virtually all of their material, except for an early chapter on symbols and codes, is contained in Volumes I and III of Knuth, and their references to Knuth are plentiful.

Chapters 1 and 2 deal with symbols and their encodings and include a treatment of error detection and correction and Huffman coding. These topics seem somewhat remote from the rest of the book. Chapter 3 discusses internal representations of numbers. This would be an appropriate point for a discussion of character strings, but there is none, perhaps because Algol does not have any facilities for manipulating them. Chapter 4 treats arrays and methods of accessing them, and includes a good discussion of sparse arrays (arrays that are mostly zero). Chapter 5 discusses allocation methods for queues, stacks, and deques, considering both arrays and linked lists. There is also a more general discussion of linked lists here. Chapter 6 discusses trees and how to traverse and represent them. The discussion of conversion between binary and general trees seems overblown, however. Chapter 7 discusses searching, including binary and hashcoded searches, and Chapter 8 discusses sorting methods.

There are a number of disconcerting flaws in the book. Explanations are occasionally obscure (e.g., an application of generating functions on p. 158). The justifications given for circular lists (p. 86) are incorrect; a circular (singly-linked) list saves one pointer and nothing else, compared with an ordinary list with front and rear positions. Algol is used throughout as the programming language, and the limitations of Algol (no strings, no pointers) cause these programs, as well as the choice of methods, to be unnecessarily obscure. The book is well supplied with exercises, but many of these are taken from British university examinations and require nothing more than a regurgitation of material in the preceding chapter.

On the plus side, the book is free of major idiosyncracies (a frequent problem with computer science texts) and the explanations are generally clear. One can ask students to read the book without forcing them to get involved in extraneous auxiliary material. Despite its shortcomings, I recommend this book as a text for a data structures course.

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