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Tables of Reductions of Symmetrized Inner Products ("Inner Plethysms") of Ordinary Irreducible Representations of Symmetric Groups

By N. Esper

Abstract. Decompositions of symmetrized inner products $[\alpha] \boxdot [\beta]$ of ordinary irreducible representations $[\alpha]$ of symmetric groups S_n and $[\beta]$ of S_m were evaluated on a CDC 6400. Tables were obtained for $2 \leqslant n \leqslant 10$ and $2 \leqslant m \leqslant 5$ as well as for $m = 6$ and $2 \leqslant n \leqslant 7$.

In [3] R. C. King published tables of reductions of symmetrized inner products $[\alpha] \boxdot [\beta]$ which he calls inner plethysms, of ordinary irreducible representations $[\alpha]$ of S_n and $[\beta]$ of S_m, where $n = 4$ and $m \leqslant 5$, $n = 5$ and $m \leqslant 4$, $n = 6$ and $m \leqslant 3$.*

He obtained the decomposition by restricting certain representations $\{\beta\}$ of the general linear group GL_n to symmetric subgroups.

Such decompositions can be obtained directly by evaluating the character of $[\alpha] \boxdot [\beta]$ which is

$$\chi^{[\alpha]\boxdot[\beta]}(g) = \frac{1}{|S_m|} \sum_{\pi \in S_m} \zeta^{[\beta]}(\pi) \prod_{k=1}^{n} \zeta^{[\alpha]}(g^k)^{a_k(\pi)},$$

where $g \in S_n$ and $a_k(\pi)$ denotes the number of cyclic factors of length k in $\pi \in S_m$, $1 \leqslant k \leqslant m$.

For this formula see [1], [2], and [4, p. 74]. The evaluation was carried out with the aid of a computer (CDC 6400 RWTH Aachen) by using the program described in [1], in double-precision arithmetic. The characters of the products $[\alpha] \boxdot [\beta]$ were then decomposed into their irreducible constituents via orthogonality relations by using the character table of S_n.

Tables were thus obtained of the reductions of the symmetrized inner products of the ordinary irreducible representations of the symmetric groups S_2 up to S_{10} with the ordinary irreducible representations of S_2 up to S_5 and of the characters of S_2 up to S_7 with those of S_6. These tables appear on the microfiche card in this issue.

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*Dr. King wants me to point out that in the tables of [3] two printing errors occurred in the decomposition of $[3, 1] \boxdot [2^2]$ and $[3, 1^2] \boxdot [2, 1^2]$. The correct values can be obtained from the microfiche.

Lehrstuhl D für Mathematik
RWTH Aachen
Aachen, Federal Republic of Germany

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