Mathematics of Computation

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Mathematics of Computation

A Quarterly Journal Edited by JAMES H. BRAMBLE, Chairman

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Manuscripts should be typewritten double-spaced in the format used by the journal. For journal abbreviations, see the latest *Mathematical Reviews* volume index. An author should submit the original and one copy of the manuscript and retain one copy. The author may suggest an appropriate editor for his paper. It is recommended that the author acquaint himself with the pertinent material contained in "A Manual for Authors of Mathematical Papers," which is available from the American Mathematical Society. All contributions intended for publication and all books for review should be addressed to James H. Bramble, Chairman, Editorial Committee, Mathematics of Computation, Center for Applied Mathematics, 275 Olin Hall, Cornell University, Ithaca, New York 14853. Institutions sponsoring research reported in the journal are assessed page and microfiche charges.

Each article submitted for publication must be accompanied by a brief and reasonably self-contained abstract, and by AMS (MOS) subject classification numbers. If a list of key words and phrases is included, it will be printed as a footnote on the first page. A list of the classification numbers may be found in the Index to Mathematical Reviews, Volume 39 (June 1970).

Topics in Operator Theory Edited by Carl Pearcy Mathematical Surveys, Number 13

The five articles in this volume are expository in nature, and they all deal with various aspects of the theory of bounded linear operators on Hilbert space. The volume is very timely, because in the last year or two great progress has been made on hard problems in this field, and thus operator theory today is a very exciting area of mathematical research. One particular problem on which considerable progress has been made recently is the invariant subspace problem. This is the question whether every bounded linear operator on a separable, infinite-dimensional, complex Hilbert space H has a nontrivial invariant subspace. Even though this problem remains unresolved, there are some operators T on H for which the structure of the lattice of all invariant subspaces of T is known, and the first article in this volume, "Invariant subspaces", by Donald Sarason, is devoted to a discussion of such operators. One of the interesting features of this lucid presentation is the interplay between operator theory and classical analysis.

The second article is entitled "Weighted shift operators and analytic function theory" and was written by Allen Shields. He has taken essentially all of the information presently known about weighted shift operators (with scalar weights) and incorporated it into this comprehensive article. A central theme of the exposition is the interaction between weighted shift operators and analytic function theory, and as an added bonus for the reader, the article contains a list of thirty-two interesting research problems.

The third article in the volume is a treatise entitled "A version of multiplicity theory" by Arlen Brown. The problem treated is how to decide when two normal operators are unitarily equivalent. (Unitary equivalence is the analog for operators of the concept of isomorphism for groups, rings, etc.) The unitary equivalence problem for arbitrary operators is exceedingly difficult, but the theory of spectral multiplicity, which can be approached in several different ways, furnishes a reasonable complete set of unitary invariants for normal operators. The author focuses attention on the concept of a spectral measure, and his clear presentation of this circle of ideas should lead to a better understanding of multiplicity theory by beginners and experts alike.

The fourth article in this volume, "Canonical models" by R. G. Douglas, is concerned with the theory of canonical models for operators on Hilbert space. The central underlying idea is that if T is any contraction operator on H (i.e., if the norm of T is at most 1), then there is a canonical construction that associates with T an operator M_T that is unitarily equivalent to T, called its "canonical model". One can therefore study T by studying M_T instead, and this theory has made significant progress in the past ten years. The author, who has contributed substantially to the geometrization of this theory, exposes in his article various important components of the theory, and thereby gives the reader much insight into its successes and failures.

The final article in this volume, "A survey of the Lomonosov technique in the theory of invariant subspaces" by Carl Pearcy and Allen Shields, is a survey of some new invariant-subspace theorems that resulted from the brilliant and elegant method of proof introduced by Victor Lomonosov early in 1973. Further study and refinement of this technique should lead to additional progress on the invariant subspace problem.

ISBN 0-8218-1513-X, Book Code SURV/13 List \$23.00; Member \$17.25 235 pages Orders must be prepaid AMERICAN MATHEMATICAL SOCIETY P. O. BOX 1571, ANNEX STATION, PROVIDENCE, RHODE ISLAND 02901

PROCEEDINGS OF SYMPOSIA IN APPLIED MATHEMATICS, Volume 20 The Influence of Computing on Mathematical Research and Education, Edited by Joseph P. LaSalle

This volume contains seven of the invited addresses and fourteen of the contributed papers that were presented at the joint American Mathematical Society and the Mathematical Association of America Conference on the Influence of Computing on Mathematical Research and Education held at the University of Montana, August 13–24, 1973.

The invited addresses were directed primarily to the influence of the computer on mathematical research and the applications of mathematics and secondarily on what this means for the teaching of mathematics and the education of mathematicians. The contributed papers describe more specifically some experiments in developing courses in mathematics with computing and algorithmic orientations and a few reports on computer influenced research.

The titles of the seven invited addresses and their authors follow:

The Influence of Computing on Research in Number Theory by D. H. Lehmer

The Influence of Computers on Algebra by Charles C. Sims

Computational Probability and Statistics by Ulf Grenander

An Introduction to Some Current Research in Numerical Computational Complexity by J. F. Traub

Applied Mathematics and Computing by Peter D. Lax

The Unexpected Impact of Computers on Science and Mathematics by Thomas E. Cheatham, Jr.

The titles of the fourteen contributed papers and their authors follow:

Computational Complex Analysis by Peter Henrici

Combinatorial Games with an Annihilation rule by Aviezri S. Fraenkel

The Integration of Computing and Mathematics at the Open University by F. B. Lovis and R. V. M. Zahar

Real Time Computer Graphics Techniques in Geometry by Thomas Banchoff and Charles Strauss

Visual Geometry, Computer Graphics and Theorems of Perceived Type by Philip J. Davis Dual Orthogonal Series: A Case Study of the Influence of Computing upon Mathematical Theory by Robert P. Feinerman, Robert B. Kelman and Chester A. Koper, Jr.

The Design and Use of an Undergraduate Numerical Analysis Laboratory by Myron Ginsberg Statistical and Numerical Analysis: A Computer Oriented Approach by Andre R. Brousseau Some Problems in Computational Probability by Marcel F. Neuts

The Influence of Computing on Generalized Inverse Applications in Statistical Analysis by Cecil R. Hallum

- On Using the Electronic Analog Computer to Illustrate Mathematical Concepts by Tyre A. Newton
- An Inexpensive Computer Assist in Teaching Large Enrollment Mathematics Courses by Edward L. Spitznagel, Jr.
- A new Computer Oriented (Algorithmic) Linear Algebra Course—Preliminary Report by Robert Ducharme
- Computer Supplemented Business Oriented Mathematics by Kenneth L. Hankerson and Gene A. Kemper

Only some college training in mathematics is needed to read most of the volume. It should be of some interest to high school teachers of mathematics.

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INDEX TO MATHEMATICS OF COMPUTATION, 1943-1969

Edited by

Yudell L. Luke, Jet Wimp and Wyman Fair

462 + xviii pages; list price \$19.95; institutional member price \$14.96; individual member price \$9.97 ISBN 0-8218-4000-2; to order, please specify MCOMIN/1

The INDEX TO MATHEMATICS OF COMPUTATION is a compilation, by author and by subject, of all material which has appeared in MATHEMATICS OF COMPUTATION and its predecessor, MATHEMATICAL TABLES AND OTHER AIDS TO COMPUTATION, during the vears 1943-1969—twenty-three published volumes. The INDEX contains over 7,000 entries. This is an unusual compilation because of the unique character of the journal which not only publishes research papers, but also publishes reviews of material on mathematics of computation and a table errata section covering a number of other publications. In addition, an unpublished mathematical tables (UMT) file is maintained.

A new classification system, which was developed in 1969 by a committee chaired by Yudell Luke at the Midwest Research Institute, is used in the *subject classification index*. In this section, all articles, tables, reviews, etc. are classified. The classification scheme is designed as an indexing system for retrieval of information in MATHEMATICS OF COMPUTATION, and the present index contains classification numbers for all entries beginning with 1943.

The author index has been set up so that it gives bibliographical information on all of the items published in the journal. Information in this index includes title of article; translated title of books not in English, French, German, or Italian; title of book which is a collection of articles written by a number of authors; volume, year, and number of pages; publisher of a book; journal title of periodicals, information concerning translations; MATHEMATICAL REVIEWS numbers; subject classification. Items are listed alphabetically by author and chronologically under each author's entry. Each article is identified by both an ordering numeral and an identifying code in order that the reader may tell quickly whether the information listed concerns a primary research publication, a review of the work, errata to a table, or any of the other types of information covered by the journal. The following codes are used:

P = research paper	$\mathbf{E} = \mathbf{table errata}$
$\mathbf{R} = \mathbf{review} \text{ of article or book}$	$\mathbf{Q} = \mathbf{q}$ ueries and replies
T = table in UMT (unpublished mathematical table) file	M = microfiche

These identifying codes appear in the subject classification index, also, to refer the reader to the author index.

The identifying code appearing in both indexes provides a cross-reference system that enables the reader to retrieve information rapidly. For example, if the following entry appears in the subject classification index

> 3.10 Linear Equations Smith, A. B. 1 PR, 3E

the reader knows immediately that under Smith's name in the *author index*, there will be listed both a research article on the subject of linear equations, written by Smith, and a review of the article. In addition, an errata to a table which Smith published in another journal will be listed.

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The editorial committee would welcome readers' comments about this microfiche feature. Please send comments to Professor James H. Bramble, MATHE-MATICS OF COMPUTATION, Center for Applied Mathematics, 275 Olin Hall, Cornell University, Ithaca, New York 14853.

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