REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS

The numbers in brackets are assigned according to the indexing system printed in Volume 28, Number 128, October 1974, pages 1191–1194.

9 [5.05.2.1, 5.05.3.1].—P. Brenner, B. Thomée & L. B. Wahlbin, *Besov Spaces and Applications to Difference Methods for Initial Value Problems*, Springer-Verlag, Berlin, Heidelberg and New York, 1975, 154 pp., 24 cm. Price $7.80.

The authors of this well-written volume have all made important contributions to the study of finite-difference methods for initial value problems of partial differential equations. The main question addressed by this book is the extent by which the accuracy of a finite-difference method suffers when the initial data is not smooth enough. A quite complete theory, including inverse results, is presented. A few model problems, rather than general parabolic systems, etc., are treated to simplify the presentation. The theory is developed in L_p for general p. The authors show how only a few, well-chosen, extra technical tools are required to extend the theory from L_2 to the general case.

The first two chapters contain introductory material on Fourier multipliers and Besov spaces. This highly useful material has not, to my knowledge, previously been presented in English with a comparable clarity.

Chapter 3 surveys the theory of well-posed initial value problems and stable finite-difference schemes with constant coefficients. Chapter 4 treats the heat equation in a very complete way. A discussion of the effects of smoothing of the initial data is included. The theory for hyperbolic problems is developed in the next chapter. Thomée's interesting application of Besov spaces to a semilinear problem is included. The last chapter, which includes previously unpublished material, develops a theory for the Schrödinger equation.

Olof Widlund

Courant Institute of Mathematical Sciences
New York University
251 Mercer Street
New York, New York 10012

10 [2.00, 3.00, 4.00].—G. M. Phillips & P. J. Taylor, *Theory and Applications of Numerical Analysis*, Academic Press, London and New York, 1973, x + 380 pp., 23 cm. Price $14.95 paperbound.

This is an introductory text for use at the undergraduate level. It is organized in such a way that the only prerequisite is a one-year course in calculus. Background material in linear algebra, e.g., is provided in the appropriate chapters. Although the selection and treatment of topics is fairly conventional, the exposition is exceptionally clear and, to the extent possible, fully supported by mathematical theory. Among topics not included (or only briefly mentioned) are rational and spline approximation, Fourier analysis, polynomial equations, optimization, sparse matrices, overdetermined systems of linear equations, algebraic eigenvalue problems, and partial differential equations. Advanced topics, such as best (polynomial) approximation, systems of nonlinear equations, and boundary value problems for ordinary differential equations, on the other hand, are treated in some detail.

The chapter headings are as follows: 1. Introduction, 2. Basic analysis, 3. Taylor's polynomial series, 4. The interpolating polynomial, 5. "Best" approximation, 6. Numerical differentiation and integration, 7. Solution of algebraic equations of one variable, 8. Linear equations, 9. Matrix norms and applications, 10. Systems of non-linear equations, 11. Ordinary differential equations, 12. Boundary value and other methods for ordinary differential equations. Appendix: Computer arithmetic.

Each chapter has a collection of problems. Solutions to selected problems are given at the end of the book. As is appropriate for a book on this level, there are no references to the literature; instead, there is a list of some key texts.

W. G.

11 [2.05].– V. F. DEMYANOV & V. N. MALOZEMOV, *Introduction to Minimax*, John Wiley & Sons, New York, 1974, vii + 307pp., 25cm. Price $20.00.

This book is a thorough introduction to mathematical optimization and is intended for electrical engineers in Russia. The content is outlined.

I. *Tchebycheff approximation by polynomials–discrete case.* The problem is motivated by a data analysis application, formulated precisely and the basic mathematical results (existence, uniqueness and altervation) developed. Two computational methods and the linear programming interpretation are given.

II. *Tchebycheff approximation by polynomials–continuous case.* The development is similar to Chapter I along with various convergence results. The Remes algorithm and discretization method are analyzed in detail.

III. *The discrete minimax problem.* The problem is formulated precisely and various elementary properties developed. The necessary condition (derivative equal zero) and several sufficient conditions for a solution are given. The coordinate direction and steepest descent methods are presented and then three successive approximation methods are analyzed. This is the key chapter of the book.

IV. *The discrete minimax problem with constraints.* The complications introduced by constraints are examined in an analysis somewhat in parallel with Chapter III.

V. *The generalized problem of nonlinear programming.* The generalization of the previous problems is developed along with basic results. Lagrange multipliers and the Kuhn-Tucker theorem are presented. The generalization of the descent and successive approximation methods are presented along with the penalty function method.

VI. *The continuous minimax problem.* The final level of generality and abstraction is reached and developed. Discretization is analyzed and the final two sections return to polynomial approximation.

VII. *Appendices and notes.* There are 60 pages of mathematical material and a short set of notes.

The style of the book is definitely tutorial. It goes from the concrete to the abstract and there are numerous detailed examples. New notation is frequently introduced. A student who covers this material will have a solid background in mathematical optimization.

The principal weakness of the book is that it is not up-to-date. In some areas the mathematical aspects have developed considerably beyond that presented here. For example, the Remes algorithm is shown to be linearly convergent but over 10 years ago H. Werner showed it to be quadratically convergent. The newer and more effective methods such as Davidon, variable metric, Fletcher-Powell, etc. are not mentioned for the nonlinear programming problems. The influence of high speed computers is not seen; the aim of this book is the treatment of small problems.

The translation is of high quality and no misprints were noted. The references are alphabetized according to the Russian spellings.

J. R.

12 [4.00, 5.00, 6.00].—RICHARD BELLMAN & MILTON G. WING, *An Introduction to Minimax*, John Wiley & Sons, Inc., New York, 1975, 250 pp., 23 cm. Price $18.95.

For over twenty years Bellman and Wing have devoted much effort to developing and popularizing a mathematical technique which they call the method of invariant imbedding. They have now collaborated on a textbook/monograph which gives a wide ranging exposition of what might be called the classical method of invariant imbedding. The authors view their approach as a perturbation method for general mathematical systems where the structure of the system is varied and a functional relationship is derived which describes the behavior of the system under such perturbations. For example, in the context of two-point boundary value problems for ordinary differential equations the solution is considered to be a function not only of the independent variable but also of the length of the interval of integration and of the boundary values. The related invariant imbedding equation then describes the behavior of the solution as these variables are changed. However, unlike some earlier books on invariant imbedding, this book is not restricted to boundary value problems for ordinary differential equations; instead, it is the authors' intent to provide a "toolchest of invariant imbedding methods" which will allow the reader to find the invariant imbedding formulation for a variety of applications.

The book consists of twelve chapters; nine of them deal with two-point boundary value problems for ordinary and partial differential equations. Others explore invariant imbedding for random walk problems, wave propagation and integral equations. Throughout, whenever possible the terminology of particle transport theory is used and much effort is devoted to obtaining the imbedding equations for various Boltzmann transport equations. An extensive collection of problems is provided at the end of each chapter.

The level of presentation throughout the book is fairly elementary; the emphasis is on deriving, and occasionally solving, the invariant imbedding equations through formal manipulation or on physical grounds. Indeed, it is the authors' expressed intent to avoid all "mathematical pseudosophistication" so that the book be accessible to a variety of readers.

The overall impression is not of a book with a concise new mathematical technique but of a compendium of novel applications of one and multiparameter (operator) continuation methods (with the range of integration as the key imbedding parameter). Numerical analysts, however, will likely find the book to be of limited value since the authors by choice do not explore the computational aspects of their method. Throughout, the claim is implicit that the invariant imbedding equations, which typically are of evolution type, are easier to solve than alternate formulations. This is neither true in general since the continuation may terminate prematurely nor helpful in those cases where the equations have classical solutions since numerical stability and machine memory limitations abound. There is computational and theoretical merit to the initial value formulation now associated with the name of invariant imbedding. However, the authors' uncritical exposition is not likely to dispel the reservations widely held against this method.

GUNTER MEYER

Department of Mathematics
Georgia Institute of Technology
Atlanta, Georgia 30302

13 [7.50].—THEODORE J. RIVLIN, *The Chebyshev Polynomials*, John Wiley & Sons, Inc., New York, 1974, vi + 186 pp., 24 cm. Price \$15.95.

This short monograph is an elegant presentation of most of what is known and interesting about the Chebyshev polynomials $T_n(x)$. The presentation appears leisurely in spite of the amount of material presented due to the careful organization of the material. The book contains four chapters as follows: Chapter 1 has basic definitions, then treats interpolation at the zeros and extrema of $T_n(x)$ and finishes with orthogonality properties. Chapter 2 gives the minimax approximation and extremal properties of $T_n(x)$. This chapter is divided into two distinct parts and each viewpoint is developed naturally from first principles. The result is compact introductions to Chebyshev approximation theory and the maximization of linear functionals. The theme of Chapter 3 is the use of expansions of functions in a series of $T_n(x)$. This material introduces many ideas and methods of numerical analysis and approximate computations. The final short Chapter 4 shows that $T_n(T_m(x)) = T_{nm}(x)$ (i.e. the Chebyshev polynomials are closed under composition) and develops the consequences of this.

The book is written so as to be useful as a text and there are over two hundred exercises. These vary from easy to difficult and also serve to present many facts without lengthening the text. The primary use of this book is as a reference for the working applied mathematician and numerical analyst. It gathers together, as no other book does, the variety of material that one needs from time to time in the analysis of approximate methods or the search for counterexamples. A brief survey of the related literature (or, at least, a more complete bibliography) and a more detailed index would have enhanced its value in this respect.

The book is recommended as a welcome and unique addition to an applied mathematics library.

J. R.

14 [8.00].—J. LAURIE SNELL, *Introduction to Probability Theory with Computing*, Prentice-Hall, Inc., Englewood Cliffs, N. J., 1975, x + 294 pp. Price \$8.95 paperbound.

This is an introductory course in probability theory, with major emphasis on computer simulations and applications. The level is quite elementary; for instance, only finite probability spaces are considered. As each new concept is introduced, the student is presented with a simple computer program and sample run illustrating the concept. Thus, after only a few pages of preliminaries, the five-line program "RANDOM" is introduced, showing how to generate twenty random numbers. Such computer generated random numbers are the tool for illustrating stochastic phenomena throughout the remainder of the book. By the next page the student learns how to generate a sequence of outcomes of coin tossing, and before the course's end one can simulate the arc sine distribution of fluctuation theory and the ergodicity of a regular finite Markov chain. All programs are written in the language BASIC, with actual run printouts immediately following the program listings. Needless to say, the text will only be effective in conjunction with access to a computer facility equipped with BASIC. Also, a certain familiarity with the language (which may be acquired in a few hours) is a prerequisite. To a certain extent, though, the student is able to learn program writing skills concurrently with the theoretical material.

Chapter 1 contains the basic framework and terminology of probability, some elementary combinatorics, then conditional probability. Among the computer applications are numerical solutions to the famous "birthday" and "hat check" problems. In

Chapter 2 the author introduces random variables, expectation and variance. The important notion of a martingale is also given, and illustrated with a simple "stock market" model. Chapter 3 deals with limit theorems, but only by approximation to finite range experiments. The discussion includes the weak law of large numbers, central limit theorem and arc sine law. Each is illustrated with illuminating computer graphics and several simulations. Key ideas, e.g. Chebyshev's inequality and the reflection principle, are discussed, but details of proofs are often omitted. The final chapter gives the basic theory of finite Markov chains, culminating in the limit theorem for regular chains. The text is complemented by many problems, of greatly varying difficulty, often involving the writing of a BASIC program.

The book constitutes a novel approach to elementary probability theory, which should appeal to students and teachers interested in a computer oriented perspective. The tenor of the discussion is casual, with an emphasis on ideas rather than formalities. The computer simulations add a dimension of tangibility to the subject matter, a dimension often lacking in the modern, abstract approach to mathematics.

DAVID GRIFFEATH

Department of Mathematics
Cornell University
Ithaca, New York 14853

15 [4.10.4, 5.05.4, 5.10.3, 5.15.3, 5.20.4].—J. R. WHITEMAN, *A Bibliography for Finite Elements*, Academic Press, Inc., London, New York and San Francisco, 1975, 26 cm. Price $9.25.

16 [4.00, 5.00].—R. ANSORGE, L. COLLATZ, G. HÄMMERLIN & W. TÖRNIG, Editors, *Numerische Behandlung von Differentialgleichungen*, International Series of Numerical Mathematics, Birkhäuser Verlag, Basel, Switzerland, 1975, 355 pp., 25 cm. Price approximately $18.00.

The volume contains papers presented at a meeting organized by R. Ansorge, L. Collatz, G. Hämmerlin and W. Törnig. This meeting took place at the Mathematical Research Institute at Oberwolfach, Germany from June 9—June 14, 1974.

J. B.

17 [2.05].—L. COLLATZ & G. MEINARDUS, Editors, *Numerische Methoden der Approximationstheorie*, Band 2, International Series of Numerical Mathematics, Birkhäuser Verlag, Basel, Switzerland, 1975, 199 pp., 25 cm. Price approximately $14.00.

This volume contains papers presented at a meeting organized by L. Collatz and G. Meinardus. This meeting took place at the Mathematical Research Institute at Oberwolfach, Germany from June 3—June 9, 1973.

J. B.

18 [9].—G. SCHRUTKA v. RECHTENSTAMM, *Tabelle der (Relativ)-Klassenzahlen der Kreiskörper deren ϕ-Funktion des Wurtzelexponenten (Grad) nicht grösser als* 256 *ist,* Deutschen Akad. Wiss. Berlin, Abhandlungen, Kl. Math. Phys. Tech., 1964, No. 2, 64 pp.

This remarkable work, a labor of some twenty-eight years, has apparently gone unreviewed and unnoticed for more than a decade. It is an extension of a small table of H. Hasse [1] which in turn is an elaboration of an original work of E. Kummer [2] on cyclotomic fields.

As the title indicates, it covers fields and subfields generated by $\exp(2\pi i/f)$ whenever Euler's $\phi(f) \leqslant 256$. The tables of Kummer and Hasse are for $f \leqslant 100$. Schrutka gives data on three-hundred and thirty-eight different fields. His format is the same as that of Hasse except that he puts all those fields for which $\phi(f)$ are the same under one heading.

For each such f and each appropriate subfield is given the generating character, the order, the degree, and relative class number of the subfield and finally the product $h^*(f)$ of these class numbers, the so-called first factor of the field. This last is often a 30–50 digit integer. With each class number is given its factorization when known. Otherwise an indication "keine primzahl" after a number N means that $2^{N-1} \not\equiv 1 \pmod{N}$. Whenever $2^{N-1} \equiv 1 \pmod{N}$, Schrutka enters N as a prime although he admits in a footnote that the probability that N is composite is positive but of "the order of 10^{-6} to 10^{-10}". It would be useful to complete the proof of primality in all such cases, and some steps in this direction have already been taken.

Finally, there is a small table of $\phi(n)$ for $n = 1(1)1059$ to aid the user in entering the table.

Most of the computational effort in obtaining the huge class numbers is spent in the "norming" of character sums of the form $M_1 = \Sigma k\chi(k)$. Schrutka does not indicate precisely what method he used for norming. Metsankyla [3] and Spira [4] have suggested the use of multiprecise floating point approximations to M_1 but apparently this is pretty expensive [5]. Recently, Newman [6], unaware at the time of Schrutka, published a table of $h^*(p)$ for all primes under 200 in which he used a determinant method. A comparison of the Newman method with that of Schrutka seems to favor the latter since the class numbers appear already algebraically partially factored into numbers whose prime power divisors belong to predictable arithmetic progressions. But there is considerable room for improvement of Schrutka's algorithm.

The tables seem to be quite accurate. Newman's table is in complete agreement wherever it intersects the one under review. In testing an improved algorithm, the reviewer obtained Schrutka's 55D value of

$$h^*(257) = 54524\ 8502341923\ 0873223822\ 6255559644\ 6147642285\ 4662168321$$
$$= 257 \cdot 20738946049 \cdot \text{prime}$$

whose factorization is due to John Selfridge.

Schrutka has attempted to fit a conjectured asymptotic formula of the form

$$\log h^*(mp) = \tfrac{1}{4}\phi(mp) \log(a_m + b_m p),$$

where m is small, to the data in his table. For $m - 1$ he suggests $a_1 = .56$ and $b_1 = .0257$. This gives for $p = 257$ the value $h^*(257) = 5.37 \cdot 10^{54}$.

D. H. LEHMER

Department of Mathematics
University of California
Berkeley, California 94720

1. H. HASSE, *Über die Klassenzahl abelscher Zahlkörper,* Akademie-Verlag, Berlin, 1952, Tafel I, pp. 139–141.

2. E. KUMMER, "Über die Klassenzahl der aus n-ten Einheitswurzeln gebildeten komplexen Zahlen," *Monatsh. Preuss. Akad. Wiss. Berlin*, 1861, pp. 1051–1053.

3. T. METSANKYLA, "Calculation of the first factor of the class number of the cyclotomic field," *Math. Comp.*, v. 23, 1969, pp. 533–537.

4. R. SPIRA, "Calculation of the first factor of the cyclotomic class number," *Computers in Number Theory*, Academic Press, New York and London, 1971, pp. 149–151.

5. R. SPIRA, Personal communication.

6. M. NEWMAN, "A table of the first factor for prime cyclotomic fields," *Math. Comp.*, v. 24, 1970, pp. 215–219.

19 [7].—JEFFREY SHALLIT, *Calculation of* $\sqrt{5}$ *and* ϕ *(the Golden Ratio) to* 10,000 *Decimal Places*, ms. of 12 typewritten sheets deposited in the UMT file.

In a two-page introduction the author briefly describes his method of calculating these related numbers to 10015D on an IBM 360/75 system. He states that he successfully compared the first 4599D of his approximation to ϕ with the value given to that precision by Berg [1].

Following the tabulation of $\sqrt{5}$ and ϕ to 10000D, there appear tables of the frequency distribution of the decimal digits in each number.

As a further check on this calculation, this reviewer has successfully compared the present approximation to $\sqrt{5}$ with more extended, unpublished values of Jones [2] and of Beyer, Metropolis and Neergaard [3], which were carried to 22900D and 24576D, respectively.

J. W. W.

1. MURRAY BERG, "Phi, the golden ratio (to 4599 decimal places) and Fibonacci numbers," *Fibonacci Quart.*, v. 4, 1966, pp. 157–162.

2. M. F. JONES, 22900D *Approximations to the Square Roots of the Primes less than* 100, reviewed in *Math. Comp.*, v. 22, 1968, pp. 234–235, UMT 22.

3. W. A. BEYER, N. METROPOLIS & J. R. NEERGAARD, *Square Roots of Integers* 2 *to* 15 *in Various Bases* 2 *to* 10: 88062 *Binary Digits or Equivalent*, reviewed in *Math. Comp.*, v. 23, 1969, p. 679, UMT 45.

20 [9].—P. BARRUCAND, H. C. WILLIAMS & L. BANIUK, *Table of Pure Cubic Fields* $Q(\sqrt[3]{D})$ *for* $D < 10^4$, University of Manitoba, 1974, 133 pages computer output deposited in the UMT file.

There are 8122 distinct pure cubic fields $Q(\sqrt[3]{D})$ for $1 < D < 10^4$. They are listed here in order of D, not in order of their discriminants $-3k^2$. For the calculation of k, see the paper [1] in this issue for which this table was computed. There are listed here D; k; J, the period length of Voronoi's algorithm for computing the fundamental unit; R, the regulator to 10S; h, the class number; and $\Phi(1) = 2\pi hR/\sqrt{3}\,k$, Artin's function, to 10D.

Concerning Tables 1–5 in [1], the following comments may be of interest. In Table 1, for every natural number $n < 53$, there is at least one D for which $n|h$. But there are none here for $n = 53, 55, 59, \ldots$. In analogy with the results of Yamamoto [2] and Weinberger [3] for real quadratic fields, it is reasonable to conjecture that every n will be a divisor as $D \longrightarrow \infty$. One finds no less than 142 D here with $81|h$, but since the class groups are not computed in [1], nor even the 3-rank r_1 (see Section 7), it is left open whether $r_1 = 4$ or 5 occurs for $D < 10^4$.

Table 2 shows that the density of D with $h = 1$ declines as D increases. Of course, the density must $\longrightarrow 0$ since almost all D will have $3|h$ (and even $3^n|h$) as $D \longrightarrow \infty$. But if one restricts D to the primes $q \equiv 2 \pmod 3$, then $3 \nmid h$, and it is reasonable to ask if the number of $Q(\sqrt[3]{q})$ having $h = 1$ has an asymptotic density as $q \longrightarrow \infty$. That is plausible. I find that 294 of the 617q here have $h = 1$ and the density remains close to 48%. It would be of interest to extend the table of such $Q(\sqrt[3]{q})$ having $h = 1$ for $q > 10^4$ to study this further. Since the Euler product method (see Section 5) should be able to distinguish $h = 1$ and $h \geqslant 2$ with a modest value of Q, this extension could be done very efficiently.

Tables 3 and 4 are analogous to the *lochamps* and *hichamps* of [4] for quadratic fields. Note that all D in Table 3 are $\equiv \pm 2, \pm 4$, or $\pm 6 \pmod{18}$. That guarantees that 2 and 3 ramify completely and thereby contribute the minimal factor 1 to $\Phi(1)$. In Table 4 all $D > 29$ are $\equiv \pm 1 \pmod{18}$, and now 2 and 3 contribute the maximal factor

2. Note that the largest and smallest $\Phi(1)$ here have the very modest ratio $3.81191/0.61997 = 6.14850$. That is much smaller than the ratios obtainable in quadratic fields with comparable discriminants, cf. [4]. The reason is that all primes $\equiv 2 \pmod 3$ split the same way in every $Q(\sqrt[3]{D})$, unless they divide D, and so the variation possible in $\Phi(1)$ is much diminished.

Note that one cannot assure an exceptionally large $\Phi(1)$ merely by selecting D that are cubic residues of all small $p \equiv 1 \pmod 3$. Thus, $(D/p)_3 = 1$ for $D = 1546$ and $p = 7, 13, 19, 31, 37, 43$. No D in Table 4 has such a long run, but $1546 \equiv -2 \pmod{18}$, its $\Phi(1)$ loses a factor of 2 as above, and so $D = 1546$ does not appear in Table 4. The $\Phi(1)$ in Table 4 are also somewhat restrained by the competition of their D with perfect cubes, cf. [4, p. 275].

In contrast to *pure* cubic fields, *cyclic* cubic fields have discriminants d that are perfect squares and all primes either split completely in the field or are inert. Thus, [5], one finds

$$\Phi(1) = 0.17377 \quad \text{and} \quad \Phi(1) = 0.16850$$

for $d = 139^2$ and 2557^2, respectively. These $\Phi(1)$ are *even smaller* than occur in comparable quadratic fields. Correspondingly, the polynomial $f(x) = x^3 - 49x^2 - 52x - 1$, having $d = 2557^2$, has a very high density of primes considering the fact that $f(x)$ is cubic. At the other extreme, $Q(x)$ for $x^3 + x^2 - 1332x + 15840 = 0$ having $d = (7 \cdot 571)^2$ has the astonishingly large $\Phi(1) = 11.63136$. This is far larger than occurs in comparable quadratic fields. As H. Stark pointed out to me, this can occur since cyclic cubic fields have Artin functions $\Phi(s)$ that are the products of *two* L functions. It would be desirable for someone to extend Littlewood's analysis [6], [4] to such cyclic (and other) algebraic fields and thereby determine bounds on their $\Phi(1)$ when the Riemann hypothesis holds.

Table 5 gives the D having champion values of R. All $D > 15$ there have $h = 1$ and one notes that the ratio R/J always remains close to 1.12 when R and J are large. For quadratic fields the analogous ratio is [7] Lévy's constant: $\pi^2/12 \ln 2 = 1.18657$. It would be interesting to obtain an analytic expression for the ratio (≈ 1.12) here, but Voronoi's algorithm is quite intricate. That makes any such analysis quite complicated relative to the quadratic case which is based upon regular continued fractions.

As stated in [1], this table was computed using a formula of Barrucand for $\Phi(1)$; and this method is said to be much faster than Dedekind's method based upon Epstein zeta functions. But there are different ways of doing the latter: if the quadratic forms and their weights are determined by trial and error factorizations, then Dedekind's method is certainly very slow for large D. But if one used *group-theoretic* methods of generating the forms and determining their weights [8, pp. 278, 281], it may go much faster. Nonetheless, it would take time: these are large discriminants and the number of forms needed goes as $O(k)$.

D. S.

1. P. BARRUCAND, H. C. WILLIAMS & L. BANIUK, "A computational technique for determining the class number of a pure cubic field," *Math. Comp.*, v. 30, 1976, pp. 312–323.

2. Y. YAMAMOTO, "On unramified Galois extensions of quadratic number fields", *Osaka J. Math.*, v. 7, 1970, pp. 57–76.

3. P. J. WEINBERGER, "Real quadratic fields with class numbers divisible by n", *J. Number Theory*, v. 5, 1973, pp. 237–241.

4. DANIEL SHANKS, *Systematic Examination of Littlewood's Bounds on* $L(1, \chi)$, Proc. Sympos. Pure Math., vol. 24, Amer. Math. Soc., Providence, R.I., 1973, pp. 267–283.

5. DANIEL SHANKS, "The simplest cubic fields", *Math. Comp.*, v. 28, 1974, pp. 1137–1152.

6. J. E. LITTLEWOOD, "On the class-number of the corpus $P(\sqrt{-k})$", *Proc. London Math. Soc.*, v. 28, 1928, pp. 358–372.

7. P. LÉVY, "Sur le développement en fraction continue d'un nombre choisi au hasard", *Compositio Math.*, v. 3, 1936, pp. 286–303.

8. DANIEL SHANKS, "Calculation and applications of Epstein zeta functions", *Math. Comp.*, v. 29, 1975, pp. 271–287.

21 [9].—RICHARD P. BRENT, *Tables Concerning Irregularities in the Distribution of Primes and Twin Primes to* 10^{11}, Computer Centre, Australian National University, Canberra, August 1975, 2 pp. + 12 computer sheets deposited in the UMT file.

These tables supersede the author's earlier incomplete UMT [1], which one can see for further detail. The previous Tables 1 and 2 are here extended to $n = 10^{11}$, and the author thereby also completes two tables in his paper [2] as follows. To Table 1, page 45, add a final row:

8×10^{10}	10^{11}	8176	16088	−5618	3037	−9881	1786

and to Table 4, page 51, add two more rows:

9×10^{10}	203710414	−6872	1.797468808649	1.90216053
10^{11}	224376048	−7183	1.797904310955	1.90216054

While these tables required a great amount of machine time, the author expresses confidence in their accuracy since the counts of $\pi(n)$ obtained here for $n = 10^{10}(10^{10})10^{11}$ agree with earlier values computed by Lehmer's method. In the extension here, from $n = 8 \times 10^{10}$ to $n = 10^{11}$, of $r_1(n) = \langle L(n)\rangle - \pi(n)$, nothing extraordinary occurs, it being a melancholy feature of these computations that computation time goes as $O(n)$ while points of interest occur as $O(\log n)$.

The downward trend of $s_3(q)$ in Fig. 3 of [2] that began at $\log_{10}(q) \approx 10.6$ continues throughout this extension with one consequence that the estimate for Brun's constant is now up to 1.9021605. But the earlier value 1.9021604 may really be more accurate according to the discussion in the previous review [1]. Of course, it still is "unknown" that there are infinitely many twin primes; there are only 224376048 pairs here. Perhaps in all mathematics there is no conjecture for which there is more supporting data. Further, this data makes it almost certain that the Hardy-Littlewood conjecture is true. On the other hand, the second-order fluctuations, observed in Fig. 3, are a complete mystery; to my knowledge they have no rational interpretation whatsoever. It is a highly repetitive feature in the history of physics that the investigation of very small second-order effects (the perihelion of Mercury, the fine-structure of the hydrogen spectrum, etc.) have repeatedly led to a radically new understanding of the main phenomenon. If that is relevant here, let the reader draw the proper inference.

D. S.

1. RICHARD P. BRENT, UMT 4, *Math. Comp.*, v. 29, 1975, p. 331.

2. RICHARD P. BRENT, "Irregularities in the distribution of primes and twin primes", *ibid.*, pp. 43–55.

22 [9].—WILLIAM J. LEVEQUE, Editor, *Reviews in Number Theory,* Amer. Math. Soc., Providence, R. I., 6 vols., 2931 pp. Price $76.00 for individual AMS members.

This collection contains all reviews of papers of an arithmetical nature which have appeared in Volumes 1–44 (1940–1972) of *Mathematical Reviews.* As such, its value to anyone interested in recent research in number theory is hard to overestimate.

The reviews are classified by a modification of the 1970 MOS classification scheme of such a nature that most of the three-hundred thirty-six sections contain about fifty reviews, the reviews appearing chronologically within each section. Since the classification is inevitably based primarily on results obtained rather than on methods used, papers involving computational methods or related to mathematics of computation are not all collected in any one place. However, the section entitled "Tables and Computation; Evaluation of Constants" contains many of them and refers to many of the others. Needless to say, several other sections, such as the section "Mersenne, Fermat Numbers", contain reviews of many computational articles. Volume 6 includes both a subject index and a name index.

While it is easy to express regret that these volumes do not cover the entire period since Dickson's *History of the Theory of Numbers* (1920), the reviewer believes that such regrets exhibit an unwillingness to face the fact that completeness is an impossible dream. After all, Dickson's history itself is incomplete since, for example, it does not cover the law of quadratic reciprocity. As it is, these volumes contain reviews of practically every article in number theory since the demise of the *Jahrbuch über die Fortschritte der Mathematik*, which is no mean accomplishment. The arithmetical community owes Professor LeVeque a tremendous debt of gratitude for his dedication in fashioning this important research tool.

PAUL T. BATEMAN

Department of Mathematics
University of Illinois
Urbana, Illinois 61801

23 [12.00].—HEWLETT PACKARD ADVANCED PRODUCTS DIVISION, HP-45 *Applications Book*, Hewlett Packard Co., Cupertino, Calif., 1974, 218 pp., 22 cm. Price $10.00 spiral-bound.

For two reasons this book will have a cursory interest to readers outside its intended audience of users of HP-series calculators.

First, keystroke sequences and examples are listed alphabetically for more than two-hundred purposes, including applications from algebra, geometry, statistics and numerical methods, among other areas. With a minimal understanding of the Polish logic of HP-series calculators, most of these sequences can easily be converted to use on other scientific calculators.

Secondly and primarily, these sequences are of interest more for their nature than for their specific solution. For they illustrate particularly graphically the recent innovations and inherent limitations of nonprogrammable calculators. To illustrate the advances in calculator capacities, they include directions for such calculations as Bessel and Gamma functions, multiple linear regression, and Gauss-Legendre quadrature, and readily suggest other potential extensions of hand-calculator usage.

At the same time, even in their most efficient form the most interesting of these routines require so many keystrokes as to be impractical in real use and to discourage efforts to create counterparts for unlisted topics. The longest sequence in the book, for three-variable linear regression, requires $155 + 32n$ keystrokes to process n 3-tuples of data, a number so large as practically to insure key misstroking.

C. MCMANUS

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