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Manuscripts should be typewritten double-spaced in the format used by the journal. For journal abbreviations, see the latest *Mathematical Reviews* volume index. An author should submit the original and one copy of the manuscript and retain one copy. The author may suggest an appropriate editor for his paper. It is recommended that the author acquaint himself with the pertinent material contained in "A Manual for Authors of Mathematical Papers," which is available from the American Mathematical Society. All contributions intended for publication and all books for review should be addressed to James H. Bramble, Chairman, Editorial Committee, Mathematics of Computation, Center for Applied Mathematics, 275 Olin Hall, Cornell University, Ithaca, New York 14853. Institutions sponsoring research reported in the journal are assessed page and microfiche charges.

Each article submitted for publication must be accompanied by a brief and reasonably self-contained abstract, and by AMS (MOS) subject classification numbers. If a list of key words and phrases is included, it will be printed as a footnote on the first page. A list of the classification numbers may be found in the Index to Mathematical Reviews, Volume 39 (June 1970).

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Edited by

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R. Tyrrell Rockafellar, *Lagrange multipliers in optimization*

O. L. Mangasarian, *Unconstrained methods in nonlinear programming*

J. E. Dennis, Jr., *A brief survey of convergence results for quasi-Newton methods*

These Proceedings are based on lectures delivered at the symposium on Nonlinear Programming held March 23 and 24, 1975, as part of the American Mathematical Society's annual New York meeting. This event was the ninth in a series of Symposia in Applied Mathematics jointly sponsored by the Society for Industrial and Applied Mathematics and the American Mathematical Society with financial support from the Energy Research and Development Agency (formerly the Atomic Energy Commission) and the National Science Foundation.

The organizing committee for the symposium consisted of R. W. Cottle (chairman), C. E. Lemke, S. M. Robinson, and J. B. Rosen. The committee's intent was to help bring to the attention of a larger mathematical audience some of the history, theory, applications and vigorous research activity of the Nonlinear Programming field. The editors feel that the results included in these Proceedings can be recommended as well to the worker in the field as to the interested initiate.

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This volume is the proceedings of a symposium held in New York City on April 18 and 19, 1973, under the joint sponsorship of the American Mathematical Society and the Society for Industrial and applied Mathematics.

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The Influence of Computing on Mathematical Research and Education,

Edited by Joseph P. LaSalle

This volume contains seven of the invited addresses and fourteen of the contributed papers that were presented at the joint American Mathematical Society and the Mathematical Association of America Conference on the Influence of Computing on Mathematical Research and Education held at the University of Montana, August 13—24, 1973.

The invited addresses were directed primarily to the influence of the computer on mathematical research and the applications of mathematics and secondarily on what this means for the teaching of mathematics and the education of mathematicians. The contributed papers describe more specifically some experiments in developing courses in mathematics with computing and algorithmic orientations and a few reports on computer influenced research.

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INDEX TO MATHEMATICS OF COMPUTATION, 1943 — 1969

Classified and Edited by

Yudell L. Luke, Jet Wimp and Wyman Fair

462 + xviii pages; list price \$19.95; institutional member price \$14.96; individual member price \$9.97

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The INDEX TO MATHEMATICS OF COMPUTATION is a compilation, by author and by subject, of all material which has appeared in MATHEMATICS OF COMPUTATION and its predecessor, MATHEMATICAL TABLES AND OTHER AIDS TO COMPUTATION, during the years 1943–1969—twenty-three published volumes. The INDEX contains over 7,000 entries. This is an unusual compilation because of the unique character of the journal which not only publishes research papers, but also publishes reviews of material on mathematics of computation and a table errata section covering a number of other publications. In addition, an unpublished mathematical tables (UMT) file is maintained.

A new classification system, which was developed in 1969 by Yudell L. Luke, Jet Wimp and Wyman Fair is used in the *subject classification index*. In this section, all articles, tables, reviews, etc. are classified. The classification scheme is designed as an indexing system for retrieval of information in MATHEMATICS OF COMPUTATION, and the present index contains classification numbers for all entries beginning with 1943.

The *author index* has been set up so that it gives bibliographical information on all of the items published in the journal. Information in this index includes title of article; translated title of books not in English, French, German, or Italian; title of book which is a collection of articles written by a number of authors; volume, year, and number of pages; publisher of a book; journal title of periodicals, information concerning translations; MATHEMATICAL REVIEWS numbers; subject classification. Items are listed alphabetically by author and chronologically under each author's entry. Each article is identified by both an ordering numeral and an identifying code in order that the reader may tell quickly whether the information listed concerns a primary research publication, a review of the work, errata to a table, or any of the other types of information covered by the journal. The following codes are used:

P = research paper

R = review of article or book

T = table in UMT (unpublished mathematical table) file

E = table errata

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These identifying codes appear in the *subject classification index*, also, to refer the reader to the *author index*.

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3.10 Linear Equations

Smith, A. B. 1 PR, 3E

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