REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS

The numbers in brackets are assigned according to the revised indexing system printed in Volume 28, Number 128, October 1974, pages 1191–1194.

24 [2.00, 3.10, 3.20, 4.00].—F. B. HILDEBRAND, *Introduction to Numerical Analysis*, 2nd ed., McGraw-Hill Book Co., New York, 1974, xiii + 669 pp., 24 cm. Price $15.50.

The first edition of this well-known introductory text was published in 1956. The present edition preserves not only the basic character of the original work, but also pretty much its content. While many changes have been made, most of them are relatively minor. Among the more substantive additions are new sections on machine errors, recursive computation, Romberg integration, and cubic spline interpolation. Also, the number of problems has increased substantially, from 513 to 670. On the whole, however, the text reflects the state of the art as it existed in the mid-fifties, when the first edition appeared. Sections entitled "Supplementary References", which accompany each chapter, serve to direct the reader to newer developments.

W. G.

25 [2.05.1].—PH. TH. STOL, *Nonlinear Parameter Optimization*, Centre for Agricultural Publishing and Documentation, Wageningen, The Netherlands, 1975, 197 pp., 24 cm. Price 49.40 Dutch guilders.

This interesting book is the author's doctoral thesis, and his abstract which we give below is more accurate than usual.

Nonlinear parameter optimization in least squares was studied from a point of view of differential geometry. Properties of curvilinear coordinates, scale factors and curvature were investigated. Parameters of the condition function were expressed as functions of algorithm parameter to generalize the formulas. The analysis of the convergence process cumulated in the development of procedures that accelerate convergence. Scale factors were used as weights to the differential correction vector to improve the direction of search. A method to correct for curvature, called back projection method, was developed. Use was made of the tangent plane on which the path of search on the fitting surface was projected. Deviations from the original direction were corrected by optimizing the angle of deviation and step factor. The correspondence between rate of convergence and curvature of the path of search was illustrated with an example. A small geodesic curvature at the starting point indicates fast convergence. Curvature properties of the parametric curves appeared to be of more influence than those of the fitting surface. To avoid heavy oscillation of intermediate parameter values a method was developed that required the intermediate points to be the foot of a perpendicular from the terminal point of intermediate observation vectors thus producing paths of controlled approach. Since condition functions may have a complicated structure in that they can be implicit functions, sequential functions or can consist of mathematical models involving alternative functions, it was treated how first derivatives can be calculated and programmed systematically for these functions. Methods introduced were made operational by means of a FORTRAN program. A description of the use of the subprograms and instructions to modify the main program to suit the various algorithms and procedures developed are given in the Appendices.

The strong point of this work is its heavy geometric flavor. Its weakness is in the failure to incorporate good numerical linear algebra into the suggested modifications of the Gauss-Newton algorithm. The author not only forms the normal equations at each step, but he even solves the system by inverting the Gram matrix he should not have formed. The algorithm thus seems inefficient in general and inaccurate for ill-conditioned problems.

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26 [2.05.1].—CHARLES L. LAWSON & RICHARD J. HANSON, *Solving Least Squares Problems*, Prentice-Hall, Inc., Englewood Cliffs, N. J., 1974, 340 pp., 24 cm. Price $16.00.

This book is intended both as a text and a reference on solving linear least squares problems. It is written from the numerical analyst's point of view and not only brings together a lot of information previously scattered in research papers, but also contains some original contributions.

The authors evidently have a great deal of hard earned experience from solving least squares problems. The strongest feature of the book is that it covers all aspects of the solution up to a set of field tested portable Fortran programs. For a reader whose immediate concern is with solving problems, it is possible to bypass the first half of the book and pass directly to the last two chapters where the practical aspects are discussed.

The first half of the book develops basic theory and algorithms both for under- and overdetermined systems. Detailed perturbation bounds for the pseudoinverse and the least squares solution are given here. Algorithms based on Householder transformations and the singular value decomposition are then described thoroughly. An algorithm based on sequential Householder reduction for the case when A has a banded structure, is given in a later chapter. Problems when A is more generally sparse are not specially treated.

Two other methods for solving linear least squares problems (normal equations and modified Gram-Schmidt) are briefly described. A more extensive coverage of these and other alternative methods (e.g. the method of Peters and Wilkinson) would have been appropriate and made the book more useful as a textbook. Another topic, which this reviewer thinks should have been included is iterative refinement of a solution.

Linear least squares problems with linear equality or inequality constraints are, however, exhaustively treated. A solution of the problem to minimize $\|Ex - f\|$ subject to $Gx \geqslant h$ is given, which depends on transforming this problem in two steps into a nonnegative least squares problem. This solution gives an elegant modularity in the algorithms for different constrained problems. Unfortunately the transformation described in Chapter 23, Section 5, contains an error, and does not work when the matrix E is rank deficient. Recently in an ICASE report A. K. Cline has shown how to perform a corresponding reduction in the general case.

The last part of the book contains descriptions and ANSI Fortran listings of subroutines for most of the algorithms described earlier in the book. This includes the Householder method, the singular value analysis, the sequential solution of a problem with a banded matrix, the nonnegative least squares solution and the least distance problem. A set of six main programs are also given for validation of these subroutines. The codes can now also be obtained in machine readable form from IMSL.

This is a very useful book, which also sets a new style for books in numerical analysis. Similar books are needed for many other problem areas.

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27 [2.05.03].—HERBERT E. SALZER, NORMAN LEVINE & SAUL SERBEN, *Hundred-Point Lagrange Interpolation Coefficients for Chebyshev Nodes*, 47 computer printout sheets, 1969, deposited in the UMT file.

Tables of Lagrange interpolation coefficients $L_i^{(100)}(x)$, where

$$L_i^{(100)}(x) = \prod_{j=1, j\neq i}^{100} (x - x_j) \Big/ \prod_{j=1, j\neq i}^{100} (x_i - x_j),$$

are given for the Chebyshev nodes

$$x_i = -\cos[(2i - 1)\pi/200], \qquad i = 1(1)100,$$

for $x = 0(0.01)1.00$, to 26S. For negative arguments, we have

$$L_i^{(100)}(-x) = L_{101-i}^{(100)}(x).$$

$L_i^{(100)}(x)$ is tabulated so that there is a separate block of four columns for each i, and is read horizontally. The argument x is not printed, and the 2nd through 26th digits are unseparated.

Three functional checks,

$$\sum_{i=1}^{100} L_i^{(100)}(x) = 1, \quad \sum_{i=1}^{100} x_i L_i^{(100)}(x) = x \quad \text{and} \quad \sum_{i=1}^{100} x_i^2 L_i^{(100)}(x) = x^2,$$

for $x = 0(0.01)1.00$, were performed upon the entries on tape before final printout, the greatest relative deviation from a true answer being $< \frac{1}{4} \cdot 10^{-21}$. The user is cautioned that these checks upon the 26S entries, prior to printout, cannot guarantee the correctness of digits on tape which occur beyond the twenty-first decimal place, or the accuracy of the printout in any place. However, it appears likely that all entries are correct to around 23S.

It was not noticed until 1975 that the printout was defective in that minus signs were not printed in all the first columns, making uncertain twenty-five percent of the entries. As the means and opportunity for reproducing a corrected version of the printout were no longer available, a careful determination was made of the locations of the missing minus signs, which were then inserted by hand.

AUTHOR'S SUMMARY (H. E. S.)

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28 [2.10].—PHILIP J. DAVIS & PHILIP RABINOWITZ, *Methods of Numerical Integration*, Academic Press, New York, 1975, xii + 459 pp., 24 cm. Price $34.50.

This book is an expanded and updated successor to the previous works on this subject by the same authors, *Numerical Integration*, Blaisdell Publishing Co., Waltham, Mass., 1967 (see *Math. Comp*, v. 22, 1968, pp. 459–460; *Math. Reviews*, v. 35, 1968, #2482). The new version is almost exactly twice the size of the old, yet retains the sparkle of the original version. The overall organization is the same, with about sixty-four new sections and subsections added, some of the latter being interpolated two deep. Even to list these would go beyond the limits of this review, so only a few high points will be noted. Chapter 1, Introduction, has been augmented by material on orthogonal polynomials and extrapolation and speed-up. Chapter 2, Approximate Integration over a Finite Interval, has been augmented by a discussion of spline interpolation with applications to numerical integration, the Kronrod scheme, and a number of other methods developed recently. Chapter 3, Approximate Integration over Infinite Intervals, contains a wealth of new material on the Fourier transform, including the discrete Fourier transform and fast Fourier transform methods, and the Laplace transform and its numerical inversion. Chapter 4, Error Analysis, in addition to other new topics, contains a greatly expanded treatment of the applications of functional analysis to numerical integration. Chapter 5, Approximate Integration in Two or More Dimensions, has a new section on the state of the art in this extremely difficult field. Chapter 6, Automatic Integration, has been supplemented by a number of new results. As one might expect, a number of programs (about eight) have been added to Appendix 2, FORTRAN Programs, and Appendix 3, Bibliography of ALGOL, FORTRAN, and PL/I Procedures has been increased by about seventy-two items. Additions have been made to Appendix 4, Bibliography of Tables, and about six hundred and forty-nine additional entries have been made to Appendix 5, Bibliography of Books and Articles, showing the feverish activity in this field, as well as the scholarly diligence of the authors.

The previous version was an excellent example of mathematical typography at its best; the present book, if anything, is even easier to read. A random inspection finds an "l" missing from Zweifel's name on p. 180, but nothing serious in the way of misprints was noted.

A mere recitation of details does not do justice to this book. Each section and subsection gives a clear statement of the basic idea discussed, its theoretical foundation, proofs (if needed), examples, and references. It is a rare achievement to produce a book which is an inspiration to the student, useful to the occasional as well as the frequent practitioner, and invaluable to the theoretician as a resource; but that is what the authors have done.

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29 [3, 13].—William C. Maguire, *Rotation Matrices $d^j_{m'm}$ for Argument $\pi/2$ in Numerically Factored Form*, ms. of seventy computer pages deposited in the UMT file, May 1975.

This unpublished table gives the rotation matrices $d^j_{m'm}(\beta)$ for arguments $\pi/2$ for integer values j from 1 to 30 in the form $2^{-k}\Pi p_i\sqrt{\Pi p_{i'}}$, p prime. The matrices are defined as in Edmonds [1]. An effort has been made to see that all integers are prime (except for powers indicated by ** powers), but the seventy-five largest integers, each greater than 100,000, have not been checked. A test calculation has been made and a third separate calculation [2] shows no differences to the latter's five available decimal places. The computations were performed at NASA/Goddard Space Flight Center on an IBM 360/91 with the main algorithms written in FORMAC and PL/I.

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1. A. R. EDMONDS, *Angular Momentum in Quantum Mechanics*, Princeton Univ. Press, Princeton, N. J., 1960.

2. B. KROHN, Private communication, 1975.

30 [7].—RICHARD P. BRENT, *Knuth's Constants to* 1000 *Decimal and* 1100 *Octal Places*, Technical Report no. 47, Computer Centre, The Australian National Univ., Canberra, A.C.T. 2600, Australia, 1975, 25 pp., 30 cm.

In appendices to the three volumes published to date of *The Art of Computer Programming* [1], Knuth lists 33 mathematical constants to 40D and 44 octal places, and suggests in Volume 2 (Exercise 4.3.1.36) that it would be worthwhile to compute them to much higher precision.

The present author has followed this suggestion by extending the precision to that stated in the title, using his Fortran multiple-precision arithmetic package on a UNIVAC 1108 computer. Each constant was computed twice, once with base 10000 and 260 floating-point digits, and once with base 11701 and 250 digits. Each run required approximately 25 minutes of computer time, and both runs for each constant produced identical results. The results were also checked by comparison with available published values, cited in the appended list of 17 references.

Specifically, the constants are the square roots of 2, 3, 5, and 10; the cube roots of 2 and 3; the fourth root of 2; the natural logarithms of 2, 3, 10, π, and ϕ (the golden ratio); the reciprocals of ln 2, ln 10, and ln ϕ; π; $\pi/180$; π^{-1}; π^2; $\pi^{1/2}$; $\Gamma(1/3)$; $\Gamma(2/3)$; e; e^{-1}; e^2; γ; e^γ; ϕ; $e^{\pi/4}$; sin 1; cos 1; $\zeta(3)$; and ln ln 2.

J. W. W.

1. D. E. KNUTH, *The Art of Computer Programming*, v. 1, *Fundamental Algorithms*; v. 2, *Seminumerical Algorithms*; v. 3, *Sorting and Searching*, Addison-Wesley, Reading, Mass., 1968–1973.

31 [8].—THE INSTITUTE OF MATHEMATICAL STATISTICS, Editors, and H. L. HARTER & D. B. OWEN, Coeditors, *Selected Tables in Mathematical Statistics*, Vol. II, Amer. Math. Soc., Providence, R. I., 1974, viii + 388 pp., 26 cm. Price \$16.40.

In a discussion of the contents of the first volume [1] of this series of statistical tables this reviewer directed his remarks to their applications and their importance to the practicing statistician. Although attention was drawn to the adequacy of the background explanation provided by the authors for specific mathematical procedures followed in developing the tables, the important questions regarding convergence properties of the relevant mathematical approaches were not addressed. The present review is written in the same vein.

As in the first volume, the tables herein relate to real problems that somehow have been neglected in the main stream of statistical literature. Perhaps the best example of this is the fixed-effect analysis-of-variance model usually discussed in the literature. It is generally assumed that the denominators of the F ratios are valid $\chi^2(\sigma^2)/f$ statistics (f being the number of degrees of freedom), and therefore, under the null hypothesis of no fixed effects, the F statistic is the correct one. Most practicing statisticians, in reality, feel very uncomfortable about this assumption; they are usually aware that the assumed model is not correct in that all the effects have *not* been accounted for, thereby truly making the denominator of the F ratio a multiple of a *noncentral* χ^2. The tables herein of Doubly Noncentral F Distribution, by M. L. Tiku, and one of the accompanying examples directly address this extremely important point. The other examples accompanying these particular tables also address problems that require more realistic models than those usually presented in the literature.

Tables 1 and 2 of the doubly noncentral F distribution give to 4D the values of the probability $P(f_1/2, f_2/2, \lambda_1, \lambda_2, u_0)$ for values of u_0 for which type I error of the

F-test equals 0.05 and 0.01 and for the ranges $f_1 = 1(1)8, 10, 12, 24, f_2 = 2(2)12, 16, 20, 24, 30, 40, 60, \phi_1 = 0(0.5)3$, and $\phi_2 = 0(1)8$. Table 3 gives 4D values of $P(f_1/2, f_2/2, \lambda_1, \lambda_2, u_0)$ for the same values of ϕ_1 and ϕ_2 and for $f_1 = f_2 = 4(2)12$, $u_0 = 0.02(.08)0.50, 0.60, 0.75, 0.95$.

While the same points could have been made about the examples accompanying the tables herein of the Doubly Noncentral t-Distribution, by William G. Bulgren, it seemed to this reviewer that a very serious problem in terminology occurs, for which there is inadequate background explanation. In particular, the exact meaning of the symbol $\bar{\mu}_i$, as contrasted to the symbol μ_i, is not made perfectly clear. As a consequence, this reviewer believes that the very important accompanying examples will not provide proper guidance for the potential user of the tables. It is hoped that this fault can be corrected in later editions because these tables can be extremely important in solving problems where the customary Student t-distribution cannot be realistically applied.

The probability integral to 6D of the doubly noncentral t-distribution with degrees of freedom n and non-centrality parameters δ and λ is tabulated over the following ranges of the parameters:

$$t = 0, \quad \delta = -4(1)5, \quad \text{any } n \text{ and } \lambda,$$
$$t = 0.1, \quad 0.2(0.2)9.0, \quad \delta = -4(1)5, \quad \lambda = 0(1)2(2)8, \quad n = 2(1)20.$$

The importance to the practicing statistician of Tables of Expected Sample Size for Curtailed Fixed Sample Size Tests of a Bernoulli Parameter, by Colin R. Blyth and David Hutchinson cannot be overemphasized. The direct benefits of these tables in attribute acceptance sampling and in reliability problems are quite obvious. They provide an entire class of sampling plans with a highly desirable minimax property (namely, that of minimizing the maximum expected sample size subject to known producer and consumer risks), and then provide an extensive tabulation of the expected sample size of these plans as functions of percent defects, sample size, and number of rejects.

The tabulation of Zonal Polynomials of Order 1 Through 12, by A. M. Parkhurst and A. T. James is an exceedingly praiseworthy undertaking and provides the means of solving a large class of multivariate problems where the distribution function or moments of the distribution function can be expressed as symmetric functions of the latent roots involved in the expression.

For the convenience of the user, two alternative sets of tables have been tabulated for evaluating the zonal polynomials. Table I gives the coefficients of the zonal polynomials in terms of the sum of the powers of the latent roots, while Table II gives the coefficients of the zonal polynomials in terms of the elementary symmetric functions of the latent roots.

It is the opinion of this reviewer, however, that the authors did not do themselves justice; their explanations of the use of these tables seemed a bit too concise and therefore may not appeal to those who would most need to use them. The authors do indicate that most expressions involving zonal polynomials are extremely complicated and it is therefore difficult to illustrate the use of the tables without burdening the reader with secondary calculations. Nevertheless, it seems to this reviewer that a middle ground could have been accomplished that would be more satisfying to those who want to use these tables. If these tables are to have a more general appeal, more examples relatable to the more familiar literature in multivariate analysis (Anderson's *Introduction to Multivariate Statistical Analysis,* for example) will have to be provided.

In summary, it can be stated that the tables in this volume, as those in the first, are addressed to a number of extremely pertinent problems confronting the practicing statistician for which tables were not previously available. However, as mentioned earlier, it is important that the terminology relating to the noncentral t tables be fully clarified so that these valuable tables can be properly understood and applied. In addition, more familiar examples are recommended to illustrate the use of the zonal polynomials, so that they will appeal to a wider class of users.

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1. THE INSTITUTE OF MATHEMATICAL STATISTICS, Editors, and H. L. HARTER & D. B. OWEN, Coeditors, *Selected Tables in Mathematical Statistics*, Vol. I, American Mathematical Society, Providence, R. I., second printing, 1973. (See *Math. Comp.*, v. 29, 1975, p. 661, RMT 32.)

32 [9].—I. O. ANGELL, *A Table of Totally Real Cubic Fields*, Royal Holloway College, Univ. of London, Surrey, England, 1975. 80 computer sheets deposited in the UMT file.

This is the table referred to in Angell's paper [1]. The 4794 nonconjugate totally real cubic fields $Q(x)$ having discriminants $D < 10^5$ are listed here in the format

$$D \quad I \quad A \quad B \quad C \quad H \quad P \quad Q \quad R \quad S \quad U \quad V \quad W \quad T.$$

Here, H is the class number and $(Px^2 + Qx + R)/S$, $(Ux^2 + Vx + W)/T$ is a fundamental pair of units. (In thirty-five fields here, one or both units have coefficients that are too large for this format and they are given in an appendix at the end of the table.) The three conjugate fields are generated by the three real roots of the polynomial

$$f(x) = x^3 - Ax^2 + Bx - C = 0 \tag{1}$$

which has index I and discriminant I^2D. The fifty-one self-conjugate (cyclic) fields included here are, of course, generated by any of the three roots.

The reader is referred to my longish review [2] of Angell's complex cubic fields for comparison with the discussion that follows. The class numbers tend to be very small here since the existence of two units implies that the regulators are relatively large. The number # of fields with class number H are as follows:

H	1	2	3	4	5	6	7	8	9
#	4184	287	268	20	19	7	7	1	1

Note the curious two-step wherein each even H has about the same population as the subsequent odd H.

The polynomial (1) follows Godwin's convention [3]; A, B and C are positive and the three roots satisfy

$$0 < x_0 < 1, \quad x_0 < x_1 < x_2, \quad 2x_1 > x_0 + x_2.$$

In the reviewer's opinion, the altered polynomial $g(x') = -f([x_2] + 1 - x')$, which has $2x_1' < x_0' + x_2'$ instead, is preferable. Since the polynomial coefficients are symmetric functions of the three roots, the smaller x_1', instead of the larger x_1, implies that the coefficients of $g(x')$ will generally be smaller than those of $f(x)$ (and sometimes they will be much smaller). In Table 2 below, we follow this AG (anti-Godwin) convention.

As in [2], the index I is not always minimized here. Of the first eight cases of $I = 2$ listed, $f(x)$ for $D = 1304, 1772, 2292, 2589$ and 2920 can be easily transformed into other equations with $I = 1$. But $D = 2089$ and the cyclic $D = 31^2$ and 43^2 must have $I = 2$ since the prime 2 splits completely in these fields. This $D = 2089 = 51^2 - 2^9$, together with subsequent examples such as $4481 = 67^2 - 2^3$ and $9281 = 97^2 - 2^7$ are of a form $D = n^2 - 2^{2m+1}$ that frequently has this property; see [4, Table 2]. On the other hand, 2 is a cubic residue of 31 and 43 and therefore splits completely in those cyclic fields.

Davenport and Heilbronn [5] proved that the nonconjugate totally real cubic fields have an asymptotic density of $[12\zeta(3)]^{-1} = 0.069326$ while the empirical average density δ here is notably smaller:

TABLE 1

$D/5000$	δ	$D/5000$	δ	$D/5000$	δ	$D/5000$	δ	$D/5000$	δ
1	.0346	5	.0426	9	.0447	13	.0462	17	.0471
2	.0382	6	.0433	10	.0451	14	.0463	18	.0474
3	.0402	7	.0442	11	.0455	15	.0469	19	.0476
4	.0418	8	.0442	12	.0459	16	.0469	20	.0479

While δ is obviously increasing with D, at $D = 10^5$ it has only attained 69% of its limit. In [2], the density of the complex fields attained 76% of its limit at $|D| = 2 \cdot 10^4$. The slow convergence in [2] and even slower convergence here do not now have a good quantitative explanation but no doubt are mostly due to the delayed appearance of D having large multiplicity m. In [1], as in [2], there are D having m distinct nonconjugate fields for $m = 2, 3$, or 4, but none with $m > 4$. (For larger D, beyond these tables, there will be D with m arbitrarily large.)

While the first $m = 4$ in [2] is for $D = -3299$, $m = 4$ does not occur here until $D = 32009$. In [2], there are twenty-two D with $m = 4$ while here there are only five such D even though there are more fields and $|D|$ can be five times as large. But for $D > 10^5$, as we show below, the proportion of D having $m = 4$ increases strongly, and if this proportion has a limit as $D \longrightarrow \infty$, cf. [5, p. 406], the slowness in attaining this limit correlates with the slow convergence of δ above.

Prior to the computation of this table there were three known cases of $m = 4$ for $D < 10^5$. Two are prime [4, p. 161]:

$$32009 = 5^6 + 4 \cdot 4^6 = 179^2 - 2^5; \quad 62501 = 1^6 + 4 \cdot 5^6$$

and one is even [6, p. 540]:

$$94636 = 4 \cdot 23659 = 4\Delta(-5).$$

The table was computed because the reviewer suggested to Professor Godwin that it would be desirable to extend his earlier table [3] in order to verify that $D = 32009$ and 94636 are indeed the smallest D and smallest even D having $m = 4$, That is true; the only new cases found here are two odd, composite D related to 32009:

$$42817 = 47 \cdot 911 = 207^2 - 2^5; \quad 72329 = 151 \cdot 479 = 269^2 - 2^5.$$

But for $10^5 < D < 2 \cdot 10^5$ there are at least eight more cases and probably about 10. There are four primes: $151141 = \Delta_2(-7)$ was given in [6, Table II] and Lakein [9, Table 5] gave

$$114889 = 339^2 - 2^5; \quad 142097 = 377^2 - 2^5,$$

together with $D = 153949$ of no known series. I found that there are exactly two even D:

$$4 \cdot 43063 \quad \text{and} \quad 4 \cdot 2 \cdot 17 \cdot 1279.$$

The odd composite D were not systematically examined. Two are known: $\Delta_6(5) = 3 \cdot 17 \cdot 2999$ is due to me and $130397 = 19 \cdot 6863$ is due to Heilbronn [3, p. 109]. Probably there are at least two or three others. So the relative proportion of D having $m = 4$ about doubles in this next interval. Other $m > 1$ will also become relatively more numerous.

In Table 2 of [2], I showed that for three known series of $D < 0$, with $m = 4$, it was possible to give the four cubic polynomials a priori. For the present fields with $D > 0$ that is no longer the case. But in the *nonescalatory* cases in [6] and [7] we can give *one* polynomial, but only one, a priori. For example, for the

$$D = A^6 + 4B^6, \quad 3 \nmid B,$$

of [7],

$$x^3 - (A^2 + B^2)x + A(A^2 + 2B^2)/3 = 0$$

gives one field. This is suitable for the $D = 32009$ and 62501 above. For the Series 1 and 2 and Complementary Series 3 and 6 of [6], one can also give one polynomial. Further, these polynomials can even be put into AG form a priori. They would give one field for the examples $D = 4\Delta(-5)$, $\Delta_2(-7)$ and $\Delta_6(5)$ above.

The smallest known [8] real $Q(\sqrt{D})$ having 3-rank = 3 is $D = 44806173$. So this D gives the smallest known case of $m = 13$. In Table 2, I give its thirteen polynomials in AG form and show how thirteen primes split (shown as S) in these thirteen fields. Compare Tables 3 and 4 in [2]. The reader is invited to transform these cubics into Angell's form and to note the effects of this upon the coefficients.

TABLE 2

$D = 44806173$

I	A	B	C	11	13	17	29	41	43	107	113	131	137	151	163	179
3	61	697	330	S	S	–	–	–	–	–	–	–	–	–	S	S
3	279	441	170	S	–	S	S	–	–	S	–	–	–	–	–	–
3	63	423	8	S	–	–	–	S	–	–	S	S	–	–	–	–
3	69	435	216	S	–	–	–	–	S	–	–	–	S	S	–	–
3	63	603	494	–	S	S	–	–	S	–	–	S	–	–	–	–
3	83	297	54	–	S	–	S	–	–	–	S	–	S	–	–	–
3	63	837	494	–	S	–	–	S	–	S	–	–	–	S	–	–
3	257	477	216	–	–	S	–	S	–	–	–	–	S	–	–	S
3	87	273	36	–	–	S	–	–	–	–	S	–	–	S	S	–
3	62	546	261	–	–	–	S	–	–	–	–	S	–	S	–	S
3	60	660	97	–	–	–	–	–	–	S	–	S	S	–	S	–
3	165	273	90	–	–	–	S	S	S	–	–	–	–	–	S	–
1	127	185	62	–	–	–	–	–	S	S	S	–	–	–	–	S

Finally, a couple of words on an erroneous first version of this table. It was instructive precisely because it was erroneous. The four class numbers for the $D = 62501$ above came out $H = 3, 3, 4, 9$. Since all H for the other four cases of $m = 4$ were divisible by 3, it did appear A) that that $H = 4$, and presumably other H, were wrong; and B) that the Gras-Callahan Theorem referred to in [2] was also valid in the real case. Georges Gras subsequently proved this B) but Frank Gerth III had already done that independently. While the thirteen H for Table 2 are not known to me, they must all be divisible by 9. The errors in A) were confirmed and corrected.

There were also errors in some units. The Artin function at argument 1 equals

$$\Phi(1) = 4RH/\sqrt{D} \tag{2}$$

where R is the regulator. Since $\Phi(1)$ is easily estimated by a determination of how all small primes split, (2) is a very powerful check on the consistency of R and H, and one can detect an error in one if the other is known. So the erroneous units were also detected and corrected. If ϵ_1 and ϵ_2 are a fundamental pair of units, then so are $\epsilon_3 = \epsilon_1^2\epsilon_2$ and $\epsilon_4 = \epsilon_1\epsilon_2$. But ϵ_3 and ϵ_2 are not a fundamental pair. Is ϵ_3 a "fundamental unit"? The moral is that it is erroneous and dangerous to speak of "a pair of fundamental units." One must say "a fundamental pair of units."

D. S.

1. I. O. ANGELL, "A table of totally real cubic fields," *Math. Comp.*, v. 30, 1976, pp. 184–187.

2. DANIEL SHANKS, UMT Review 33 of I. O. Angell, "Table of complex cubic fields," *Math. Comp.*, v. 29, 1975, pp. 661–665.

3. H. J. GODWIN & P. A. SAMET, "A table of real cubic fields," *J. London Math. Soc.*, v. 34, 1959, pp. 108–110.

4. DANIEL SHANKS, "On Gauss's class number problems," *Math. Comp.*, v. 23, 1969, pp. 151–163.

5. H. DAVENPORT & H. HEILBRONN, On the density of discriminants of cubic fields. II," *Proc. Roy. Soc. London Ser. A*, v. 322, 1971, pp. 405–420.

6. DANIEL SHANKS, "New types of quadratic fields having three invariants divisible by 3," *J. Number Theory*, v. 4, 1972, pp. 537–556.

7. DANIEL SHANKS & PETER WEINBERGER, "A quadratic field of prime discriminant requiring three generators for its class group, and related theory," *Acta Arith.*, v. 21, 1972, pp. 71–87.

8. DANIEL SHANKS, "Class groups of the quadratics fields found by F. Diaz y Diaz," *Math. Comp.*, v. 30, 1976, pp. 173–178.

9. RICHARD B. LAKEIN, "Computation of the ideal class group of certain complex quartic fields. II," *Math. Comp.*, v. 29, 1975, pp. 137–144.

33 [12.15].—R. J. ORD-SMITH & J. STEPHENSON, *Computer Simulation of Continuous Systems,* Cambridge Univ. Press, New York & London, 1975, vi + 327 pp., 23 cm. Price $9.95.

The authors have presented a good introduction to analog and hybrid computation techniques. The book is written so that students without an electronic background can follow the material. In the first chapter, for example, the operation of analog and logic components is adequately presented without detailed electronic circuitry. A more detailed description of the analog components is covered in the Appendix for those who are interested. Another favorable point is the variety of good, basic problems given at the end of several chapters.

The method of implementing a differential equation on the analog computer and the method of amplitude scaling presented in Chapter 2 are not the most convenient techniques for large scale systems. The change of variables suggested is neither necessary nor desirable when simulating a large system. However, the techniques set forth are adequate for an introductory course where simple systems are considered. The method of time scaling is well presented.

Perhaps the two chapters on function generation are too lengthy when compared with the time allotted to other more important topics. However, the material is well presented and is indeed a strong part of the text. Similarly, the chapter on analog memory is a welcome variation from most analog computer texts. More advanced analog techniques such as integration with respect to a variable other than time are also presented.

Before presenting hybrid computation, the authors discuss digital simulation of second order differential equations. A basic knowledge of computer programming is assumed. The comparison of digital and analog methods is made.

The introductory chapter on hybrid computing is excellent. The information relative to the software necessary to utilize the interface components is well presented. In the following chapters, sequential and parallel hybrid computation techniques are demonstrated by examples. Split boundary value problems and parameter optimization are given as examples of sequential operation. Examples of parallel operation include axes rotation and time delays. In the final chapter the application of simulation to the study and design of feedback control systems is introduced.

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