REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS

The numbers in brackets are assigned according to the revised indexing system printed in Volume 28, Number 128, October 1974, pages 1191–1194.

8 [3, 3.25].—JAMES R. BUNCH & DONALD J. ROSE, Editors, *Sparse Matrix Computations,* Academic Press, New York, San Francisco, London, 1976, xi + 453 pp., 24 cm. Price $15.00.

This volume contains the proceedings of a Symposium at Argonne National Laboratory on September 9–11, 1975. Twenty-six papers are presented, collected under the six headings of: I. Design and analysis of elimination algorithms, II. Eigenvalue problems, III. Optimization, least squares and linear programming, IV. Mathematical software, V. Matrix methods for partial difference equations, VI. Applications. The collection gives a comprehensive view of the field at the time.

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9 [3.10, 3.15, 5, 12.05.1].—KLAUS-JÜRGEN BATHE & EDWARD L. WILSON, *Numerical Methods in Finite Element Analysis,* Prentice-Hall, Englewood Cliffs, N. J., 1976, xv + 528 pp., 23.5 cm. Price $28.95.

This book treats most aspects of the formulation and construction of a computer program for linear finite element analysis using a conforming displacement approach. The topics involving choice of shape functions, formulation and numerical evaluation of element matrices, global assembly and solution of the equilibrium equations are given in detail, frequently with computer subroutines as examples.

One special feature which distinguishes this book from many others is the major portion devoted to eigenproblems.

I shall next reproduce a list of contents.

The exposition is clear and readable but not concise. Generally true to the title, basic numerical principles are discussed in some detail, and thereafter applied to finite element analysis. Topics not treated at length in the text (like nonconforming methods, assumed stress field and other formulations, bandwidth reducing algorithms) are adequately referenced. Practical hints are given, sometimes without much motivation (e.g., p. 102 for averaging of stresses, p. 165 for recommendation of orders for numerical integration).

The book is mainly intended as a textbook for upper class or graduate courses in engineering. The combined exphasis on basic numerical methods and their use in existing computer codes should serve that purpose well.

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10 [7].—ELDON R. HANSEN, *A Table of Series and Products,* Prentice-Hall, Englewood Cliffs, N. J., 1975, xviii + 523 pp., 28.6 cm. Price $74.00.

Transcendents arise in research in several forms—for instance, integrals, differential equations and series. Given a transcendent in one form, we often desire its other properties, and in particular we would like to know if there exists an equivalent closed or simple form to facilitate its evaluation and to simplify analytic expressions involving the transcendent.

Quite often, we are given a function either analytically or by name (for example, the Bessel function $K_\nu(z)$). Then many compendiums give series expansions in powers of z, both convergent and divergent though asymptotic. However, one can also come upon series by solving a functional equation or by expanding an integrand in series and integrating termwise. With a series in hand, one desires an equivalent simple form or identification of the series as a named function. For such a situation the author states that in his experience extant tables of series were inadequate. Either the tables were limited, or location of material was difficult. To correct these deficiencies, the author decided to develop a comprehensive handbook on the subject, and the volume under review is the fruition of his labors.

Primarily, the present volume is for use where the input is a given series and the output is its sum simply expressed or some equivalent representation. However, the volume can also be used in the reverse sense, as follows. Suppose we are given the error function $\mathrm{erf}(x)$. Then there is provided an "Index of Symbols" which directs one to input entries in the tables where the output is or involves $\mathrm{erf}(x)$. Again, suppose we are given $\cos xy \sec x$; and we desire its expansion in powers of x. Then the desired reference is given by consulting the "Index of Elementary Functions Expanded as a Series or Product".

Collecting data on series is straightforward enough, but organization of the data demands considerable effort. To produce an efficient table, it is imperative that series be given in a standard or canonical form, and that a system be developed so that if a series is in the table, then its location is easily spotted. To guide the user, the author devotes four introductory chapters where topics like references, canonical forms, errors, and ordering of tables are discussed.

We now consider the tables. Each entry is given an equation number and what would ordinarily be called Chapter X, X an integer, with a title, is composed of several sections, and each section contains a number of subsections. Subsection 5.1 has 6 entries (5.1.1–5.1.6). The 27 sections 5.1–5.27 comprise the 'chapter' "Numerical Power Series", but this chapter is given no number. It contains many entries of the form

$$\sum_{k=0}^{n} x^k \left[\prod_{i=1}^{m} (a_i k + b_i)\right]^{-1},$$

n finite or infinite, m ranges from 2 to 11, but $m = 2$ is dominant. The parameters a_i and α_i are always specified. Clearly, such series are related to integrals of the form $\int_0^x t^\gamma (1 + t)^{-1}\, dt$ and the closed expressions involve ln, arc tan, or both. Higher transcendental forms like those for incomplete gamma functions and Bessel functions are also given.

The next chapter, Sections 6–13, is called "Series Involving Rational, Factorial and Power Functions", and treats in part expressions like $\Sigma_{k=0}^{n}(\pm 1)^k z^k q_s(k)/P_r(k)$ where $q_s(k)$ and $P_r(k)$ are polynomials in k of degree s and r, respectively; and n may be finite or infinite. Other forms are also treated, as indicated by the chapter title. Many of the series are also hypergeometric in character. Quite often the coefficients in the polynomials noted above are free, and it appears that this is used to differentiate the series placed in the two chapters thus far described.

Sections 14–41 are called "Trigonometrical Series". They are of the form $\Sigma_{k=0}^{n}(\pm 1)^k a_k[\sin(kx+y)]^m$, m a positive integer, n finite or infinite. Here a_k can have various forms, e.g., rational in k, involve k factorially, etc. This chapter also treats series like the above with sin replaced by cos. This is needless duplication since one type follows from the other upon replacing y by $y + \pi/2$. Further, it is not necessary to have both $(\pm 1)^k$ since the series for one follows from the other upon replacing x by $x + \pi$.

Series involving arc tan are given in Section 42. Section 43, with 10 subsections, is concerned with series of hyperbolic functions. Series involving logarithms, e.g., $\ln(kx + y)$, $\ln\Gamma(kx + y)$, are detailed in Section 44 with 13 subsections.

"Orthogonal Polynomials" is the title of a chapter comprising Sections 45–49 with numerous subsections. Here the orthogonal polynomials are the classical ones of Jacobi (which includes Legendre and Gegenbauer or ultraspherical polynomials), Laguerre and Hermite polynomials. This chapter contains much more information than is described. In the first place, a given result involving Jacobi polynomials with parameters α and β independent of the summation index immediately gives results for Legendre, Chebyshev (both kinds) and Gegenbauer polynomials by specialization of α and β. If at least one of the parameters α, β is independent of the summation index, then from a result involving the Jacobi polynomials, the corresponding formula for the generalized Laguerre polynomials follows by application of the confluence principle. One also has results for Hermite polynomials, as they are special cases of the generalized Laguerre polynomials. Secondly, series involving the Chebyshev polynomials $T_k(x)$ and $U_k(x)$ are also trigonometrical sums, see Sections 14–41, since with $x = \cos\theta$, $T_k(x) = \cos k\theta$ and $U_k(x) = \csc\theta\sin(k+1)\theta$. Thirdly, when α and β are independent of the summation index, these tables are essentially integral transforms involving the product of the output function, the orthogonal polynomial and its associated weight function in view of the definition of orthogonality. An analogous statement also applies to many of the forms in the chapter "Trigonometrical Series".

The title of the chapter comprising Sections 50–52 with numerous subsections is "Bernoulli, Euler, and Stirling Polynomials and Numbers and Neumann Polynomials".

The chapter, "Series Involving Higher Transcendental Functions," is comprised of Sections 54–66, each with numerous subsections. This is the longest chapter and contains a wide assortment of material. There are series involving the generalized Riemann zeta function, the psi function, Legendre functions (both kinds), Bessel functions (Neumann, Schlömilch and Kapteyn series), Struve functions, incomplete gamma functions, and confluent, Gaussian and generalized hypergeometric functions. The chapter, "Series Involving Two or More Functions," (Sections 68–87, each with numerous subsections) is much akin to the chapter just described.

Sections 88.1–88.6 deal with "Multiple Series" and Sections 89–93 with "Products".

To assist in the location of material, there is a detailed table of contents and a 'subject index' which is in two parts–Index of Symbols and Index of Elementary Functions Expanded as a Series or Product. Examples of these indices have already been described.

A valuable asset of these tables is that virtually all of the entries are tagged with a reference. This facilitates checking the source and proof of the equation. Any series where a reference is lacking was derived by the author and no reference source is known to him.

Next we turn to the bibliography. We illustrate with examples. In the main text, a given series is tagged with a reference, say BY(900.07). In the bibliography, BY signifies the reference Byrd, Paul F. and M. D. Friedman, *Handbook of Elliptic Integrals for Engineers and Physicists*, Springer-Verlag, Berlin, 1954. The data 900.07 is the pertinent equation number of this reference. (The 1971 revised edition of this volume is not noted.) All references to books and reports are in a similar vein. The situation with respect to journal articles is different. Suppose a series is tagged NE 65 A p. 50 (3.4). From the bibliography, we find that NE stands for Nederl. Akad. Wetensch. Proc. The data 65 A p. 50 (3.4) stands for Vol. 65 (Series A), page 50, equation 3.4. The reader is left with no knowledge of the author and title of the publication and year of publication. I can appreciate the need for streamlining a list of references and in my own work I have often omitted titles of journal articles (titles are often deceptive). But the author and year of publication is very informative and should never be omitted. The reference list is quite lengthy, and the overall coverage of books is good, but many important references are not noted. It is difficult to judge at a glance the journal coverage because of the scanty reference data given. There are at least 145 references, and of the books and reports, all but two are dated prior to 1970. The two noted are dated 1972 and 1973. I am therefore surprised to find that the reviewer's volumes, *The Special Functions and Their Approximations*, Vols. 1, 2, Academic Press, 1969, are not listed.

This tome contains a wealth of information and should prove of much value to both pure and applied workers. The volume could have been considerably reduced in size by avoiding the duplications noted and by the introduction of other economic measures. The duplications per se are not detracting, but in today's economy, it is very important since it is reflected in the high cost of the volume. This will no doubt deter many research workers from purchasing the book. Mostly, certainly university and major reference libraries should have a copy, the price notwithstanding. This leads me to make some comments on the production of reference books where sales are not likely to be large and where the cost of setting type in print is expensive. I believe we have to give up the notion of producing volumes by conventional means, that is, books that are set in printer's type. This procedure has aesthetic appeal. However, I should like to make the point that we can do with less art and preserve usefulness and accuracy. High quality typewriters with provisions for symbols, etc. are rather commonplace. So, too, are typists who are efficient at producing technical papers. It is ridiculous to have

1. A typist prepare a manuscript which the author must proofread;
2. A copy editor mark the typed copy for the printer and perform other duties;
3. An author proofread the printed matter and the page proofs,

when, with little extra effort, the typist could prepare the manuscript for camera copy. This method produces a volume just as serviceable as one made by conventional means, eliminates at least two proofreadings, and necessitates minimal use of a copy editor. Granted the end product is not a fine piece of art. But are you looking for usefulness or art at the expense of cost? The camera copy approach is much faster and my guess is that the cost savings is at least 30%. The foregoing also applies to the production of journals.

Y. L. L.

11 [7.15, 9.10].—F. D. CRARY & J. B. ROSSER, *High Precision Coefficients Related to the Zeta Function,* MRC Technical Summary Report #1344, Univ. of Wisconsin, Madison, May 1975, 64 pp. text and 107 pp. tables.

The Riemann-Siegel formula [4], [5], [7] is the most efficient procedure known for the computation of the Riemann zeta function on the critical line $R(s) = \frac{1}{2}$. This report gives accurate coefficients for the first seven terms in the Riemann-Siegel formula.

Let $s = \frac{1}{2} + it$, $t = 2\pi\tau > 0$, $m = \lfloor \tau^{1/2} \rfloor$, $z = 2(\tau^{1/2} - m) - 1$, $n \geqslant -1$, $\theta(t) = \arg[\pi^{-\frac{1}{2}it}\Gamma(\frac{1}{4} + \frac{1}{2}it)]$, and $Z(t) = \exp(i\theta(t))\zeta(s)$. It is known that $Z(t)$ is real, so a search for zeros of $\zeta(s)$ on the critical line reduces to a search for changes of sign of $Z(t)$. The Riemann-Siegel asymptotic expansion for $Z(t)$ is

$$Z(t) = \sum_{k=1}^{m} 2k^{-\frac{1}{2}}\cos(t \cdot \ln(k) - \theta(t)) - R(t),$$

where

$$R(t) = (-1)^m \tau^{-\frac{1}{4}} \sum_{j=0}^{n} \Phi_j(z)(-1)^j \tau^{-\frac{1}{2}j} + O(\tau^{-(2n+3)/4}) \tag{1}$$

as $\tau \longrightarrow \infty$, and the $\Phi_j(z)$ are certain entire functions which may be expressed in terms of the derivatives of

$$\Phi_0(z) = \Phi(z) = \cos[\pi(4z^2 + 3)/8]/\cos(\pi z).$$

Expressions for Φ_1 to Φ_4 were given in [3]. The present report gives the derivation of (1) and expressions for Φ_1 to Φ_8. These have been verified by the reviewer, using the representation

$$\Phi_j(z) = \sum_{0 \leqslant k \leqslant 3j/4} \frac{r_{k,j}\Phi^{(3j-4k)}(z)}{4^j \pi^{2(j-k)}(3j-4k)!},$$

where

$$r_{k,j} = \begin{cases} \rho_{j/4} & \text{if } 3j = 4k \geqslant 0, \\ 0 & \text{if } k < 0 \text{ or } 3j < 4k, \\ r_{k-1,j-1} + (3j-4k-1)(3j-4k-2)r_{k,j-1} & \text{otherwise,} \end{cases}$$

and ρ_0 to ρ_{12} are given in [1] ($\rho_0 = 1$, $\rho_1 = 2$, $\rho_2 = 82$, $\rho_3 = 10572$, $\rho_4 = 2860662$, etc.).

For computational purposes the explicit expressions for $\Phi_j(z)$ are unwieldy. Much more convenient are the Taylor series coefficients $c_{j,k}$ defined by $\Phi_j(z) = \sum_{k=0}^{\infty} c_{j,k}z^k$. (Note that $c_{j,k} = 0$ if $j + k$ is odd.) Since $\Phi_j(z)$ is entire, $|c_{j,k}|$ decreases rapidly as k increases. Haselgrove and Miller [3] gave $c_{j,k}$ for $j = 0(1)4$ with an accuracy ranging from almost 20D (for $c_{0,k}$) to almost 11D (for $c_{4,k}$). The present table gives $c_{j,k}$ to 70D for $j = 0(1)6$ and $k = 0(1)(100 - 3j)$. The entries are correctly rounded to 70D (verified by the reviewer by comparison with [1]). The range includes all $c_{j,k}$ with $j \leqslant 6$ and $|c_{j,k}| > 10^{-41}$: the largest $|c_{j,k}|$ omitted is $c_{6,84} = 1.155 \times 10^{-42}$. Some auxiliary quantities (not checked by the reviewer) are also tabulated to 70D. To compensate for cancellation the computation was carried out, using the equivalent of 157 decimal digits, with Crary's multiple-precision package [2].

The report under review does not give any rigorous bounds for the error in (1).

(A subsequent report is promised.) However, we can estimate the accuracy obtainable by using (1) with $n = 6$ and the 70D tables of $c_{j,k}$. Let $M_j = \max_{-1 \leqslant z \leqslant 1} |\Phi_j(z)|$. From [1] (which gives M_j for $j = 0(1)64$) we have $M_7 < 2 \times 10^{-5}$ and $M_8 < 3 \times 10^{-6}$. Thus, so long as τ is large enough for exponentially decreasing terms to be negilgible, the error in the computed $R(t)$ should be bounded by $A\tau^{-15/4} + B$, where A is of order 3×10^{-5} and B of order 10^{-40}. For $\tau \simeq 1000$ this gives an accuracy of about 16D. To obtain greater accuracy, more terms (given in [1]) could be used in (1), or the Euler-Maclaurin formula could be used [4], [5]. For $\tau \simeq 3 \times 10^5$ (at the limit of the computation of [6]) an accuracy of about 25D is obtainable, and this should be more than enough for most applications.

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1. R. P. BRENT, *Numerical Investigation of the Riemann-Siegel Approximation*, manuscript, 1976.

2. F. D. CRARY, *Multiple-Precision Arithmetic Design With an Implementation on the Univac* 1108, MRC Technical Summary Report #1123, Univ. of Wisconsin, Madison, May 1971.

3. C. B. HASELGROVE in collaboration with J. C. P. MILLER, *Tables of the Riemann Zeta Function*, Roy. Soc. Math. Tables No. 6, Cambridge Univ. Press, New York, 1960; RMT **6**, *Math. Comp.*, v. 15, 1961, pp. 84–86. MR **22** #8679.

4. D. H. LEHMER, "Extended computation of the Riemann zeta-function," *Mathematika*, v. 3, 1956, pp. 102–108; RMT **108**, *MTAC*, v. 11, 1957, p. 273. MR **19**, p. 121.

5. D. H. LEHMER, "On the roots of the Riemann zeta-function," *Acta Math.*, v. 95, 1956, pp. 291–298; RMT **52**, *MTAC*, v. 11, 1957, pp. 107–108. MR **19**, p. 121.

6. J. B. ROSSER, J. M. YOHE & L. SCHOENFELD, *Rigourous Computation and the Zeros of the Riemann Zeta-Function*, Information Processing 68 (Proc. IFIP Congress, Edinburgh, 1968), v. 1: Mathematics, Software; North-Holland, Amsterdam, 1969, pp. 70–76. MR **41** #2892.

7. C. L. SIEGEL, *Über Riemanns Nachlass zur analytischen Zahlentheorie*, Quellen Studien zur Geschichte der Math. Astron. und Phys. Abt. B: Studien 2, 1932, pp. 45–80. (Also in "Gesammelte Abhandlungen", v. 1, Springer-Verlag, New York, 1966.)

12 [13.20].—OVE SKOVGAARD, IB A. SVENDSEN, IVAR G. JONSSON & OLE BRINK-KJAER, *Sinusoidal and Cnoidal Gravity Waves—Formulae and Tables*, Institute of Hydrodynamics and Hydraulic Engineering, Technical University of Denmark, Lyngby, Denmark, 1974, 8 pp., 21 cm. Price Dkr. 5.

This is an eight-page fold-out booklet made of plastic-covered cardboard. It contains basic formulas derived from linear theory (sinusoidal) and from nonlinear theory (cnoidal) pertaining to progressive surface water waves. The formulas provide expressions to calculate various water wave properties such as phase velocity, group velocity, mean energy density, and pressure. Evaluation of the formulas requires the use of tables of complete elliptic integrals of the first and second kind. In particular, the formula for the wave profile of a cnoidal wave is expressed in terms of the Jacobian elliptic function $\mathrm{cn}(\theta, m)$, hence the term cnoidal, analogous to sinusoidal.

For the case of sinusoidal waves basic formulas are given together with deep-water and shallow-water approximations. For the case of cnoidal waves only the basic formulas are given, as cnoidal wave theory is applicable only for shallow water (water depth small compared to wave length). In addition to the formulas there are tables (to 3 and 4S) of functions that are used to evaluate the formulas for various parameters such as wave period and wave length. Furthermore, directions are provided for using the formulas and tables to determine wave properties such as length and celerity, given other properties, as water depth, wave height, and wave period. The directions apply to the use of the formulas for waves progressing over water of constant depth (referred to as "local parameters" by the authors) as well as waves progressing over water of one depth to water of a different depth (shoaling).

It seems appropriate here to point out that, for the most part, similar and more comprehensive formulas and tables can be found elsewhere, as for example in publications of Wiegel [1], [2]. Although nothing new appears to have been presented in this booklet, nevertheless the convenient assembly in condensed form of the information herein should prove very useful to those engaged in water-wave calculations.

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1. R. L. WIEGEL, *Oceanographical Engineering,* Prentice-Hall, Englewood Cliffs, N. J., 1964.

2. R. L. WIEGEL, "A presentation of cnoidal wave theory for practical application," *J. Fluid Mech.,* v. 7, 1960, pp. 273–286.