

## REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS

The numbers in brackets are assigned according to the indexing system printed in Volume 28, Number 128, October 1974, pages 1191–1194.

**13[2.10.1]** .—MARIA ODETE RODRIGUES CADETE, *Cálculo Automático de Integrais Definidos* (Automatic Calculation of Definite Integrals), Gulbenkian Institute of Science, Scientific Computation Center, Lisbon, 1975, 173 pp., 23 cm. Price \$2.50.

This booklet was prepared in connection with expository lectures on Operations Research, held from 1972 to 1974 at the Scientific Computation Center, and courses on Automatic Computation at the Higher Technical Institute. It presents a conventional treatment of classical quadrature formulas, including those of Newton-Cotes, Gauss, Chebyshev, and Euler-Maclaurin. Notwithstanding the title, there is no discussion of automatic integration, as the term is understood today, and references to more modern treatments (e.g., [1]–[4]) are conspicuously absent from the bibliography. The appendix, however, contains FORTRAN programs for generating abscissas and weights of the classical Gauss-type quadrature formulas.

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1. P. J. DAVIS & P. RABINOWITZ, *Numerical Integration*, Ginn (Blaisdell), Boston, Mass., 1967.
2. P. J. DAVIS & P. RABINOWITZ, *Methods of Numerical Integration*, Academic Press, New York, 1975.
3. A. GHIZZETTI & A. OSSICINI, *Quadrature Formulae*, Academic Press, New York, 1970.
4. A. H. STROUD & D. H. SECREST, *Gaussian Quadrature Formulas*, Prentice-Hall, Englewood Cliffs, N. J., 1966.

**14[2.20, 13.30, 3.25]** .—MICHAEL J. TODD, *The Computation of Fixed Points and Applications*, Lecture Notes in Economics and Mathematical Systems, No. 124, Springer-Verlag, Berlin, 1976, vii + 129 pp., 25 cm. Price \$7.40.

The Brouwer fixed point theorem and its relatives are important tools in analyzing problems in applied mathematics. In particular, these theorems have proved crucial for obtaining existence results for models in mathematical economics. The problem of calculating fixed points is therefore important. In recent years, beginning with the work of H. Scarf, there has been developed a class of approximate methods for finding fixed points of a map, based on combinatorial techniques. The book under review gives a very readable account of these methods. Following is a brief description of the material in the eleven chapters.

In Chapter I there is a discussion of the Brouwer theorem, the famous lemmas of Sperner and Knaster, Kuratowski, and Mazurkiewicz, and the notions of simplex and triangulation. Chapter II contains examples of the use of fixed point theory in mathematical economics and nonlinear programming. Chapters III and IV give several triangulations of the  $n$ -simplex and determine the computational algorithms corresponding to these triangulations. Chapters V and VI contain a discussion of the Kakutani fixed point theorem and its applications in economics and nonlinear programming. Chapter VII presents an algorithm for the calculation of a Kakutani fixed point. The next three chapters present the “variable size” algorithms, in which a homotopy argument is used to automatically refine the simplicial approximation. The final chapter contains an analysis of which triangulations lead to more efficient algorithms.

The book contains little discussion of numerical experience with the various

algorithms, or with difficulties that may arise in their computer implementation. Aside from this, the book gives a useful introduction to the calculation of fixed points by simplicial methods. It is carefully and clearly written, and contains a number of examples and exercises.

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15 [2.30, 7.05].—R. WILLIAM GOSPER, JR., *Table of the Simple Continued Fraction for  $\pi$  and the Derived Decimal Approximation*, Artificial Intelligence Laboratory, Stanford University, October 1975, ms. of 248 pages,  $8\frac{1}{2}'' \times 11''$ , deposited in the UMT file.

This attractively printed table of the first 204,103 partial quotients in the simple continued fraction for  $\pi$ , conveniently arranged in consecutively numbered blocks of 100 entries (with 25 on each line), supersedes an earlier unpublished table by the same author, referred to in [1].

The derived value of  $\pi$ , displayed to 210,130D, has been successfully compared with the value to 1,000,000D, given in [2]. From theory due to Khintchine and Lévy it may be deduced that for almost all real numbers the number of decimal places corresponding to  $n$  partial quotients is approximately equal to  $K^{2n+1}$ , where  $K = \exp[\pi^2/(12 \ln 2)]$  is Lévy's constant. In particular, this yields 210,356D corresponding to the number of partial quotients listed in the present table, which is in acceptably good agreement with the accuracy actually attained.

A detailed examination of the table by this reviewer revealed a total of 30 partial quotients each exceeding  $10^4$ . In this case the Gauss-Kuzmin law predicts a total of  $[204103 \ln(10002/10001)]/\ln 2 \doteq 29$  for almost all real numbers.

These large partial quotients are as follows:

$i$	$a_i$	$i$	$a_i$
431	20776	120516	32080
15543	19055	125615	13245
23398	19308	130015	10419
28421	78629	130089	10777
51839	17538	141705	10577
61844	52403	145491	13414
70158	36848	153278	33914
81027	15365	156381	179136
81547	10528	159492	14727
85619	27192	160927	19472
86841	20567	165898	45181
96278	47475	167582	19983
109404	15366	175623	12167
109526	12586	181517	26532
115122	11295	182291	37554

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1. MTE 521, *Math. Comp.*, v. 30, 1976, p. 381.

2. J. GILLOU & M. BOUYER, *Un Million de Décimales de  $\pi$* , Commissariat à l'Energie Atomique, Paris, 1974.

16[3.25]. —SAUL I. GASS, *Linear Programming: Methods and Applications* (4th Edition), McGraw-Hill, New York, 1975, x + 406 pp., 24 cm. Price \$18.50.

Linear Programming is an elegant multifaceted subject and, nowadays, of much practical importance. It permits much flexibility in its manner of presentation, each alternative approach emphasizing a different aspect of the subject.

Dr. Gass' book, now in its fourth edition (earlier editions appearing in 1958, 1964 and 1969) has become established as one of the standard classroom texts. It is a book worth possessing. In format this edition follows the previous one; and in accordance with historical development in this area, it emphasizes practical aspects and computational procedures. Applications of Linear Programming are dealt with substantively (in a chapter expanded at the expense of some material on L.P. and the Theory of Games), as are the various forms of the Simplex Method. Perhaps less emphasis than one might hope for is placed on theorems of the alternative and the elegant theory of duality, and the nice cleavage that this provides between algorithms, namely, that the primal simplex method maintains primal feasibility and complementary slackness and seeks dual feasibility, the dual simplex method maintains dual feasibility and complementary slackness and seeks primal feasibility, while Dantzig-Wolfe decomposition techniques maintain primal and dual feasibility of the original system and seek complementary slackness. In contrast to previous editions, the revised Simplex Method is now presented before the chapter on duality.

Parametric Linear Programming, degeneracy procedures and advanced computational techniques which include new material on Generalized Upper Bounding, are dealt with at length. Many new problems have been added to each chapter, and in several cases there is new explanatory material.

Some of the more recent developments in the area of stable implementations of the Simplex Method using matrix factorizations and other numerically sound techniques of computational linear algebra are not discussed, and hopefully these will appear in the next edition. In general, this book does not deal with computer codes and the extensive interface that Linear Programming shares with Computer Science.

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