# TABLE ERRATA

**548.**—MILTON ABRAMOWITZ & IRENE A. STEGUN, Editors, *Handbook of Mathematical Functions with Formulas, Graphs, and Mathematical Tables,* National Bureau of Standards, Applied Mathematics Series, No. 55, U. S. Government Printing Office, Washington, D. C., 1964, and all known reprints.

In Table 9.5, on p. 409, the final digit in the 10D values of $j_{0,s}$ should be decreased by a unit when $s = 4, 5$, and 8, as implied by a calculation of Gerber [1]. Also, the 10D value of $J_0'(j_{0,s})$ for $s = 2$ should be increased by a unit in the last place.

A recalculation to 15S of those parts of Table 9.8 (pp. 416–422) involving $I_n(x)$ revealed the following terminal-digit errors in addition to those announced by Fettis and Caslin [2], [3]. The 10D values of $e^{-x}I_0(x)$ should each be increased by a final unit when $x = 10.4, 16.0, 18.6, 19.4$, and 19.6. The 10D values of $e^{-x}I_1(x)$ should each be similarly increased when $x = 10.4, 12.2, 13.2, 14.6, 16.0, 16.8$, and 18.0, and they should be decreased by a like amount when $x = 13.0, 15.0, 16.4, 17.2$, and 19.0. The 10D values of $x^{-2}I_2(x)$ should be increased by a final unit when $x = 0.4, 1.5, 1.6, 1.9, 3.9$, and 4.0, and they should be decreased by the same amount when $x = 0.8, 2.3, 4.5$, and 5.0. When $x = 3.4$ an increase of two final units is necessary. The 9D values of $e^{-x}I_2(x)$ should each be increased by a final unit when $x = 6.3$ and 8.0, and the 9D value of $x^{1/2}e^{-x}I_1(x)$ should be similarly increased when $x^{-1} = 0.050$.

A similar recalculation of Table 9.9 (pp. 423–424) revealed that the 5S value of $e^{-x}I_9(x)$ should be increased by a final unit when $x = 13.5$. (It should be noted that beginning with the fourth printing the value of $e^{-x}I_6(x)$ when $x = 0.2$ has been corrected to read $(- 9)1.1388$ in place of $(- 8)1.1388$.)

In Table 9.10 (pp. 425–427) the 7D value of $10^{11}x^{-11}I_{11}(x)$ should be increased by a final unit when $x = 5.0$. The 5–6D values of $10^{24}x^{-20}I_{20}(x)$ should each be decreased by a final unit when $x = 9.8$ and 19.4. The 5–6D values of $10^{26}x^{-21}I_{21}(x)$ should be similarly decreased when $x = 0.2$ and 0.8, and increased by the same amount when $x = 0.4, 5.6$ and 9.4. The 8D values of $\ln[x^{1/2}e^{-x}I_{10}(x)]$ should be increased by a final unit when $x^{-1} = 0.048, 0.047, 0.040, 0.039, 0.037, 0.025, 0.020, 0.019, 0.013, 0.008, 0.007, 0.005, 0.004$, and 0.002. The 6D values of $\ln[x^{1/2}e^{-x}I_n(x)]$ should be similarly increased for $n = 20, x^{-1} = 0.004$ and for $n = 21, x^{-1} = 0.049$.

It should be noted that all these end-figure corrections are within the error tolerance specified on p. ix of the Introduction.

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1. HENRY GERBER, "First one hundred zeros of $J_0(x)$ accurate to 19 significant figures," *Math. Comp.*, v. 18, 1964, pp. 319–322.
2. MTE 418, *Math. Comp.*, v. 22, 1968, p. 244.
3. RMT 13, *Math. Comp.*, v. 24, 1970, p. 233.

**549.**—W. A. BEYER & M. S. WATERMAN, *Decimals and Partial Quotients of Euler's Constant and* ln 2, UMT 19, *Math. Comp.*, v. 28, 1974, p. 667.

The value of $\gamma$ here to 7114D is correct only to 4879D. Starting at 4880D the

value here is ...2349 while it should be ...9424. Correspondingly, the continued fraction for $\gamma$ here is correct only to the partial quotient $a_{4793} = 3$. Beyond this it is wrong. The error was discovered when I compared these tables with Brent's work — see UMT **1** in this issue.    The authors traced it to an error in their value of ln 2 at 4879D, and attribute it to a machine error. The long history of $\pi$, $\gamma$, etc. shows that it is necessary to compute them twice, using different formulas, if the results are to be really trustworthy. These errors affect the text in the review above and also in their paper [1].

The authors have now recomputed these numbers. The new copy is marked "Revision of UMT **19**, Jan. 21, 1976." It is placed in the same folder as the original copy.

<div align="right">D. S.</div>

1. W. A. BEYER & M. S. WATERMAN, "Error analysis of a computation of Euler's constant," *Math. Comp.*, v. 28, 1974, pp. 599–604.

**550.**—I. S. GRADSHTEYN & I. M. RYZHIK, *Tables of Integrals, Series, and Products*, 4th ed., Academic Press, New York, 1965.

On p. 516, the right side of formula 4.117.5 (and its source in the tables of Bierens de Haan [1]) should read

$$- ae^{-a} - \operatorname{ch} a \ln(1 - e^{-2a}).$$

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1. D. BIERENS DE HAAN, *Nouvelles Tables d'Intégrales Définies*, Hafner, New York, 1957 (corrected reprint of the edition of 1867), p. 557, Table 388.7.

**551.**— A. GRAY, G. B. MATHEWS & T. M. MACROBERT, *A Treatise on Bessel Functions and Their Applications to Physics*, 2nd ed., Macmillan, London, 1922, reprinted by Dover Publications, New York, 1966.

Recalculation to 15S of Tables VII and VIII, using Algorithm BESLRI by Sookne [1], has revealed a total of 248 terminal-digit errors in the 9D values of $I_0(x)$ and $I_1(x)$ therein. Just three of these errors occur in the values of $I_0(x)$; namely, the final decimal should be increased by a unit for $x = 1.09$, 1.15, and 4.31.

In the 9D table of $I_1(x)$ the final decimal should be increased by a unit in 80 entries, corresponding respectively to $x = 0.03$, 0.07, 0.09, 0.14, 0.16, 0.19, 0.24, 0.32, 0.43, 0.48, 0.50, 0.55, 0.64, 0.70, 0.77, 0.80, 0.85, 0.91, 1.02, 1.03, 1.04, 1.07, 1.08, 1.09, 1.18, 1.21, 1.30, 1.33, 1.35, 1.36, 1.37, 1.39, 1.42, 1.49, 1.50, 1.51, 1.55, 1.59, 1.70, 1.83, 1.94, 1.98, 1.99, 2.03, 2.29, 2.35, 2.43, 2.44, 2.77, 2.79, 2.82, 2.83, 2.95, 3.04, 3.16, 3.23, 3.25, 3.31, 3.36, 3.40, 3.41, 3.45, 3.58, 3.59, 3.77, 3.84, 3.94, 3.97, 4.19, 4.21, 4.23, 4.25, 4.43, 4.61, 4.63, 4.77, 4.79, 4.82, 4.84, and 5.05; it should be decreased by a unit in the 124 entries corresponding to $x = 0.05$, 0.11, 0.13, 0.18, 0.21, 0.23, 0.26, 0.31, 0.33, 0.41, 0.53, 0.61, 0.62, 0.66, 0.67, 0.75, 0.99, 1.12, 1.14, 1.17, 1.29, 1.46, 1.53, 1.57, 1.64, 1.68, 1.69, 1.73, 1.75, 1.77, 1.82, 1.84, 1.85, 1.87, 1.97, 2.07, 2.10, 2.12, 2.13, 2.17, 2.27, 2.33, 2.45, 2.48, 2.49, 2.50, 2.54, 2.57, 2.58, 2.62, 2.64, 2.68, 2.69, 2.70, 2.72, 2.84, 2.85, 2.87, 2.89, 2.98, 3.03, 3.12, 3.14, 3.29, 3.30, 3.32, 3.33, 3.35, 3.44, 3.47, 3.48, 3.49, 3.51, 3.54, 3.57, 3.67, 3.68, 3.71, 3.72, 3.73, 3.74, 3.78, 3.82, 3.85, 3.88, 3.89, 3.92, 3.93, 3.95, 3.98, 4.01, 4.03, 4.04, 4.08, 4.09, 4.10, 4.12, 4.15, 4.18, 4.26, 4.29, 4.31, 4.32, 4.33, 4.34, 4.35, 4.46, 4.47, 4.49,

4.51, 4.54, 4.55, 4.56, 4.62, 4.64, 4.65, 4.66, 4.68, 4.73, 4.76, 4.83, 4.85, 4.94, and 5.10.

An increase of two final units in the values of $I_1(x)$ is required when $x = 0.87$, 0.88, 0.89, and 2.91, and a decrease of the same amount is required when $x = 0.72$, 0.73, 1.11, 1.13, 1.15, 1.16, 1.25, 2.67, 3.27, 3.52, 3.91, 4.07, 4.11, 4.27, 4.28, 4.48, 4.53, 4.69, 4.71, 4.74, 4.86, 4.89, 4.91, 5.06, and 5.09.

Furthermore, the final decimal digit of $I_1(x)$ should be increased by three units when $x = 0.86$, and it should be decreased by a similar amount when $x = 0.74$, 4.06, 4.52, 4.67, 4.72, 4.87, 4.88, 4.92, 4.93, 5.07, and 5.08.

The 41 errors amounting to two or three final units confirm the remark of Comrie on p. xv of [2] regarding the accuracy of the corresponding tables computed by Lodge, which in abridged form constitute the present tables.

The only gross error in these tables (occurring in the value of $I_1(x)$ for $x = 4.86$) has been announced in [3, p. 825].

Reference may also be made here to the more extensive 15S tables of these functions in [4].

OVE SKOVGAARD

1. D. J. SOOKNE, "Bessel functions of real argument and integer order," *J. Res. Nat. Bur. Standards Sect. B*, v. 77, 1973, pp. 125−132.

2. BRITISH ASSOCIATION FOR THE ADVANCEMENT OF SCIENCE, *Mathematical Tables*, v. VI: *Bessel Functions, Part* I, Cambridge Univ. Press, Cambridge, 1937.

3. A. FLETCHER, J. C. P. MILLER, L. ROSENHEAD & L. J. COMRIE, *An Index of Mathematical Tables*, v. II, 2nd ed., Addison-Wesley, Reading, Mass., 1962.

4. H. E. FETTIS & J. C. CASLIN, *Table of Modified Bessel Functions*, Report ARL-69-0032, Aerospace Research Laboratories, Wright-Patterson Air Force Base, Ohio, February 1969. (See *Math. Comp.*, v. 24, 1970, p. 233, RMT 13.)

**552.**—JOHN F. HART, E. W. CHENEY, CHARLES L. LAWSON, HANS J. MAEHLY, CHARLES K. MESZTENYI, JOHN R. RICE, HENRY G. THACHER, JR. & CHRISTOPH WITZGALL, *Computer Approximations*, Wiley, New York, 1968.

The following corrections should be made in this book:

| page | line | change |
|------|------|--------|
| 97 | −7 | *For* cos(x) + i sin(x), *read* cos(y) + i sin(y). |
| 100 | −5 | *For* $k^\beta$, *read* $\beta^k - 1$. |
| 102 | −2 | *For* [1, 1/16], *read* [0, 1/16]. |
| 103 | 2 | *For* 2x, *read* $2^x$. |

On p. 103, left column, lines 4−6, the three precisions quoted for the range [0, 1/20] are all less than that which can be obtained. Compare, for example, the precisions given for the wider range [0, 1/16] on the preceding page of the text.

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**553.**—W. MAGNUS, F. OBERHETTINGER & R. P. SONI, *Formulas and Theorems for the Special Functions of Mathematical Physics*, Springer-Verlag, New York, 1966.

On p. 114 the left side of the formula should read $\Gamma(\frac{1}{2} + \nu) [I_\nu(z) + L_\nu(z)]$ in place of $\Gamma(\frac{1}{2} + \nu) [I_\nu z - L_\nu(z)]$.

Also, at the top of page 170, the formula should read:

$$P^{\mu}_{-\nu-1}(x) = P^{\mu}_{\nu}(x), \quad \text{in place of} \quad P_{-\nu-1}(x) = P^{\mu}_{\nu}(x).$$

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EDITORIAL NOTE: For notices of additional errors in this and earlier editions, see *Math. Comp.*, v. 24, 1970, p. 505, MTE **464**; v. 25, 1971, p. 201, MTE **477**; v. 29, 1975, p. 672, MTE **519**; v. 30, 1976, pp. 677—678, MTE **527**.