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# NUMERICAL ANALYSIS

*edited by*

**Gene H. Golub and Joseph Oliger**

*Lecture Notes from the Short Course  
sponsored by the AMS, Atlanta, January 3–4, 1978*

"The papers given here are mainly of a mathematical nature. The results presented describe properties of computational methods that are only relevant in the context of that computation. It is the need to perform the computation which presents the problems to the subject and justifies it. For example, in the emerging field of Computational Physics methods are developed as they are *needed* for various problems. These methods are usually constructed via physical reasoning, experience, and intuition. They are often tested on problems with known solutions, but their validity is often judged on their behavior in physical terms. It is then the numerical analyst who attempts to give error estimates and describe the numerical behavior of these methods. The convergence results needed here differ from those of classical constructive analysis. Error estimates which hold for finite values of the

discretization parameters are what are really needed, as opposed to asymptotic estimates as these parameters tend to zero. The effect of rounding errors is a central issue in numerical analysis and is a unique aspect of the subject. Algorithms which are otherwise exact may be useless because of rounding errors.

Though applications are discussed here, the important relationships between the problems, the algorithms, and the machines used for the computation which are vital to the spirit of the field cannot be found here. Numerical analysis is not a textbook subject; computational experience is essential.

We hope that these manuscripts and their bibliographies will prove useful to those who wish to learn something of the nature of numerical analysis and what some of the current problems of interest are."

*— From the Preface*

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