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edited by

Gene H. Golub and Joseph Oliger

*Lecture Notes from the Short Course
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This is the collection of texts prepared by the lecturers of the Numerical Analysis Short Course given at the A.M.S. meeting in Atlanta, Georgia in January 1978. Computational linear algebra, optimization and the solution of nonlinear equations, the approximation of functions and functionals, and approximations for initial and boundary value problems for ordinary and partial differential equations are discussed. Methods such as the QR factorization, singular value decomposition, quasi-Newton and secant methods, finite difference, finite element and collocation methods are included in these discussions.

The subject matter was chosen to emphasize prominent research areas and attitudes in numerical analysis. These are introductory lectures on the subject matter for presentation to an audience of scientists from other areas or disciplines. Typically, there is an introduction to a given problem area and to techniques used, an application to applied problems, and a discussion of current research questions or directions.

Several trends in modern numerical analysis are discussed in these lectures. There has always been the quest to find the best way to do things. More realistic notions of "best" are evolving which in-

corporate the classical notions and realistic costs of producing the desired result. The discussion of good vs. best approximation is an example. More attention is being given to providing not only an answer, but a computed guarantee that it is a good answer—or a poor one. Easily computed and sharp a posteriori estimates are needed. The discussion of estimates of condition numbers is an example. There is progress being made in algorithm design based on operator splittings which allow one to take advantage of being able to solve simpler subproblems very efficiently. Updating strategies for optimization and splitting methods for differential equations are examples.

These texts should be useful to the practicing users of numerical methods, programmers, scientists, and engineers who would like to know what progress is being made on the theoretical and developmental side of the subject. They should be useful to numerical analysts to review progress in areas other than their own, and to mathematicians in general who would like to understand what the concerns of numerical analysts are. The texts should be useful for the development of seminars and reading courses in the academic environment. Many will probably find the bibliographies of current work most useful.

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