REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS

The numbers in brackets are assigned according to the revised indexing system printed in Volume 28, Number 128, October 1974, pages 1191–1194.

1[2.05.6].—CARL DE BOOR, *A Practical Guide to Splines*, Springer-Verlag, New York, 1978, xxiv + 392 pp., 23 cm. Price $14.80.

Up to now, there has been no comprehensive source of sound, practical information on how to use splines. Many scientists and engineers have recognized the advantages of using splines and piecewise polynomials and then end up with awkward, unstable or inefficient implementations. This book is truly a "practical guide to splines" and the long missing comprehensive reference work for those who use splines. A strong point of the book is that all of the basic algorithms are presented in Fortran so that the accuracy or efficiency of the calculations need not be ruined by sloppy programming. The book is not, however, just a cookbook for using these algorithms. The underlying theory is carefully and concisely developed and even the expert will find new insights and understanding of piecewise polynomials. This book is essential reading for anyone involved in spline calculations.

The first two chapters present background information about polynomial approximation and interpolation. The shortcomings of classical polynomial approximation are presented. The next two chapters present piecewise linear and cubic polynomial interpolation and form a basis for understanding the power and subtleties of piecewise polynomials. Chapter 5 presents the basic theory of approximation for splines, and this is followed by a chapter on parabolic splines which illustrates how to handle the slightly more difficult situation of even degree splines. At the end of these six chapters the reader will have learned a lot about splines and their potential, but he will not yet know how to use them effectively.

The next three chapters address the central problem for applications: the representation of piecewise polynomials. This problem is neglected by theoreticians because they do not intend to use splines, and it is overlooked by most practitioners because they do not appreciate the possibilities and pitfalls inherent in the choice of a basis. Unfortunately, there is no representation which is uniformly superior; there are times when the splines should be represented redundantly by a collection of polynomials and associated domains. The B-spline representation is better, often essential, for most calculations of splines and piecewise polynomials. These topics are presented carefully with illuminating examples.

B-splines are somewhat mysterious at first, and it is not easy to see how to manipulate them effectively. This question is addressed in Chapter 10, and three key algorithms are presented for their stable and efficient manipulation. It is shown how B-splines are equally effective for general piecewise polynomials as for splines.

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Chapter 11 presents the *B*-spline series and establishes the properties (well-conditioned, variation diminishing, etc.) of this series which make splines with this basis so effective in practice. The next chapter further explores the approximation theoretic properties of splines and introduces the important topic of knot placement. This line of investigation is continued in Chapter 13 where interpolation is studied in more depth. By this point all the machinery has been established for all the basic operations with splines.

The next three chapters present the more specialized topics of data smoothing, spline collocation for ordinary differential equations and special splines (taut, periodic and cardinal). Each of these chapters is a thorough development of the problem area complete with Fortran programs. The topics are developed carefully to illustrate how to accomplish things with the previously developed machinery, and they further illustrate the strengths and weaknesses of various ways to apply splines. These chapters contain a lot of original and important new material.

The final chapter introduces the important problem of surface approximation. Only tensor product methods are considered, but these are very important in applications. A very clever scheme is presented to take the tensor product of *Fortran programs* and thus easily extend 1-dimensional algorithms to several dimensions.

The book closes with a short discussion of "things not covered". We can hope that some day de Boor will cover these other important topics with the same elegant and penetrating manner that he used to introduce us to the use of splines.

J. R.

2[2.35].—CLAUDE BREZINSKI, *Algorithmes d'Accélération de la Convergence Etude Numérique*, Editions TechniP, 27 Rue Ginoux, 75735 Paris, France, 1978, xi + 392 pp., 24 cm. Price 195 French francs (≈ U. S. $45.00).

The author's intentions are announced at the beginning of the preface: "This book is addressed to engineers, researchers and students who need to use methods for accelerating convergence in the course of their work; it has, moreover, been written at the suggestion of a number of them. This is then an essentially practical book which presents the algorithms and their applications as well as the corresponding computer programs. . ."

In my opinion the presentation is very clear and easy to understand so that the book can serve as a textbook or a handbook for anyone in the intended audience for whom the French language is not a serious barrier. I recommend it highly.

The publisher has paid considerable attention to a pleasing and uncrowded page layout and the typing is nearly perfect—I noted only a very few errors and they were trivial. Physically, the book is a high quality paperback; its price reflects the current high costs of publishing, especially for a limited audience.

Five basic acceleration algorithms are presented. They are (1) Wynn's epsilon algorithm based on Shanks' transforms, (2) Wynn's rho algorithm based on Thiele's reciprocal differences, (3) Brezinski's theta algorithm which provides a link between the epsilon and rho algorithms, (4) Overholt's algorithm, and (5) Richardson's algorithm.

Chapter 1 presents these algorithms, first in their basic scalar form, then their extensions to vector sequences, and finally their confluent forms.

Chapter 2 contains enough of the theory of these algorithms so that the reader can use them intelligently, not blindly. For each algorithm are given background, algebraic properties, and convergence theorems; proofs are omitted. The best single source for theoretical discussion with proofs is the author's lecture notes [1] which were reviewed by Evelyn Frank in *Math. Rev.*, v. 55, 1978, #13505.

Chapter 3 shows how these acceleration procedures can be used to solve actual problems. Many numerical examples are given. Applications include scalar sequences, summation of series, analytic continuation, Fourier series and Chebyshev series, solution of equations and systems of equations both linear and nonlinear, calculation of eigenvalues, numerical integration and differentiation, inversion of the Laplace transform, roots of polynomials, differential and integral equations.

Chapter 4 treats problems related to programming and computation: stability and propagation of errors, singular and near singular rules, stopping rules and economization of storage. Finally, there are subroutines, written in FORTRAN, to implement each of the algorithms. Having experimented successfully with some of these same subroutines a few years ago, I can attest to the fact that they do work.

In conclusion, a word about the author's credentials: Claude Brezinski obtained his Ph.D. under Gastinel at Grenoble in 1971. Since that time he has been a Maître de Conférence à l'Université des Sciences et Techniques de Lille and has been an active researcher, teacher, and organizer of, or participant in, conferences in the subject area and related areas. He is eminently qualified to write this book which, to the best of my knowledge, is the first of its kind.

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1. C. BREZINSKI, *Accélération de la Convergence en Analyse Numérique*, Lecture Notes in Math., Vol. 584, Springer-Verlag, Berlin, Heidelberg, New York, 1977.

3[3.00, 4.00, 5.00, 13.15].—ISAAC FRIED, *Numerical Solution of Differential Equations*, Academic Press, New York, 1979, xiii + 261 pp., 23 cm. Price $23.50.

This is a carefully written book that uses a judicious amount of engineering, mathematical, and physical intuition to describe the properties of: the physical problems, their mathematical formulations, and their numerical solution by finite difference and by finite element methods. The author has chosen examples that illuminate how and why the numerical methods work. In particular, he deals with the steady state string and beam equations as illustrations of boundary value problems for ordinary differential equations. He returns to the time dependent cases as illustrative of wave propagation problems for partial differential equations. Finite elements, energy theorems and estimates, eigenvalue problems, lumping, stiff systems, heat conduction are some of the topics treated in the book. Engineering students and others at the senior undergraduate or first year graduate level should be able to read this remarkably self-contained book, which has a wealth of good material. Engineers and other applied mathematicians should enjoy this work with its pertinent bibliographical references at the end of each chapter, and its extensive index. Many exercise problems of varying degrees of difficulty are offered to illustrate and extend the work in the text.

E. I.

4[7.20].—HENRY E. FETTIS & JAMES C. CASLIN, *Ten-Place Tables of the Voigt and Growth Functions*, Technical Report AFFDL-TR-77-86, Air Force Flight Dynamics Laboratory, Wright-Patterson Air Force Base, Ohio, August 1977, v + 161 pp., 28 cm.

The precision stated in the title of these definitive tables is somewhat misleading; more precisely, the entries in the two main tables are given to 11S in floating-point form, as calculated on the CDC 6600/Cyber 74 systems at the Air Force Flight Dynamics Laboratory.

The Voigt function $H(a, x)$, defined by the definite integral

$$H(a, x) = \frac{a}{\pi} \int_{-\infty}^{\infty} \frac{e^{-t^2} dt}{(x - t)^2 + a^2},$$

is herein tabulated for $a = .0001(.0001).001(.001).01(.01).1(.1)1$, $x = 0(.1)20$, and $x^{-1} = .001(.001).2$.

The Growth function $G(a, y)$, defined in terms of the Voigt function by the relation

$$G(a, y) = \int_{-\infty}^{\infty} [1 - e^{-yH(a,x)}]\, dx,$$

is tabulated for the same range of the parameter a and for $\log y = -2(.1)b$, say, where b ranges from 9.0 for $a = .0001(.0001).0003$ down to 4.8 for $a = .7(.1)1$. This upper limit for $\log y$ is such that for larger values the function $G(a, y)$ may be calculated conveniently to the tabular precision from its asymptotic expansion, which is presented on page 16 (Appendix 2).

Properties of the Voigt function and the method of computing its tabulated values are presented in Appendix 1; similar information for the Growth function appears in Appendix 2.

As noted in the Introduction, the Voigt function is encountered in the study of spectral line formation under the influence of Doppler broadening, and the Growth function describes the integrated absorptance.

On the final page of the report there appears a short table (whose heading should read $A_G/2b$) which corrects and extends to from 6S to 8S the four-figure table of the Growth function in [1].

Also included in this report is a list of 12 references, which includes citations of earlier tables of these functions and their applications.

J. W. W.

1. C. VAN TRIGT, T. J. HOLLANDER & C. T. J. ALKMADE, "Determination of the a'-parameter of resonance lines in flames," *J. Quant. Spectrosc. Radiat. Transfer*, v. 5, 1965, p. 813.

5[7.30].—SALVADOR CONDE & SHYAM L. KALLA, *Tables of Bessel Functions and Roots of Related Transcendental Equations*, Graduate School, Faculty of Engineering, University of Zulia, Maracaibo, Zulia, Venezuela, 1978, iv + 284 pp., 8½″ × 11″, deposited in the UMT file.

In times past extensive tables of the Bessel functions $J_\nu(x)$, $Y_\nu(x)$, $I_\nu(x)$ and $K_\nu(x)$ were constructed, mostly for ν a positive integer or zero, $\nu = \pm 1/2$, and x variable $0 < x < \infty$. We also have tables of the Airy functions which are essentially Bessel functions of order $\pm 1/3$ and $\pm 2/3$. Tables for other values of ν are virtually nonexistent. On the other hand Luke [1] has given 205 coefficients which are accurate to 20 decimals to enable evaluation of $(2z/\pi)^{1/2}e^z K_\nu(z)$ for all $z \geqslant 5$ and all ν, $0 \leqslant \nu \leqslant 1$ by means of a double series expansion in Chebyshev polynomials. Similar and more extensive coefficients for $I_\nu(x)$, $J_\nu(x)$ and $Y_\nu(x)$ are also given by Luke [2, 3]. However, in the classical sense of table making, systematic tables in both ν and x directions are wanting. One purpose of the report at hand is to fill this gap. Based on these evaluations, some tables of zeros of certain combinations of the functions are also given. These are described later.

The volume begins with a description of the functions noted above. There is a section on applications presenting a variety of problems in engineering and physics where Bessel functions arise and where values of zeros of the functions and combinations thereof are required, especially ν-wise. The bulk of the report consists of seven tables. Where pertinent, the ν range is 0(0.1)1 and except for Table 1, each table is to 12 significant figures. The evaluations are based on appropriate use of the usual power series and asymptotic series expansions. The tables are as follows:

Table 1. $J_\nu(x)$, $Y_\nu(x)$, $I_\nu(x)$, $K_\nu(x)$, $x = 0(0.01)1.0(0.1)6.0(0.25)20$. J_ν and Y_ν are to 11 significant figures, I_ν and K_ν are to 9 significant figures.

Table 2. First 260 positive zeros of $J_\nu(x)$.

Table 3. First 260 positive zeros of $Y_\nu(x)$.

Table 4. First 28 positive roots of $hJ_0(x) - J_1(x) = 0$, $h = 0.01(0.01)2.0(0.05)5.0(0.5)30.0(2)100(10)500(50)2000$.

Table 5. First 28 positive roots of $J_\nu(x)Y_\nu(\beta x) - J_\nu(\beta x)Y_\nu(x) = 0$, $\beta = 0.1(0.1)2.0$.

Table 6. First 28 positive roots of $J_\nu(x)Y_{\nu-1}(\beta x) - J_{\nu-1}(\beta x)Y_\nu(x) = 0$, $\beta = 0.1(0.1)2.0$.

Table 7. First 28 positive roots of $J_{\nu-1}(x)Y_\nu(\beta x) - J_\nu(\beta x)Y_{\nu-1}(x) = 0$, $\beta = 0.1(0.1)2.0$.

Y. L.

1. Y. L. LUKE, "Miniaturized tables of Bessel functions," *Math. Comp.*, v. 25, 1971, pp. 323–330.

2, 3. Y. L. LUKE, "Miniaturized tables of Bessel functions," *Math. Comp.*, v. 25, 1971, pp. 789–795; v. 26, 1972, pp. 237–240.

6[7.45].—S. CONDE & S. L. KALLA, *A Table of Gauss' Hypergeometric Function* ${}_2F_1(a, b; c; x)$, Facultad de Ingeniería, División de Postgrado, Universidad del Zulia, Maracaibo, Venezuela. Text of 8 pages plus computer output of tables. Deposited in the UMT file.

The function in the title is tabulated for $a = 0.5(0.5)5.0$, $b = 0.5(0.5)5.0$, $b \geqslant a$, $c = 0.5(0.5)12$ and $x = -2.50(0.05)0.95$ to 8S. The introduction explains the method of computation and checks applied, Wronskians continuation formulas, special cases, etc. to insure the stated accuracy.

Aside from tables of the first two complete elliptic integrals, Legendre functions and polynomials, Chebyshev polynomials (both kinds) and the incomplete beta function, tables of the ${}_2F_1$ are scant. From this point of view, the present tables fill a noted gap. On the other hand, in this day of computers together with rational approximations, Padé approximations and expansions in series of Chebyshev polynomials, I believe workers will program a machine with an appropriate algorithm and generate needed values of the ${}_2F_1$ as required.

Y. L. L.

7[7.45].—S. CONDE & S. L. KALLA, *On the Zeros of* ${}_2F_1(a, b; c; x)$, División de Postgrado, Facultad de Ingeniería, Universidad del Zulia, Maracaibo, Venezuela. Twelve page report including computer output of 7 pages. Deposited in the UMT file.

The x zeros of the title function are tabulated to 7D for $a = 1.0(0.5)5.0$, $b = 1.0(0.5)5.0$, $b \geqslant a$, $c = 0.5(0.5)12.0$, $-8 < x < 1$. On the basis of the tables of ${}_2F_1(a, b; c; x)$ (see preceding review) noted above and some finer tabulations in $x(\Delta x = 0.01)$, an approximation for a zero is readily defined and then improved by the recent methods. To the best of my knowledge the tables are new.

Y. L. L.

8[9.10, 9.15].—D. W. MACLEAN, *Residue Classes of the Partition Function*, University of Saskatchewan, Saskatoon, Canada, 1979, 2 folders of approximately 80 pages each deposited in the UMT file.

These tables were computed in connection with [1].

Let $A(m, k, x) = \{n: p(n) \equiv k \pmod m \text{ and } n \leqslant x\}$, $s(m, k, x) = \text{card } A(m, k, x)$, where $p(n)$ is the number of unrestricted partitions of n.

Tables of values of:

(a) $s(m, k, 19000)$, $0 \leqslant k \leqslant m - 1$, m a prime between 29 and 97,

(b) $A(m, 0, 19000)$, $m = 5^2, 5^3, 5^4, 5^5, 7^2, 7^3, 7^4, 11^2, 11^3, 11^4$,

(c) $p(n)$, $n \leqslant 1435$,

were computed on a PDP 11/60 using Euler's pentagonal number equation $p(n) = p(n - 1) + p(n - 2) - p(n - 5) - p(n - 7) + \cdots$ with a program written in the PASCAL language.

AUTHOR'S SUMMARY

1. D. W. MACLEAN, "Residue classes of the partition function," *Math. Comp.*, v. 34, 1980, pp. 313–317.