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## Compiled and Edited by William G. Brown <br> McGILL UNIVERSITY <br> DEPARTMENT OF MATHEMATICS

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A classification scheme containing over 500 categories was developed for the purpose. Every review has been assigned one primary classification and, on the average, one secondary classification. Reviews are reprinted in strict chronological order of Mathematical Review numbers in their primary subject area, with a brief citation at each secondary location.

The final volume provides a detailed author index, which can serve as an effective bibliography of the subject.

These volumes are a research tool. They are directed to anyone who has occasion to consult the literature of graph theory: mathematicians, computer scientists, engineers, and management scientists, as well as students, teachers, and practicing researchers.

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