REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS

The numbers in brackets are assigned according to the revised indexing system printed in Volume 28, Number 128, October 1974, pages 1191–1194.

15[12.20].–PETER HENRICI, *Computational Analysis with the* HP-25 *Pocket Calculator*, Wiley, New York, 1977, xi + 280 pp., 23 cm. Price $11.50.

It might appear inappropriate to publish a review of a collection of programs for an obsolete pocket calculator, but this is not true. At a time when most interest is in algorithms which minimize the number of operations at the expense of program length and data storage, Henrici explores the domain of problems which can be solved with no more than eight data registers, a four-register stack, and a program of no more than 50 instructions. The choice of algorithm is further complicated by the fact that, like most pocket calculators, complicated functions like square root or the trigonometric functions and their inverses require the same number of program steps as an addition, and less than a two-digit constant. Although the possibilities are only indicated by example, the programs suggest a new criterion for computational complexity.

The programs solve problems from number theory(5), iteration(4), properties of polynomials(8), power series(3), integration (both quadrature and differential equations)(6), and special functions(10). Each program includes a statement of purpose, a discussion of the method, a flowchart, the program for the HP-25 with register allocations, operating instructions, and several examples with timings and results. These examples are often of considerable interest in themselves. The flowcharts are detailed enough to make programming for another calculator or computer relatively simple, even without a knowledge of the HP-25.

Almost all the programs and examples are based on methods developed in three of the author's books [1], [2], [3]. The major exception is in the programs for finding reciprocals, powers, and exponentials of formal power series, where storage limitations rule out the conventional algorithms.

Even with the increase in interactive time-sharing computer systems, many research workers and students may prefer the informality and convenience of a programmable calculator, at least for exploratory calculations. This is particularly so with the rapid increase in power for a given cost calculator. For such users, this collection of programs has much to offer. Many of the programs solve common practical problems, particularly when they are adapted to current calculators with much more storage than the HP-25. Some of the programs also illustrate the value of complex analysis in producing numerical results, and thus will provide good motivation for students of applied complex analysis, as well as a feeling for the

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domain of applicability of the techniques. The programs of this type might well be implemented on standard computers as well as on pocket calculators.

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1. P. HENRICI, *Elements of Numerical Analysis*, Wiley, New York, 1964.
2. P. HENRICI, *Applied and Computational Complex Analysis*, Vol. 1, Wiley, New York, 1974.
3. P. HENRICI, *Applied and Computational Complex Analysis*, Vol. 2, Wiley, New York, 1977.

16[5.10.3].—R. GLOWINSKI, E. Y. RODIN & O. C. ZIENKIEWICZ (Editors), *Energy Methods in Finite Element Analysis*, Wiley, New York, 1979, xviii + 361 pp., $23\frac{1}{2}$ cm. Price $43.95.

This volume is dedicated to Professor Fraeijs de Veubeke. A short biography of Professor Fraeijs de Veubeke is given at the beginning and a list of his main publications at the end.

The book contains nineteen chapters with thirty-three authors. The following description of its contents is taken from the editors' preface.

"Chapter 1, by J. T. Oden, gives the mathematical foundations of variational mechanics. It describes the various formulations existing for a given problem with a detailed discussion of the classical variational principles, the dual principles and their applications to elastostatics.

In Chapter 2, by P. G. Ciarlet and P. Destuynder, it is proved *without a priori assumptions*, that the classical two-dimensional linear models in elastic plate theory are indeed limits of the standard three-dimensional models of linear elasticity. This result is proved using variational formulations of the elastic problems and singular perturbation methods.

In Chapter 3, A. Samuelsson introduces the concept of 'global constant strain condition' to study non-conforming finite elements and shows its relationship to the well-known 'patch test'.

In Chapter 4, G. B. Warburton gives a survey of the recent developments in structural dynamics computational methods via finite elements. Modal methods and numerical integration methods are described with their main properties, and their use is discussed with many details.

In Chapter 5, by O. C. Zienkiewicz, D. W. Kelly, and P. Bettess, one studies how standard finite element methods and boundary integral methods can be coupled in order to solve, for example, boundary value problems on unbounded domains; various examples from fluid mechanics, electrotechnics, etc., illustrate the possibilities of this new class of methods.

The chapters from 6 to 11 are all related to complementary energy methods and dual variational principles applied to finite element approximations.

In Chapter 6, D. J. Allman discusses the use of compatible and equilibrium models and finite elements, applied to the stretchings of elastic plates. A new triangular equilibrium element is introduced and the properties of the associated matrix is studied in detail.

In Chapter 7, L. S. D. Morley, starting from Koiter's first approximation shell theory, develops an approximation of elastic shell problems, based on a new finite element stiffness formulation.

In Chapter 8, R. L. Taylor and O. C. Zienkiewicz show that the computational difficulties associated with complementary energy methods can be overcome by an appropriate penalty technique. Numerical examples illustrate the feasibility of the method. The two following chapters are more mathematical in nature.

In Chapter 9, P. A. Raviart and J. M. Thomas give the theoretical foundations of the dual finite element models for second order linear elliptic problems.

In Chapter 10, F. Brezzi does the same for fourth order linear elliptic problems taking the biharmonic plate bending problem to illustrate the various possible approaches.

Chapter 11, by C. Johnson and B. Mercier, is dedicated to a class of mixed equilibrium finite element methods. Applications to problems in linear elasticity, plasticity, and fluid dynamics (Navier-Stokes equations) are given. The next two chapters deal with incompressible media.

Chapter 12, by J. H. Argyris and P. C. Dunne describes a new finite element approximation for incompressible or near-incompressible materials. The method is based on displacement models and allows the use of low order elements.

Chapter 13, by R. Glowinski and O. Pironneau, discusses the approximation of the Stokes problem for incompressible fluids, by means of low order conforming elements to approximate velocity and pressure. This method is based on a new variational formulation of the Stokes problem and leads to approximate problems which can be easily solved by standard finite element Poisson solvers. Chapters 14 to 18 are related to nonlinear problems.

In Chapter 14, P. G. Bergan, I. Holand, and T. H. Soreide present a new incremental method for solving problems in nonlinear finite element analysis. The method is based on the use of a 'current stiffness parameter' which is a normalized measure of the incremental stiffness in the direction of motion. Several numerical tests illustrate the feasibility of the method.

In Chapter 15, S. Cescotto, F. Frey, and G. Fonder give a unified approach for Lagrangian description in nonlinear, large displacement, and large strain structural problems. Total and updated Lagrangian descriptions appear as particular cases of this more general theory.

In Chapter 16, B. M. Irons analyzes some of the difficulties arising from curve fitting and shows how nonlinear effects can make difficult the numerical solution of problems apparently easy to solve.

In Chapter 17, H. Matthies, G. Strang, and E. Christiansen analyze from a mathematical point of view a fairly difficult infinite dimensional saddle-point problem, which describes the duality between the static and kinematic theories of limit analysis. The theoretical difficulty lies in the fact that the natural functional spaces to study this class of problems, of great interest in perfect plasticity, are L_1 and spaces of functions of bounded variation. They introduce the space of functions of bounded deformation, required because Korn's inequality fails in L_1.

In Chapter 18, A. R. S. Ponter and P. Brown discuss a new finite element method for computing the deformation of creeping structures. The finite element formulation is discussed in detail for a strain-hardening creep relationship and computed solutions are presented for a thermally loaded plate.

The last chapter by M. Geradin concerns modal analysis. The biorthogonal Lanczos algorithm is proposed as a very efficient tool to compute the upper eigenvalue spectrum of an arbitrary square matrix."

17[4.00, 5.00].–M. K. Jain, *Numerical Solution of Differential Equations*, Wiley, New York, 1979, xiii + 443pp., 25cm. Price $16.95.

This book attempts the impossible: It is concerned with the numerical solution of ordinary as well as partial differential equations, with the associated initial value as well as boundary value problems (even eigenvalue problems are not omitted), and it assumes virtually no knowledge of the theory of differential equations or of numerical analysis. Consequently, it has to cover many pages with introductory material but cannot afford to do it in a rigorous or even seriously intuitive form for sake of brevity. There are a good number of farther reaching results; but they are often unmotivated or their context is not clear. A good deal of important material is missing and the present state of the art is not well represented. Computational aspects (e.g. step size control) are virtually unreflected.

Even as a gateway to the more specialized literature on the subject the volume is not suitable since the references are unsystematic and largely outdated. Under these circumstances, I cannot imagine for whom the book could be of any value.

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18[3.35].–F. J. Peters, *Sparse Matrices and Substructures with a Novel Implementation of Finite Element Algorithms*, Mathematical Centre Tracts 119, Mathematisch Centrum, Amsterdam, 1980, ii + 98pp., 24cm. Price Dfl. 12,–.

This short book, a revision of the author's Ph.D. thesis, takes a fresh look at the problem of solving sparse linear systems. The main result is that the finite element technique of factoring a sparse matrix as it is generated, building up in the process larger and larger factored substructures, is applicable to any sparse matrix problem. Moreover, the resulting code needs only to manipulate dense submatrices (or submatrices with a dense profile).

Unfortunately, the book is sprinkled with the claim that this new viewpoint will make all previous work on sparse equation solvers obsolete, a thesis not proved by the contents. There is no serious attempt, either analytical or experimental, to gauge the efficiency or simplicity of the methods in comparison with standard software. Only a very sketchy outline of a program is provided.

The reader wanting a thorough treatment of sparse matrix methods should look elsewhere–to George and Liu's new book, for example.

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19[3.15].—BERESFORD N. PARLETT, *The Symmetric Eigenvalue Problem*, Prentice-Hall, Englewood Cliffs, N. J., 1980, xix + 348 pp., $23\frac{1}{2}$ cm. Price, cloth $25.00.

In his preface Professor Parlett describes his book as a sequel to Chapter 5 of my 1965 book *The Algebraic Eigenvalue Problem*. As such it is of particular interest to me. Seeing the material assembled in one place, I was struck by how much progress has been made in the last fifteen years on a topic which one might have felt was already worked out.

Most of the 'classical' material in the A.E.P. is covered, but Parlett is adept at giving it a new slant. I was constantly struck by his ability to present standard material from an individualistic and challenging point of view. The old material is interwoven with numerous new results in a way which keeps the reader familiar with the topic constantly on his toes. In this area I would like to single out for special mention:

(i) The concentration on the use of Sylvester's inertia theorem for the location of eigenvalues by spectrum slicing.

(ii) The presentation of Kahan's proof that Rayleigh quotient iteration converges for almost all starting vectors.

(iii) The proof of the global convergence of the QL tridiagonal algorithm using Wilkinson's shift.

Chapter 10, on eigenvalue bounds, presents a great deal of material, some of which was available but widely scattered in the literature, but much of which lay buried in notes by Kahan. I am grateful to have a unified and lucid exposition of these results. Chapters 11, 12, and 13 concentrate mainly on large scale problems, a topic scarcely mentioned in the A.E.P.; research in this area is bound to dominate future work on the symmetric problem. They cover the material generally associated with the Rayleigh-Ritz process, Krylov subspaces, and the rejuvenated Lanczos algorithm. The latter includes a highly readable account of Paige's theorem; this is not as widely known as it should be.

A final chapter is devoted to the generalized eigenvalue problem. Although it covers much new material, including a brief account of the Fix-Heiberger reduction, singular pencils, the generalized Jacobi algorithm, and subspace iteration, I found it more fragmentary than the rest of the book. Perhaps the author could be excused for feeling that he was starting rather late in the day on a large topic and it was time the manuscript was sent to the printer.

Mention must be made of the numerous excellent exercises provided throughout the book. The reader will neglect these at his peril. They constitute an essential part of the argument.

Professor Parlett is to be congratulated on providing a book which maintains a sense of excitement from start to finish. It is scholarly without being pedantic and yet should make the material accessible to anybody capable of appreciating the meaning of the results. Last, but not least, at a time when books in general seem to be getting progressively more stodgy, this by contrast is really fun to read. It is highly recommended to all who have an interest in the eigenvalue problem.

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20[2.00].—J. STOER & R. BULIRSCH, *Introduction to Numerical Analysis*, Translated by R. Bartels, W. Gautschi, and C. Witzgall, Springer-Verlag, New York, 1980, ix + 609 pp., 24 cm. Price $24.00.

This textbook, which has been translated from a German edition, is a gold mine for anyone looking for facts concerning a large variety of methods in numerical mathematics. It is modern, not only with respect to analysis but also in selection of methods. For example, there are good surveys of the fast Fourier transform, spline functions, the simplex method, minimization methods, stiff differential equations, and the finite element method; several of these subjects are not generally covered in similar literature. On the other hand, partial differential equations as well as integral equations are missing for reasons which are explained but perhaps not too convincingly.

Looking at details, one can express criticism in some respects. Much effort and space is devoted to interpolation in spite of the fact that with general access to computers few people would nowadays perform this kind of calculation. The main interest is actually concentrated on the use of interpolation as a theoretical tool in connection with, e.g., numerical differentiation and integration. To cite one specific example, no one would perform polynomial interpolation on the function $y = \cot x$ for small x (p. 72). If such an interpolation has to be done, the auxiliary function $z = x \cot x$ should be used instead. It gives even better accuracy than rational interpolation (whose supremacy is supposed to be illustrated).

In several places clumsy notations are irritating; as an example, the theorem on page 69 could be mentioned. Quite often "recursive" notations would give a much better overview and even better insight. Far too often, readability is an underestimated quality with textbooks.

The exercises are numerous and as a rule very good and illustrative, most of them original. No answers are given; if supplied, they would add considerably to the value of the book, especially for students who study this course on their own. An appealing feature is the large number of references at the end of each chapter.

As is understood from the review above, this textbook represents an excellent modern and welcome addition to the literature in numerical mathematics.

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21[6.35].—JOHANN SCHRÖDER, *Operator Inequalities*, Academic Press, New York, 1980, xvi + 367 pp., $23\frac{1}{2}$ cm. Price $39.50.

The subject of this book, operator inequalities, is one which has wide application to various branches of analysis. The author sets up the abstract framework in the first chapter, where he introduces the notions of ordered linear spaces and positive linear operators. There he also states the fixed point theorems and the iterative procedures he will need in the sequel. In the subsequent chapters he extends the abstract ideas and presents a number of applications.

For example, in Chapter II, he introduces the notions of inverse positive linear operators and connected sets of operators. There he applies these ideas to the theory of M and Z matrices and gives some numerical applications. He also applies these ideas to linear second order ordinary differential inequalities and places the maximum principle in this framework. Other applications are made to oscillation theory and eigenvalue problems for second order o.d.e.s.

In Chapter III, the concept of inverse monotone operators is introduced. Here the emphasis is directed toward the study of two joint boundary value problems for *nonlinear* second order o.d.e.s. Chapter IV is concerned with an estimation theory for linear and nonlinear operators, the principal applications being to functional differential equations and nonlinear functions on finite-dimensional spaces. Chapter V deals with vector valued differential operators and systems of ordinary differential equations.

This book has several pleasant features. The inclusion of problems at various points in the text not only helps the reader toward understanding but also makes the book suitable for use in a special topics graduate course. There are also very informative notes at the end of each chapter which refer to a bibliography of approximately three hundred and fifty items. The English, while stilted in places, is otherwise fine.

There are only two criticisms to be made of the book. First, there is no discussion of operator inequalities of the form

$$\|du/dt - A(u)\| \leqslant \varphi(t)\|u(t)\|,$$

where $\|\ \|$ is a Banach space norm and $A(u)$ is a linear operator. Secondly, as the author readily confesses, there are no applications to partial differential equations. (The first criticism may partially be included in this one as well.) The author gives sizes constraint as the reason for this latter omission. This is reasonable considering the length (348 pages) of the book. However, some mention could have been made at various points in the text as to which ideas are used in partial differential equations, especially in elliptic theory (maximum principle, Schauder estimates, comparison functions etc.) (although some of this is done in the chapter notes).

All in all this is a well written book that can be read by upper-level graduate students. It does contain a wealth of material of interest to numerical analysts as well as people in ordinary differential equations. It is worthy of a sequel dwelling on applications to partial differential equations.

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