REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS

The numbers in brackets are assigned according to the revised indexing system printed in Volume 28, Number 128, October 1974, pages 1191–1194.

1[9.00].—Daniel Shanks, *Solved and Unsolved Problems in Number Theory*, Chelsea, New York, 1978, viii + 258 pp., $23\frac{1}{2}$ cm. Price $11.95.

The original edition of this book, published by Spartan Books in 1962, has become a minor classic. It was a tightly organized presentation of elementary number theory built around the historical treatment of past successful attacks on number-theoretic problems and around extensive discussion of conjectures and open questions still awaiting solution. Each of the three chapters had a central theme, namely (Chapter 1) perfect numbers and the quadratic reciprocity law, (Chapter 2) the structure of the multiplicative group formed by the coprime residue classes modulo a given positive integer, and (Chapter 3) Pythagorean triples and quadratic irrationalities. As a result of the plan of building the book around certain central themes and their history, the work had a rather idiosyncratic character which militated against its popular acceptance as a textbook in elementary number theory. (For example, some number theorists were put off by the fact that the quadratic reciprocity law was discussed and proved before congruences were introduced, even though this unusual order of presentation had a sound historical basis.) In spite of the modest success of the first edition in the textbook derby, it achieved a firm place in the literature of number theory, being of particular interest to those working in computational number theory.

It is a pleasure to see this work reissued. No changes were made in the stimulating original text, but the value of the new edition has been immeasurably increased by the insertion of a new Chapter 4 dealing with the progress made in the intervening years on the topics considered in Chapters 1 to 3. Although this additional material may not increase the appeal of the work as an elementary textbook, it greatly enhances its value to more professional readers.

While the book is not a work on computational number theory as such, it does have a healthy emphasis on algorithms and a stress on the importance of efficient algorithms. Oddly enough, there is very little emphasis on the grand-daddy of all efficient algorithms, namely, Euclid's algorithm. However, the author gives a thorough treatment of Pepin's test for the primality of a Fermat number, Lucas's test for the primality of a Mersenne number, and the Jacobi-symbol algorithm for determining quadratic residuacity modulo an odd prime. It is refreshing to see a textbook on number theory which does *not* contain the frequently-made false assertion that the use of the quadratic reciprocity law for the Legendre symbol per se is superior to the use of Euler's criterion for determining quadratic residuality modulo an odd prime. There is some good discussion, but unfortunately not enough, of pseudoprimes to a given base, Euler pseudoprimes to a given base, Carmichael numbers, and strong pseudoprimes to a given base. Among the

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unsolved problems considered are Artin's conjecture on primitive roots and the Hardy-Littlewood conjectures on primes of special forms.

In discussing the frequency of Mersenne primes, the author in 1962 wrote on page 198, "A reasonable guess is that there are about 5 new (prime) M_p for $5000 < p < 50000$". In the 1978 additional chapter, Shanks stood by that guess even though four prime M_p had been found for $5000 < p < 21000$. Since 1978 three new prime M_p have been found for $21000 < p < 50000$, so that Shanks's 1962 prediction erred on the low side. Even Shanks has feet of clay.

It is natural to compare the work under review with another book on elementary number theory which has both a historical and an algorithmic emphasis, namely, Ore's Number Theory and its History. While Ore's book makes considerably easier reading and is therefore probably more suitable for beginners, Shanks's book penetrates more deeply and gives a better feeling for contemporary research, both theoretical and computational.

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2[3.10, 3.35].—MAGNUS HESTENES, *Conjugate Direction Methods in Optimization*, Springer-Verlag, New York, 1980, x + 325 pp., 24 cm. Price $29.80.

Conjugate direction methods are playing an increasingly important role in solving optimization problems and large sparse linear systems of equations. Magnus Hestenes' book on *Conjugate Direction Methods in Optimization* concentrates on deriving algorithms for finding critical points of real valued functions. The book gives a unified treatment of a large number of conjugate direction methods and their properties and is a rich source of research material and ideas.

The text is based on lectures given at the University of California, Los Angeles, and several other conferences. Valuable as it is, it will probably not be widely used for graduate or advanced undergraduate students. The typographical layout and especially the treatment of indices makes the reading difficult. An attempt to distinguish between vectors and scalars would have improved the readability.

The text consists of four chapters. The first, *Newton's Method and the Gradient Method*, contains introductory material only partially needed in the subsequent chapters. The results in the introductory chapter are somewhat misleading, like Theorem 4.1 and its proof. This result shows that a quasi-Newton method (called a modified Newton method)

$$x_{k+1} = x_k - H_k g(x_k)$$

for solving $g(x) = 0$ is (Q-) superlinear convergent if and only if $L^* = 0$ provided $\{x_k\}$ converges to a solution x^*. Here

$$L^* = \limsup_{k\to\infty} \|I - H_k G(x^*)\|$$

and $G(x^*)$ is the Jacobian matrix. This is wrong and could have been corrected by defining superlinear convergence in terms of the errors $\{x_k - x^*\}$ and not in terms of L^*.

The remaining three chapters are: *Conjugate Direction Methods*, *Conjugate Gram-Schmidt Processes* and *Conjugate Gradient Algorithms*. These chapters contain a unified treatment of each class of methods first derived for quadratic problems and then generalized to nonquadratic problems. The algorithms are stated in a concise manner with equivalent formulations mentioned in the text. The exercises at the end of each section are a valuable supplement and an integral part of the text.

There has long been a need for a book making this material available. Rich as the book is in algorithms, it is a valuable contribution to both theoretician and practitioner. Rounding errors are frequently discussed and alternative choices of the parameters are suggested. However, there are no complete error analyses. This is even more apparent when the matrix-vector product of the Hessian matrix and a vector is replaced by a finite difference approximation involving only gradients. This points to an open research area.

A textbook is still needed to cover a major area omitted; namely, solving large and sparse linear systems of equations and the use of preconditioning matrices. The references are incomplete and contain numerous misprints. The index should have been expanded to serve as a cross-reference and an author index should have been included.

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3[2.10, 4.00].—A. H. STROUD, *Numerical Quadrature and Solution of Ordinary Differential Equations*, Appl. Math. Sciences, Vol. 10, Springer-Verlag, New York, 1974, xi + 338 pp., $25\frac{1}{2}$ cm. Price \$12.50.

The subtitle of this book reads "A Textbook for a Beginning Course in Numerical Analysis"; therefore one must not expect a monograph on the subject area specified in the title.

As an introduction, the text has a number of definite didactic merits. The level of mathematics used does not go beyond basic calculus and algebra, there are motivating and explanatory analytic and numerical examples throughout the book, proofs are either presented in all details, or omitted altogether (with references to relevant presentations). Thus, the text should be suitable for self-study as well as in the classroom. On the other hand, for an introduction to Numerical Analysis, the near neglection of round-off and the total neglection of the concept of condition is an essential weakness.

The selection of the material shows a wise restriction to fundamental problems and methods; even so there are a number of commendable features: One is the systematic use of Peano kernel error terms, with a detailed discussion of their implications, including a large number of graphs of Peano kernel functions for the situations under discussion; this also leads to a more qualified evaluation of the merits of Gauss quadrature formulas. The Riemann sum character of reasonable quadrature formulas is pointed out.

In the section on ordinary differential equations (initial value problems only), the emphasis is on explicit Runge-Kutta methods. The discussion of stepsize control remains unsatisfactory; there is neither sufficient motivation nor a serious justification for the suggested control mechanism (due to Zonneveld). In the treatment of multistep methods, D-stability is not distinguished from relative stability.

There are a number of complete Fortran programs for various tasks; the use of library programs is not emphasized. On the whole, the author has succeeded in composing an instructive and balanced "Textbook for a Beginning Course in Numerical Analysis", which is not at all an easy task.

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4[9.05].—WALTER E. BECK & RUDOLPH N. NAJAR, *A Lower Bound For Odd Triperfects—Computational Data*; a typed manuscript of 61 pages deposited in the UMT file.

The data contained in this manuscript constitute a tree, each node of which corresponds to a restriction on the canonical decomposition of an odd integer n such that $3 \mid n$ and $\sigma(n) = 3n$. The branching process is dependent on the determination of the prime factors of $\sigma(p^{2\alpha})$ where p is a prime factor of n and α runs through the set of natural numbers. In most cases the complete factorization of $\sigma(p^{2\alpha})$ is given. Roughly speaking, the nodes immediately "following" $p^{2\alpha}$ are those involving q where q is the greatest prime factor of $\sigma(p^{2\alpha})$. When a node (or case) is reached for which either $n > 10^{50}$ or $3^t \parallel n$ while $3^{t+2} \mid \sigma(n)$, an obvious contradiction, the tree is truncated. Since the nodes considered exhaust the logical possibilities and since it is easy to show (see [2]) that $n > 10^{108}$ if $(6, n) = 1$ and $\sigma(n) = 3n$, the finiteness of the tree generated establishes a lower bound of 10^{50} for the set of odd triperfect numbers. This set may, of course, be empty since no odd multiperfect numbers (integers n such that $\sigma(n)/n$ is an integer greater than 2) have, as yet, been found. A list of more than 200 even multiperfect numbers, including the six known triperfect numbers, may be found in [1]. The present paper is very well organized and the details are easy to follow. Mathematicians doing research on perfect or amicable numbers will find this manuscript a valuable source of data on the factors of $\sigma(p^{2\alpha})$.

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1. ALAN L. BROWN, "Multiperfect numbers—cousins of the perfect numbers—No. 1," *Recreational Mathematics Magazine*, Jan.—Feb. 1964, Issue No. 14, pp. 31–39.

2. WALTER E. BECK & RUDOLPH M. NAJAR, "A lower bound for odd triperfects," *Math. Comp.*, v. 38, 1982, pp. 249–251.