Mathematics of Computation

NATHEMATICATE OF THE PROPERTY OF THE PROPERTY

EDITED BY James H. Bramble, Managing Editor Carl de Boor Todd Dupont Walter Gautschi Donald Goldfarb Eugene Isaacson Heinz-Otto Kreiss Yudell L. Luke James N. Lyness Morris Newman John E. Osborn Beresford Parlett Philip Rabinowitz John R. Rice Daniel Shanks Charles C. Sims Hans J. Stetter Vidar C. Thomée Hugh C. Williams John W. Wrench, Jr.

January 1982 Volume 38, Number 157, Pages 1-337

Published by the American Mathematical Society Providence, Rhode Island USA

ISSN 0025-5718

Editorial Committee

JAMES H. BRAMBLE, Chairman. Dept. of Mathematics, White Hall, Cornell Univ., Ithaca, NY 14853 CARL DE BOOR, Mathematics Research Center, Univ. of Wisconsin, Madison, WI 53706 MORRIS NEWMAN, Dept. of Mathematics, Univ. of California, Santa Barbara, CA 93106 DANIEL SHANKS, Dept. of Mathematics, Univ. of Maryland, College Park, MD 20742

Technical Editor

ANITA WAHLBIN, Dept. of Mathematics, White Hall, Cornell Univ., Ithaca, NY 14853

Board of Associate Editors

TODD DUPONT, Dept. of Mathematics, Univ. of Chicago, Chicago, IL 69637

WALTER GAUTSCHI, Computer Sciences Dept., Purdue Univ., West Lafayette, IN 47907

DONALD GOLDFARB, Dept. of Computer Sciences, School of Engineering, The City College of the City Univ. of New York, 139th Street & Convent Avenue, New York, NY 10031

EUGENE ISAACSON, Courant Institute of Mathematical Sciences, New York Univ., 251 Mercer Street, New York, NY 10012

HEINZ-OTTO KREISS, Dept. of Applied Mathematics, California Inst. of Technology, Pasadena, CA 91125

YUDELL L. LUKE, Dept. of Mathematics, Univ. of Missouri at Kansas City, Kansas City, MO 64110 JAMES N. LYNESS, Argonne National Laboratory, 9700 South Cass Avenue, Argonne, IL 60439

JOHN E. OSBORN, Dept. of Mathematics, Univ. of Maryland, College Park, MD 20742

BERESFORD PARLETT, Dept. of Computer Science, Univ. of California, Berkeley, CA 94720

PHILIP RABINOWITZ, Dept. of Applied Mathematics, The Weizmann Institute of Science, Rehovot, Israel

JOHN R. RICE, Division of Mathematical Sciences, Purdue Univ., Lafayette, IN 47907

CHARLES C. SIMS, Dept. of Mathematics, Rutgers Univ., New Brunswick, NJ 08903

HANS J. STETTER, Institut für Numerische Mathematik, Technische Universität Wien, Karlsplatz 13, A-1040, Wien, Austria

VIDAR C. THOMEE, Mathematics Dept., Chalmers Univ. of Technology, Göteborg, Sweden

HUGH C. WILLIAMS, Dept. of Computer Science, Univ. of Manitoba, Winnipeg, Manitoba, Canada R3T 2N2

JOHN W. WRENCH, JR., 6310 Jefferson Blvd., Frederick, MD 21701

SUBSCRIPTION INFORMATION: MATHEMATICS OF COMPUTATION is published quarterly, with issues numbered serially since Volume 1, Number 1. Subscription prices for Volumes 38 and 39 (1982) are \$80.00 list; \$56.00 institutional member; \$40.00 member of CBMS organizations; \$32.00 individual AMS member. Combination paper and microform (microfiche or microfilm) subscription prices are \$107.00 list; \$75.00 institutional member; \$54.00 member of CBMS organizations; \$43.00 individual AMS member. Microfiche of each issue will be mailed the fastest way before the camera copy is sent to the printer.

BACK NUMBER INFORMATION: Back number prices per volume are for Volumes 1-29, \$64.00 list, \$48.00 member; for Volumes 30-33, \$100.00 list, \$75.00 member; for Volumes 34-37, \$60.00 list, \$48.00 member. Beginning with Volume 32, back volumes are also available on 16mm positive or negative microfilm or on microfiche; Volumes 1-31 are available on microfilm only, not microfiche. The microfilm may be mounted on spools or in Kodak or 3M cartridges. Only current subscribers are eligible to purchase back volumes on microform. Write to the AMS for a detailed price list.

UNPUBLISHED MATHEMATICAL TABLES: The editorial office of the journal maintains a repository of Unpublished Mathematical Tables (UMT). When a table is deposited in the UMT repository a brief summary of its contents is published in the section *Reviews and Descriptions of Tables and Books*. Upon request, the chairman of the editorial committee will supply copies of any table for a nominal cost per page. All tables and correspondence concerning the UMT should be sent to James H. Bramble, Chairman, Department of Mathematics, White Hall, Cornell University, Ithaca, NY 14853.

Orders for subscriptions and publications of the American Mathematical Society should be addressed to the AMS, P. O. Box 1571, Annex Station, Providence, R.I. 02901. All orders must be accompanied by payment. Other correspondence should be addressed to P. O. Box 6248, Providence, R.I. 02940.

MATHEMATICS OF COMPUTATION is published quarterly by the American Mathematical Society, 201 Charles Street, Providence, RI 02904. Second-class postage is paid at Providence, Rhode Island, and additional mailing offices. Postmaster: Send address changes to Mathematics of Computation, American Mathematical Society, P. O. Box 6248, Providence, RI 02940.

Mathematics of Computation

EDITED BY

James H. Bramble, Managing Editor

Carl de Boor

Todd Dupont

Walter Gautschi

Donald Goldfarb

Eugene Isaacson

Heinz-Otto Kreiss

Yudell L. Luke

James N. Lyness

Morris Newman

John E. Osborn

Beresford Parlett

Philip Rabinowitz

John R. Rice

Daniel Shanks

Charles C. Sims

Hans J. Stetter

Vidar C. Thomée

Hugh C. Williams

John W. Wrench, Jr.

VOLUME 38 · 1982 · NUMBERS 157-158

Providence, Rhode Island, USA ISSN 0025-5718

Information for Contributors

Manuscripts should be typewritten double-spaced in the format used by the journal. For journal abbreviations, see the latest *Mathematical Reviews* volume index. An author should submit the original and two copies of the manuscript and retain one copy. The author may suggest an appropriate editor for his paper. It is recommended that the author acquaint himself with the pertinent material contained in "A Manual for Authors of Mathematical Papers," which is available from the American Mathematical Society. All contributions intended for publication and all books for review should be addressed to James H. Bramble, Chairman, Editorial Committee, Mathematics of Computation, Department of Mathematics, White Hall, Cornell University, Ithaca, New York 14853. Institutions sponsoring research reported in the journal are assessed page and microfiche charges.

Each article submitted for publication must be accompanied by a brief and reasonably self-contained abstract, and by 1980 Mathematics Subject Classification numbers. If a list of key words and phrases is included, it will be printed as a footnote on the first page. A list of the classification numbers may be found in the 1978 Subject Index to Mathematical Reviews.

The research journals of the American Mathematical Society carry a page charge of \$50.00 per page to help defray the cost of publication. This amount is charged to the institution or to a contract supporting the research reported in the published paper. The publication charge policy of the United State Federal Council for Science and Technology (FCST) is reported on page 112 of the February, 1975 issue of the NOTICES of the American Mathematical Society. In no case is the author personally responsible for paying the page charge, nor is acceptance of the author's paper for publication dependent upon payment of the page charge.

Copying and Reprinting

Individual readers of this publication, and nonprofit libraries acting for them are permitted to make fair use of the material, such as to copy an article for use in teaching or research. Permission is granted to quote brief passages from this publication in reviews provided the customary acknowledgement of the source is given.

Republication, systematic copying, or multiple reproduction of any material in this publication (including abstracts) is permitted only under license from the American Mathematical Society. Requests for such permission should be addressed to the Executive Director, American Mathematical Society, Box 6248, Providence, Rhode Island 02940.

The appearance of the code on the first page of an article in this journal indicates the copyright owner's consent for copying beyond that permitted by Sections 107 or 108 of the U. S. Copyright Law, provided that the copier pay the stated per copy fee through the Copyright Clearance Center, Inc., 21 Congress Street, Salem, Massachusetts 01970. This consent does not extend to other kinds of copying, such as copying for general distribution, for advertising or promotion purposes, for creating new collective works, or for resale.

MATHEMATICS OF COMPUTATION TABLE OF CONTENTS

January 1982

A. H. Schatz and L. B. Wahlbin, On the Quasi-Optimality in L_{∞} of the H^1 -	
Projection into Finite Element Spaces	1
Douglas N. Arnold and Ragnar Winther, A Superconvergent Finite Element Method	
for the Korteweg-de Vries Equation	23
David Archer and Julio César Díaz, A Collocation-Galerkin Method for a First Order	
Hyperbolic Equation With Space and Time-Dependent Coefficient	37
Houde Han, The Finite Element Method in a Family of Improperly Posed Problems.	55
C. Canuto and A. Quarteroni, Approximation Results for Orthogonal Polynomials in Sobolev Spaces	67
Peter Alfeld, Fixed Point Iteration with Inexact Function Values	87
Arieh Iserles, Composite Exponential Approximations	99
M. J. Marsden, Spline Interpolation at Knot Averages on a Two-Sided Geometric Mesh	113
Avram Sidi, Numerical Quadrature Rules for Some Infinite Range Integrals	127
T. H. Charles Chen, Asymptotic Error Estimates for Gaussian Quadrature Formulas	143
B. N. Parlett, H. Simon and L. M. Stringer, On Estimating the Largest Eigenvalue	
With the Lanczos Algorithm	153
Ralph Kallman, A Method for Finding Permanents of 0, 1 Matrices	167
Annie A. M. Cuyt, Numerical Stability of the Halley-Iteration for the Solution of a	
System of Nonlinear Equations	171
Richard Franke, Scattered Data Interpolation: Tests of Some Methods	181
C. M. Joshi and J. P. Arya, Inequalities for Certain Hypergeometric Functions	201
Shyam L. Kalla, Salvador Conde and Yudell L. Luke, Integrals of Jacobi Functions	
Chih-Bing Ling and Ming-Jing Wu, Evaluation of Integrals of Howland Type Involv-	207
ing a Bessel Function	215
V. Belevitch and J. Boersma, On Stieltjes Integral Transforms Involving Γ-Functions	223
Harvey Cohn, An Explicit Modular Equation in Two Variables and Hilbert's Twelfth	
Problem	227
G. Greaves, An Algorithm for the Solution of Certain Differential-Difference Equations of Advanced Type	237
Walter E. Beck and Rudolph M. Najar, A Lower Bound for Odd Triperfects	249
Richard P. Brent, Succinct Proofs of Primality for the Factors of Some Fermat Numbers	253
Michael Laska, An Algorithm for Finding a Minimal Weierstrass Equation for an Elliptic Curve	257
H. C. Williams , Determination of Principal Factors in $Q(\sqrt{D})$ and $Q(\sqrt[3]{D})$	261
Michael Pohst and Hans Zassenhaus, On Effective Computation of Fundamental	
Units. I	275

Michael Pohst, Peter Weiler and Hans Zassenhaus, On Effective Computation of	
Fundamental Units. II	293
Reviews and Descriptions of Tables and Books	331
Shanks 1, Hestenes 2, Stroud 3, Beck and Najar 4	
Table Errata	335
Golomb 584, Baillie 585, Cormack and Williams 586, Petit Bois 587	
Corrigendum	337
Fettis	

.

FACULTY POSITIONS IN NUMERICAL METHODS TULANE UNIVERSITY

The Department of Mathematics has recently established a group in numerical methods. Two positions at the Assistant Professor level have been authorized for the academic year beginning August, 1982. The duties of these positions include teaching undergraduate and graduate courses, supervising graduate research, and developing a research program in his or her area of expertise. Applications are invited from persons with a Ph.D. degree in mathematics, applied mathematics, computer science or engineering with a practical knowledge of numerical methods.

To apply for these positions, please send a current vita and the names of three references to

Professor Gary A. Sod Department of Mathematics Tulane University New Orleans, Louisiana 70118

The closing date for applications is March 1, 1982.

Tulane University is an Affirmative Action-Equal Opportunity Employer. Applications from qualified minorities and female applicants are encouraged.

1982 MATHEMATICAL SCIENCES administrative directory

PROFESSIONAL MATHEMATICAL ASSOCIATIONS
ACADEMIC INSTITUTIONS—DEPARTMENTS IN THE MATHEMATICAL
SCIENCES AND OTHER FACILITIES in the U. S., Canada, Central America, and
the Caribbean, coded according to the highest degree offered
MATHEMATICAL UNITS IN NONACADEMIC ORGANIZATIONS
HEADS AND KEY PERSONNEL of a selected group of government agencies
EDITORS OF JOURNALS in the mathematical sciences
OFFICERS AND COMMITTEE MEMBERS in more than 25 professional
mathematical organizations
AN INDEX OF ACADEMIC INSTITUTIONS

\$12.60
Publication date: February 15, 1982
Code: ADMDIR82 – Prepayment Required

p. o. box 1571, annex station, providence, r. i. 02901

SELECTED TABLES IN MATHEMATICAL STATISTICS

W. J. Kennedy and R. E. Odeh, Editors James M. Davenport, Managing Editor

New

Volume 7. The Product of Two Normally Distributed Random Variables, William Q. Meeker, Jr., Larry W. Cornwell, and Leo A. Aroian.

This volume contains tables of percentiles of the distribution of two (possibly correlated) normally distributed random variables. Theoretical properties are reviewed, numerical methods are described, and a number of references are provided. This distribution has applications in the fields of physical, engineering, biomedical, and social sciences as well as in auditing and other business applications. This book will be a useful addition to statistical, technical, and scientific libraries.

1981, 264 pages, \$12.40. (Order Code: TABLES/7S)

- The Distribution of the Size of the Maximum Cluster of Points on a Line, Norman D. Neff and Joseph I. Naus.
 1980, 214 pp., \$14.40. (Order Code: TABLES/6S)
- 5. Variances and covariances of the normal order statistics for sample sizes 2 to 50, G. L. Tietjen, D. K. Kahaner and R. J. Beckman.
 - Means, variances and covariances of the normal order statistics in the presence of an outlier, H. A. David, W. J. Kennedy and R. D. Knight.
 - Tables for obtaining optimal confidence intervals involving the chi-square distribution, G. R. Murdock and W. O. Williford.
 - 1977, 263 pp., \$24.40. (Order Code: TABLES/5S)
- Dirichlet distribution Type 1, M. Sobel, V. R. R. Uppuluri and K. Frankowski. 1977, 310 pp., \$26.00. (Order Code: TABLES/4S)
- 3. Tables of the two factor and three factor generalized incomplete modified Bessel distributions, B. Harris and A. P. Soms.
 - Sample size requirement: Single and double classification experiments, K. O. Bowman and M. A. Kastenbaum.
 - Passage time distributions for Gaussian-Markov (Ornstein-Uhlenbeck) statistical processes, J. Keilson and H. F. Ross.
 - Exact probability levels for the Krushal-Wallis test, D. A. Alexander, R. L. Iman and D. Quade.

 Tables of standard confidence limits for linear functions of the normal mean and variance, C. E. Land.
 1975, 424 pp., \$33.60. (Order Code: TABLES/3S)
- Probability integral of the doubly noncentral t-distribution with degrees of freedom n and non-centrality parameters δ and λ, W. G. Bulgren.
 - Doubly non-central F distribution Tables and applications, M. L. Tiku.
 - Tables of expected sample size for curtailed fixed sample size tests of a Bernoulli parameter, C. R. Blyth and D. Hutchinson.
 - Zonal polynomials of order 1 through 12, A. M. Parkhurst and A. T. James. 1974, 388 pp., \$25.60. (Order Code: TABLES/2S)
- 1. Tables of the cumulative non-central chi-square distribution, G. E. Hayman, Z. Govindarajulu, and F. C. Leone.
 - Tables of the exact sampling distribution of the two-sample Kolmogorov-Smirnov criterion $D_{mn}(m \le n)$, P. 1. Kim and R. 1. Jennrich.
 - Critical values and probability levels for the Wilcoxon rank sum test and the Wilcoxon signed rank test, F. Wilcoxon, S. K. Katti, and R. A. Wilcox.
 - The null distribution of the first three product-moment statistics for exponential half-gamma, and normal scores, P. A. W. Lewis and A. S. Goodman.
 - Tables to facilitate the use of orthogonal polynomials for two types of error structures, K. B. Stewart. Rev. ed. 1973, 403 pp., \$20.80. (Order Code: TABLES/15)

Individual members of the Institute of Mathematical Statistics or the American Mathematical Society are entitled to a 50% discount from the list prices quoted here. Orders must be prepaid. American Mathematical Society, P. O. Box 1571, Annex Station, Providence, RI 02901

Mathematical Reviews Sections

ENJOY THE CONVENIENCE OF HAVING MATHEMATICAL REVIEWS AT YOUR DESK. HAVE YOU THOUGHT HOW REALLY HANDY IT WOULD BE TO HAVE THE SECTIONS

RELEVANT TO YOUR RESEARCH RIGHT BEFORE YOU? MR has been divided into 37 affordable Sets for individual subscribers. Each month you can receive the Section Sets you have chosen with an author index. With your June sets you will receive a semiannual author index (just as you would for MR) and in December an annual author and subject index (as with MR). Also available for 1982 Section subscribers are three-ring binders of sturdy quality in the familiar tangerine color of MR to hold your subscription. The binders have a two-inch spine and are adequate to hold 400 pages.

Section Sets are divided into Class 1 and Class 2 according to the estimated number of pages per year.

Set	Sections	Subjects	□ 21	42, 43, 44, 45	Harmonic analysis, integral
□ 1A	00, 01	General, history, biography	□ 1J	46	transforms/equations (Class 2) Functional analysis (Class 1)
	22.24	(Class 1)	□ 1K	47	Operator theory (Class 1)
□ 1B	03, 04	Logic, foundations, set theory (Class 1)		49	Calculus of variations, optimiz-
□ 1C	05	Combinatorics (Class 1)	□ 2K	51 50	ation (Class 2)
□ 2A	06, 08	Order, lattices, general systems		51, 52 53	Geometry, convex sets (Class 2) Differential geometry (Class 1)
- 45	10	(Class 2)	□ 2L	54	General topology (Class 2)
□ 1D	10	Number theory (Class 1)		55, 57	See 1E (18, 55, 57)
□ 2B	12	Algebraic number theory, field theory, polynomials (Class 2)	□ 1M	55, 57	Global analysis, analysis on
□ 2C	13, 14	Commutative rings and algebras,	_ ····	30	manifolds (Class 1)
	13, 14	algebraic geometry (Class 2)	□ 1N	60	Probability theory and stochas-
□ 2D	15	Linear and multilinear algebra,			tic processes (Class 1)
		matrix theory (Class 2)	□ 1P	62	Statistics (Class 1)
□ 2E	16, 17	Associative/nonassociative	□ 1Q	65	Numerical analysis (Class 1)
		rings, algebras (Class 2)	□ 1R	68	Computer science (including
□ 1E	18, 55, 57	Category theory, algebraic			automata) (Class 1)
		topology, manifolds (Class 1)	□ 2M	70, 73	Mechanics of particles, systems,
□ 1F	20	Group theory, generalizations		76 70 00	(Class 2)
	22	(Class 1)	□ 2N	76, 78, 80	Fluid mechanics, optics, elec- tromagnetics, thermodynam-
□ 2F	22	Topological groups, Lie groups			ics (Class 2)
□ 2G	26, 28	(Class 2) Real functions, measure, inte-	□ 1S	81	Quantum mechanics (Class 1)
□ 20	20, 28	gration (Class 2)	□ 2P	82, 83, 85, 86	Other physics, astronomy, astro-
□ 1G	30, 31, 32, 33	Complex analysis, potential		,,,	physics, geophysics (Class 2)
0	50, 51, 52, 55	theory, special functions	□ 1T	90	Economics, operations re-
		(Class 1)			search, programming, games
□ 1H	34	Ordinary differential equations			(Class 1)
		(Class 1)	□ 2Q	92	Biology and behavioral sci-
□ 11	35	Partial differential equations			ences (Class 2)
C) 211	20 40 41	(Class 1) Finite differences, sequences,	□ 1U	93	Systems theory; control (Class 1)
□ 2 H	39, 40, 41	approximations (Class 2)	□ 1V	94	Information and communica- tion, circuits (Class 1)
		approximations (class 2)			tion, circuits (Class 1)
		First Set	Eacl	n Add'l Set	Optional Binder
		Class 1 Class 2	Class 1	1 Class 2	\$5.00 each
1.	ndividual	\$36 \$27	\$30	\$21	
	Reviewer	24 18	20		
•					
USE THIS	PAGE OR A PI	HOTOCOPY TO ORDER.		Date _	
\$	enclos	ed for subscriptions selected and	marked a	bove	
		•			ECTIONS are an incomed from the contraction
			EMATIC	AL KEVIEWS S	SECTIONS on spine and front cover.
•	•	t one buy a binder.)			
\$	Total _I	orepaid order.			
Check this	box 🗆 if you		nched. I	f the box is not	checked you will receive unbound,
unpunche	• •				
Name					Your AMS code
Addross					
Address —					

Michael Pohst, Peter Weiler and Hans Zassenhaus, On Effective Computation of	
Fundamental Units. II	293
Reviews and Descriptions of Tables and Books	331
Shanks 1, Hestenes 2, Stroud 3, Beck and Najar 4	
Table Errata	335
Golomb 584, Baillie 585, Cormack and Williams 586, Petit Bois 587	
Corrigendum	337
Fettis	

No microfiche supplement in this issue

MATHEMATICS OF COMPUTATION TABLE OF CONTENTS

January 1982

A. H. Schatz and L. B. Wahlbin, On the Quasi-Optimality in L_{∞} of the H^1 -	
Projection into Finite Element Spaces	1
Douglas N. Arnold and Ragnar Winther, A Superconvergent Finite Element Method for the Korteweg-de Vries Equation	23
David Archer and Julio César Díaz, A Collocation-Galerkin Method for a First Order	
Hyperbolic Equation With Space and Time-Dependent Coefficient	37
Houde Han, The Finite Element Method in a Family of Improperly Posed Problems.	55
C. Canuto and A. Quarteroni, Approximation Results for Orthogonal Polynomials in Sobolev Spaces	67
Peter Alfeld, Fixed Point Iteration with Inexact Function Values	87
Arieh Iserles, Composite Exponential Approximations	99
M. J. Marsden, Spline Interpolation at Knot Averages on a Two-Sided Geometric Mesh	113
Avram Sidi, Numerical Quadrature Rules for Some Infinite Range Integrals	127
T. H. Charles Chen, Asymptotic Error Estimates for Gaussian Quadrature Formulas	143
B. N. Parlett, H. Simon and L. M. Stringer, On Estimating the Largest Eigenvalue	
With the Lanczos Algorithm	153
Ralph Kallman, A Method for Finding Permanents of 0, 1 Matrices	167
Annie A. M. Cuyt, Numerical Stability of the Halley-Iteration for the Solution of a	
System of Nonlinear Equations	171
Richard Franke, Scattered Data Interpolation: Tests of Some Methods	181
C. M. Joshi and J. P. Arya, Inequalities for Certain Hypergeometric Functions	201
Shyam L. Kalla, Salvador Conde and Yudell L. Luke, Integrals of Jacobi Functions	207
Chih-Bing Ling and Ming-Jing Wu, Evaluation of Integrals of Howland Type Involving a Bessel Function	215
V. Belevitch and J. Boersma, On Stieltjes Integral Transforms Involving Γ-Functions	223
Harvey Cohn, An Explicit Modular Equation in Two Variables and Hilbert's Twelfth Problem	227
G. Greaves, An Algorithm for the Solution of Certain Differential-Difference Equa-	24 1
tions of Advanced Type	237
Walter E. Beck and Rudolph M. Najar, A Lower Bound for Odd Triperfects	249
Richard P. Brent, Succinct Proofs of Primality for the Factors of Some Fermat	
Numbers	253
Michael Laska, An Algorithm for Finding a Minimal Weierstrass Equation for an Elliptic Curve	257
H. C. Williams, Determination of Principal Factors in $Q(\sqrt{D})$ and $Q(\sqrt[3]{D})$	261
Michael Pohst and Hans Zassenhaus, On Effective Computation of Fundamental	
Units. I	275