

REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS

The numbers in brackets are assigned according to the American Mathematical Society classification scheme. The 1980 Mathematics Subject Classification can be found in the December index volumes of *Mathematical Reviews*.

7[65–01].—JAMES S. VANDERGRAFT, *Introduction to Numerical Computations*, Academic Press, New York, 1978, xi + 344 pp., 23½ cm. Price \$21.00.

The title of this book conveys quite clearly the intent of author James Vandergraft to study numerical *computation*, i.e. the performance of algorithms to solve many commonly encountered problems of science and engineering as they actually behave when implemented on a (serial) digital computer. Adhering to this approach, he presents an intriguing blend of traditional mathematical analysis and floating point error analysis of many frequently used methods for problems in interpolation and approximation, differentiation and integration, linear and nonlinear equations, and ordinary differential equations. He does not aim to be encyclopedic, so he therefore limits his choice of algorithms in most cases to the most well-known, but these are analyzed “completely and in a uniform manner”, so as to provide “a solid foundation for more advanced work in algorithm development”. This approach distinguishes the text from those of the cookbook variety as well as those which attempt to be all-inclusive and heavily mathematical.

In order for the student to appreciate the qualities to be strived for in a suitable numerical algorithm, basic aspects of numerical computation, e.g., computer representable numbers, rational function computation, discretization, iteration, rounding and truncation error, are early introduced. The concepts of mathematical and numerical instability, and some of their sources, are discussed. The chapter on computer arithmetic is more extensive than will be found in most numerical analysis texts. A notation, introduced by Stewart, for assessing rounding error requires some getting used to, but it does simplify the presentation. The author makes here the very important, but often neglected, distinction between an error estimate and an error bound, which may sometimes contain almost no useful information.

Before proceeding to specific algorithms, the problem of trying to evaluate a function on the computer is studied thoroughly and several handy devices to avoid common difficulties are presented. As an example of the philosophy of the book, rational function approximation of transcendental functions motivates the section, but no attempt is made to explain how such rational functions are obtained.

Interpolation is studied thoroughly in the Lagrangian manner, and a complete truncation and rounding analysis is performed. The permanence problem, and its solution by the Newton form, are, however, overlooked. It must be pointed out that many formulas in the text are presented rather than derived; on the other hand, the author does choose in some instances, e.g., cubic spline interpolation, to provide a full derivation.

The study of numerical integration is restricted to standard methods (Newton-Cotes, Gaussian) and simple problems (no singularities, infinite intervals), and again the error analysis is emphasized. Romberg integration is presented but is not completely analyzed.

The treatment of linear equations is consistent with the author's philosophy, emphasizing error analysis, efficiency, and condition estimation (the latter at an elementary level). Convergence theory for iterative methods is presented in a manner so as to be understood without extensive matrix theory background, although matrix notation might be used more advantageously. The author has chosen to avoid eigenanalysis completely.

Many of the usual methods for the solution of a single nonlinear equation are considered; a simplified version of the Dekker-Brent algorithm nicely incorporates features of false position, secant, and bisection. The effect of rounding on interval methods, often overlooked in numerical analysis texts, is explained here. The author also discussed informatively the problem of starting iterative schemes, both in the search for real and complex roots.

The study of differential equations, initial and two-point boundary value problems, concludes the text. The presentation is not elaborate, only the most common Runge-Kutta and linear multistep methods being considered, but many important ideas of stability, error estimation and stepsize control are covered. While the difficulty of solving stiff equations is mentioned, there is no attempt to discuss methods appropriate for these problems.

This text is well-written, reasonably error free, and well-suited for a course to advanced undergraduate students, particularly those with a strong attraction to computing, since many of the exercises require significant programming for their successful completion, while many others require mathematical derivations not performed in the text.

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8[65M10, 65N10].— THEODOR MEIS & ULRICH MARCOWITZ, *Numerical Solution of Partial Differential Equations*, Appl. Math. Series 32, Springer-Verlag, New York, 1981, viii + 541 pp., 23½ cm. Price \$24.00.

This book, a translation of the German edition published in 1978, resulted from courses of lectures given at the University of Cologne. It consists of three parts and a set of appendices containing computer programs implementing some of the methods described in the text.

Much of Part I, which is entitled "Initial value problems for hyperbolic and parabolic differential equations", comprises a discussion of stability theory for difference schemes. There is an in-depth treatment of Lax-Richtmyer theory, and an extensive section devoted to Fourier transforms of difference methods for pure initial value problems with constant coefficients. The difference methods discussed are all classical and can be found, for example, in Richtmyer and Morton's book, as

can much of the stability theory. In Part I, there are also brief sections on characteristic methods and extrapolation methods for first-order hyperbolic equations.

In the second part, which is considerably shorter than the first, as also is Part III, methods for solving elliptic boundary value problems are considered. Finite difference methods are presented in a very general framework which tends, unfortunately, to conceal their relative simplicity. The discussion is devoted almost entirely to problems with Dirichlet boundary conditions; very little mention is made of the treatment of derivative boundary conditions in this chapter, indeed, throughout the book. The section on variational methods is rather superficial, the examples given of finite element methods being designed to show that, with certain choices of approximating subspaces and basis functions, one obtains well-known difference methods. The merits of the finite element method are listed but none of the difficulties associated with Dirichlet boundary conditions, regions with curved boundaries, numerical integration, etc., are mentioned. Following the section on variational methods, there is a rigorous treatment of global and piecewise Hermite polynomial interpolation. The rationale given for the inclusion of this topic, which is treated more extensively than the finite element method, is rather sketchy. The authors do use the approximation properties of piecewise Hermite polynomials of degree $2m - 1$ derived in this section to prove that the use of such functions in the finite element method yields an approximation whose error is $O(h^{2m-1})$ in the L^2 -norm. It is well known that this is a suboptimal estimate. The authors do state "In many practical cases it has been observed that the convergence order is actually $O(h^{2m})$ ", but make no reference to the derivation of the optimal estimate, which is rather straightforward. Part II concludes with a very brief section on collocation methods and boundary integral methods. No mention is made of finite element collocation methods. Moreover, the portion of this section devoted to boundary integral methods, less than four pages of text, consists solely of a discussion of a special case of the so-called indirect boundary integral equation method based on the simple layer potential, and only one reference, to a paper published in 1963, is cited.

Part III, "Solving systems of equations", begins with a lengthy discussion of Newton's method and some of its variants for nonlinear systems. For the iterative solution of linear systems, the authors describe the method of successive over-relaxation in considerable detail, deriving the optimal value of the iteration parameter. The method's popularity is attributed to the fact that "with the same programming effort as required by the Gauss-Seidel method, one obtains substantially better convergence in many important cases". The authors may feel that this is sufficient justification for not mentioning iterative methods other than the three basic ones, Jacobi, Gauss-Seidel and successive over-relaxation. The generalization of successive over-relaxation to nonlinear systems is also described. Direct methods are discussed in the latter part of this chapter. A detailed description of the Gibbs-Poole-Stockmeyer method for band width reduction for sparse systems is provided along with sections on the Buneman algorithm and the Schroder and Trottenberg reduction method.

For students unfamiliar with partial differential equations and functional analysis, four sections containing some basic material from these areas are included. It is this

reviewer's opinion that, even with this material, students lacking a strong background in these subjects will find the book extremely heavy going. In the main the presentation is unduly theoretical, and many concepts are introduced with little or no motivation. Furthermore, much notation is used without definition, there is a paucity of illustrative numerical examples, and no exercises are provided.

In the set of appendices, computer programs are provided which implement some of the techniques mentioned in the text, or minor modifications of them. The manner in which these codes are presented lacks uniformity and tends to confuse. In all but one of the appendices, subroutines defining test examples are given. On occasions, the test example coded is not the example discussed in the description of the use of the code. Also, for some of the test examples, a driving program is not provided.

The list of references is surprisingly short, containing fewer than forty papers, the remaining references being books and theses. One reason for this is that the authors fail to give explicit credit for many of the methods and theorems appearing in the text, for example, the high order correct method of Douglas (page 108) and Kahan's Theorem (page 365).

The book covers many interesting topics, but the reviewer has serious reservations regarding its suitability as a text in a numerical analysis course. The authors present a rather limited view of the subject, concentrating almost entirely on difference methods. Few techniques are presented which could be labelled as modern computational tools, and as a result it is unlikely that anyone wishing to solve partial differential equations will find much of value in this book. In short, the book does not appear to be a serious competitor to other available texts which discuss more recent developments in the numerical solution of partial differential equations.

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9[65H99].—C. B. GARCIA & W. I. ZANGWILL, *Pathways to Solutions, Fixed Points, and Equilibria*, Prentice-Hall, Englewood Cliffs, N. J., 1981, vii + 479 pp., 23½ cm. Price \$32.00.

In recent years, the subject of the numerical solution of nonlinear equations has been enriched by ideas from topology. Iterative methods such as Newton's method have been given a global setting, existence theorems from topology have been given constructive proofs, and both combinatorial and differential methods have been utilized to construct solution algorithms and computer programs for the solution of nonlinear systems. The book of Garcia and Zangwill gives a spritely survey of work in this area, with emphasis on path following as a theoretical and algorithmic tool. In addition, there is extensive material on applications to nonlinear programming, equilibrium programming, economic modelling, game theory, and network models.

The book contains 22 chapters and three appendices and is divided into four parts. Part I contains an exposition of the ideas of continuation and degree theory, including the Basic Differential Equation, a device which has been effectively

exploited by the authors to give a self-contained development of degree theory. Part II, comprising over 1/3 of the book, contains the applications. An enthusiastic discussion is given of equilibrium programming and its philosophical implications. The other applications are shown to be special cases of equilibrium programming, and, in all cases, continuation arguments are used to prove the existence theorems for these applications. Part III covers algorithms. The traditional numerical analyst will be disappointed here. In the differential case, continuation uses a mixture of techniques drawn from the numerical solution of ordinary differential equations and the local numerical solution of nonlinear systems. Both of these subjects have been developed to a high degree, and the treatment in this book does not reflect this development. On the combinatorial side, the authors do not give any specific triangulations, or discuss the relative merits of different triangulations. They give no algorithms or computer programs. What they do provide is a lucid account of the ideas that go into the simplicial and differentiable methods for path following. Part IV contains a chapter on the calculation of all complex solutions of polynomial systems, two chapters on the linear complementarity problem, a chapter on the Kakutani fixed point theorem, and a final chapter giving an intuitive discussion of the Sard and Weierstrass theorems.

The book provides a broad picture, with some technical mathematical details left to journal articles. It is well written, often using vivid images to illustrate the mathematical ideas. Each chapter concludes with a number of exercises. It should have a salutary effect in the dissemination of ideas from numerical analysis and applied topology to an audience that includes students of economics, operations research, and game theory.

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10[65-06].—P. ROZSA (Editor), *Numerical Methods*, Colloquia Mathematica Societatis János Bolyai, Vol. 22, North-Holland, Amsterdam and New York, 1980, 631 pp., 24 cm. Price \$87.75/Dfl. 180.00.

This volume contains 41 lectures delivered at the Colloquium on Numerical Methods held in Keszthely, Hungary, from September 4–10, 1977. The papers cover the following areas of numerical mathematics: ordinary and partial differential equations including initial value, boundary value, and stiff problems; numerical algebra such as matrix eigenvalue problems, generalized matrix inverses, recursive computations; unconstrained and constrained optimization; elliptic functions; applications of splines.

11[65-06].—C. A. BREBBIA (Editor), *Boundary Element Methods*, Springer-Verlag, New York, Heidelberg, Berlin, 1981, xxiv + 622 pp., 23½ cm. Price \$59.00.

This is the proceedings of the Third International Seminar held at Irvine, California, in July, 1981. It contains 39 papers divided into the following sections: I Potential and Fluid Flow Problems, II Elasticity Problems, III Geomechanics, IV

Material Problems, V Numerical Techniques and Mathematical Principles, VI Coupling of Boundary and Finite Element Methods.

12[65-06].—R. NOYE (Editor), *Numerical Solutions of Partial Differential Equations*, North-Holland, Amsterdam and New York, 1982, xii + 648 pp., 23 cm. Price \$93.00/Df. 200.00.

This is the proceedings of a conference on the Numerical Solutions of Partial Differential Equations held at Queen's College, Melbourne University, Australia, from August 23–27, 1981. It contains six invited contributions and twenty papers describing recent refinements of the various numerical techniques used to solve partial differential equations.