REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS

The numbers in brackets are assigned according to the American Mathematical Society classification scheme. The 1980 Mathematics Subject Classification (1985 Revision) can be found in the December index volumes of Mathematical Reviews.

36[65-01].—KENDALL ATKINSON, *Elementary Numerical Analysis*, Wiley, New York, 1985, xii + 416 pp., 24cm. Price $31.95.

This book is a first-rate textbook for a basic course in numerical analysis. As stated in the preface, the author has several objectives in teaching the course to students. "First, they should obtain an intuitive and working understanding of some numerical methods for the basic problems of numerical analysis (as specified by the chapter headings). Second, they should gain some appreciation of the concept of error and of the need to analyze and predict it. And third, they should develop some experience in the implementation of numerical methods using a computer." The material presented in this textbook is consistent with these objectives. The range of topics dealt with in the book is illustrated by the chapter titles: Taylor Polynomials, Computer Representation of Numbers, Error, Rootfinding, Interpolation, Approximation of Functions, Numerical Integration and Differentiation, Solution of Systems of Linear Equations, and The Numerical Solution of Differential Equations. The author provides a wide range of problems at the end of each subsection and answers to selected problems. The book includes sample programs written in Fortran 77 (the formatting of the listing could be improved, but this is a minor issue). The text even includes an appendix describing sources of numerical software packages.

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37[65N30, 65N15, 73K25].—J. TINSLEY ODEN & GRAHAM F. CAREY, *Finite Elements, Mathematical Aspects*, Vol. IV, Prentice-Hall, Englewood Cliffs, N.J., 1983, viii + 195 pp., $23\frac{1}{2}$cm. Price $37.95.

Computer modelling of physical phenomena is a discipline that is developing rapidly and gaining name recognition (known variously as scientific computation or computational mathematics, with subfields such as computational mechanics or computational fluid dynamics). Some people identify this as the emergence of a third methodology in science and engineering, complementing experimental and theoretical (pencil and paper) studies. Among the various practitioners of the discipline, one finds a wide range of philosophical perspectives that guide the particular approach taken by individual investigators. One key variable characterizing different approaches is the role that mathematical rigor plays in the research. The subject can be described, in part, as the numerical approximation of solutions to abstract physical models. Thus one could, at one extreme, demand a proof that the numerical approximation reflects the properties of the mathematical model in the full detail that is actually programmed on the computer (and that the model itself be well posed in some sense). At the other extreme, algorithms could be developed based only on heuristic or physical arguments. It is useful to consider where the book under review fits into this variable's spectrum of possibilities.

Professor Oden's background is unusual in that he has done research at a rigorous level mathematically, as well as worked directly on the finite element method as a tool in engineering practice. The book under review is one volume of a six-volume series (the Texas Finite Element Series, all published by Prentice-Hall) reflecting this broader perspective, and the book of interest here represents the rigorous side of the subject. This series presents an integration of different approaches, and a reader interested in studying the possible interplay between theory and practice could consult the other volumes. However, this reviewer has not done a study of the other volumes in the series, and the remainder of the review will focus on Volume 4 only. Moreover, this volume is quite compact, and so it seems appropriate to make this review quite short as well.

The examples in the introduction used to motivate elliptic systems of partial differential equations are very good. The book begins with one of the simplest possible (Laplace's equation) and then describes the full three-dimensional equations of linear elasticity, thus giving a good feeling for the level of complexity that can arise in applications after starting with a simple example introducing the key ideas. The book assumes a high level of mathematical sophistication on the reader's part. For example, one of the first homework problems is to prove the Riesz representation theorem! Further, important results concerning Sobolev spaces that are later used are only quoted, not derived. Thus, the book would form the basis for an advanced graduate course only if the students were sufficiently well prepared or if the instructor supplemented the book with lectures on some fundamental functional analysis and function spaces.

To this reviewer's taste, there are both weak and strong points concerning the authors' selection of material on finite element discretizations, the heart of the book. These opinions are, of course, subjective, but they are included here for readers who might wish to know topics to emphasize, supplement, etc., in using the book as a text or reference for a course. Unisolvence of elements is not discussed as extensively as might be possible. For example, systematic techniques for constructing elements are not given. More examples like the Argyris element at the end of Section 2.3 would make this part of the subject richer. On the other hand, the general discussion of mixed methods is quite good, and one of the main selling points of the book. Additional detailed calculations like the verification of the inf-sup condition in Section 4.4.3 would also be useful. Finally, the information in the chapter on hybrid methods is not to be found elsewhere in book form, as far as we know.

The intent of the book seems to be to collect in one place some mathematical results directly or indirectly relevant to finite element methodology. It can serve as a compact reference to these results, but it would be hard for students of mathematics to use this as a course text, without extensive supplements.

R.S.

38[65N99].—DEREK B. INGHAM & MARK A. KELMANSON, *Boundary Integral Equation Analyses of Singular, Potential, and Biharmonic Problems*, Lecture Notes in Engineering (C. A. Brebbia and S. A. Orszag, Editors), Springer-Verlag, Berlin, Heidelberg, New York, 1984, xiv + 173 pp., 24 cm. Price $12.50.

As the title of the book suggests, the purpose of this work is to extend the range of applicability of the boundary integral equation (BIE) method to include certain biharmonic problems and nonlinear potential problems, particularly problems which involve boundary singularities. The presence of these singularities greatly reduces the rate of convergence of standard BIE procedures. In their treatment of such problems, the authors modify the classical BIE method to take into account the analytic form of such a singularity, and demonstrate the efficacy of the modified method by comparing it with the classical BIE method on an appropriate test problem. In each problem discussed in this book, the approximate solution is piecewise constant, and, whenever possible, integrations are performed analytically, which, the authors claim, substantially reduces cpu time.

Chapter 1 is a brief introductory chapter. Chapter 2 is devoted to a discussion of the BIE solution of a biharmonic problem arising in fluid flow problems involving "stick-slip" boundary conditions, which give rise to a boundary singularity. In Chapter 3, methods similar to those developed in Chapter 2 are used to solve problems of flow near sharp corners, which involve corner singularities. Chapter 4 is concerned with certain nonlinear potential problems in which the differential equation can be linearized by applying the Kirchhoff transformation. The resulting problem not only has nonlinear boundary conditions but also boundary singularities. (It should be noted that Eq. (5) of this chapter is incorrect; the integral should be multiplied by φ^{-1}.) Viscous flow problems are also discussed in Chapters 5 and 6. In Chapter 5, problems with free surfaces, which are nonlinear, are considered, while Chapter 6 is devoted to a study of slow flow in bearings with arbitrary geometries. In Chapter 7, some conclusions are briefly stated. In Chapters 2 to 6, the results of numerical experiments are presented.

This book is an unusual publication. Not only is it a photo reproduction of the complete Ph.D. thesis of the second author, but five of its seven chapters, Chapters 2 to 6, have appeared in their entirety in five separate papers in refereed journals under the sole authorship of the second author. Since each of these chapters is self-contained, with its own abstract, introduction, conclusions, and references, there is substantial repetition, the elimination of which would considerably reduce the size of the book without any loss of information. Moreover, each of Chapters 2 to 6 is presented in the format of a preprint, with tables and figures gathered together after the references, and not inserted in the text where they are first mentioned. This, together with the lack of consistency in the notation, makes the book awkward to read.

There is no question that the material presented in this book is of much interest to researchers concerned with the development and application of BIE methods, particularly those interested in solving slow viscous flow problems. But it ought not to have been published in this form, when it has already appeared verbatim in the open literature.

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39[93B40, 93C20, 93C75].—K. L. TEO & Z. S. WU, *Computational Methods for Optimizing Distributed Systems*, Mathematics in Science and Engineering, Vol. 173, Academic Press, Orlando, Fla., 1984, xiii + 317 pp., $23\frac{1}{2}$cm. Price \$68.50.

With the rapid decrease of cost of computer power, the possibilities of using mathematical modelling in science and engineering have dramatically increased during the last decade. In particular, it is possible today to use computer-implemented numerical methods to solve complicated optimal control problems which may not be solved by classical analytical or ad hoc methods. This is particularly true for optimal control problems involving partial differential equations, and there is thus a great practical interest in having efficient numerical methods for such problems. There is a comparatively rich literature on theoretical-mathematical aspects of optimal control of partial differential equations, but much less so concerning numerical methods. The present book aims at partly filling this gap and is concerned with computational methods for optimal control of partial differential equations or distributed parameter systems. This problem area is very vast, including, in addition to analysis of the continuous problem, also discretization using finite element or finite difference methods and application of optimization methods for finite-dimensional problems, together with related convergence questions. The emphasis of the book in this wide spectrum is towards the continuous problem, particular attention being given to a mathematical convergence theory for certain gradient type methods for some optimal control problems involving (linear) parabolic equations. Discretized problems occur in the numerical examples, but are not treated theoretically. The material is based mainly on research of the authors and their associates.

A brief outline of the contents of the book is as follows: Chapters 1 and 2 contain background material from functional analysis and existence and regularity theory for linear parabolic partial differential equations. Chapter 3 is concerned with a class of optimal control problems involving linear parabolic equations with the controls occurring (nonlinearly) in lower-order derivative terms and in the forcing term. The controls are supposed to belong to a compact and convex subset of $\mathbb{R}^m$, and the cost functional is essentially a linear functional of the state at a given terminal time. This is a typical example of a stochastic optimal control problem with Markov terminal time. For this problem the authors consider a gradient type method involving, as is usual, the solution of an adjoint problem. An algorithm for constructing a sequence of controls with improving cost is presented. Theoretical aspects of the convergence of the algorithm are discussed and two numerical examples from stochastic population dynamics, involving one-dimensional parabolic problems, are presented. In Chapter 4, results of the previous chapter are extended, e.g., to problems involving also parameter selection, and two applications are made to one-dimensional parabolic problems arising in a study of choosing an optimal level of advertising in a marketing problem. Chapter 5 is concerned with the theoretical problem of existence of optimal controls, using suitable weak topologies and corresponding relaxed controls. Finally, Chapter 6 contains material similar to that of Chapter 3, now with Neumann boundary conditions occurring in the parabolic problem, and an application using a finite element discretization to a problem of optimally heating a slab of metal is made.

As indicated, the emphasis of the book is on theoretical aspects of computational procedures. The presentation is fairly technical and requires a relatively good mathematical background. The book addresses an important problem area of potentially great practical significance and presents several interesting contributions, together with brief overviews of earlier literature and an extensive bibliography.

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40[65L05, 65L20, 65M20].—K. Dekker & J. G. Verwer, *Stability of Runge-Kutta Methods for Stiff Nonlinear Differential Equations*, CWI Monograph 2, North-Holland, Amsterdam, 1984, ix + 307 pp., $24\frac{1}{2}$cm. Price \$36.50.

This book is concerned with aspects of the numerical solution of ordinary differential equations.

The work of Dahlquist, as presented in the classical book of Henrici [3], culminated in the so-called equivalence theorem: Convergence is equivalent to consistency and stability. This result covers the case when the stepsize tends to zero. However, more often we are interested in using a stepsize as large as possible. The stability behavior for a fixed step sequence is important.

Based on this vague requirement, Dahlquist [2] introduced the fundamental concept of A-stability for linear multistep methods. Later, A-stability was defined for other classes of methods, amongst them the family of Runge-Kutta methods. Contrary to linear multistep methods, whose order under the constraint of A-stability is bounded by two, A-stable Runge-Kutta methods of arbitrarily high order were shown to exist. But the implementation in efficient software was more difficult.

A-stability is based on the simple test equation $y' = \lambda y$, $\lambda \in \mathbb{C}$. In order to understand how a Runge-Kutta method would behave on a nonlinear problem, Burrage and Butcher [1] studied B-stability, i.e., the stability of the method on monotone differential systems. B-stability is a stronger requirement than A-stability. As a consequence, fewer methods are B-stable than A-stable.

The theme of this book is stability of Runge-Kutta methods for nonlinear stiff problems. As the title indicates, the exposition is largely theoretical. The book is devoted to the central question of stability. The purpose is to give a sound mathematical treatment of the development in the stability of Runge-Kutta methods from the A-stability childhood to today's B-stability manhood.

In the book's title there is one very important word, stiff. The usual way to define stiffness is through the eigenvalues of the system Jacobian. That definition links stability and stiffness. However, it has become more and more clear that there is more to stiffness than that. In the first chapter the authors therefore try to describe and identify stiff systems. As they phrase it on the first page, "The essence of stiffness is that the solution to be computed is slowly varying but that perturbations exist which are rapidly damped." Basically, that is correct, but the problem is how to describe stiffness in a firm mathematical way. As far as I can see, the authors are not able to give a precise definition. My conclusion is that we still do not know what a stiff system is.

Other related concepts, such as dissipativity and logarithmic norm, are introduced here.

In Chapters 2, 4, and 6 contractivity is defined and discussed. It is here that the results on B-stability are collected. However, although the book's title says nonlinear stability, I feel that it would have been natural to also include a discussion of A-stability, all the more so since the order star theory essentially characterizes that concept. There is no book yet that gives a treatment of this important work from 1978.

Stable Runge-Kutta methods are implicit. In Chapter 5 recent work on the existence of solutions to the respective nonlinear systems is described. Under the condition of a one-sided Lipschitz constant strictly smaller than zero the existence of a unique solution can be guaranteed.

Chapter 7 is devoted to B-convergence. The idea is to give an equivalence theorem for stiff systems.

A chapter on Rosenbrock methods and a chapter on stiff systems from semidiscretizations of partial differential equations are also included.

In summary, I will formulate my views as follows:

(1) The book is an up-to-date monograph on nonlinear stability of Runge-Kutta methods. However, too often the authors only tell where proofs can be found. In that way one needs a good collection of journals if one really wants to know how the results are obtained.

(2) The important work on order stars is mentioned only briefly. The book would have been much more complete with a chapter on that subject.

(3) The construction of Runge-Kutta methods is hardly mentioned. The implementation of the methods into good software is not considered at all. (I am aware of the fact that a discussion of that deserves a book on its own.)

(4) The book contains recent results on nonlinear stability. As such, it deserves to be read by all interested in that topic. It cannot be used as a textbook, but serves as a good reference text to the latest work in the field of stability for Runge-Kutta methods. The authors deserve our thanks for a valuable piece of work.

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1. K. BURRAGE & J. C. BUTCHER, "Stability criteria for implicit Runge-Kutta methods," *SIAM J. Numer. Anal.*, v. 16, 1979, pp. 46–57.

2. G. DAHLQUIST, "A special stability problem for linear multistep methods," *BIT*, v. 3, 1963, pp. 27–43.

3. P. HENRICI, *Discrete Variable Methods in Ordinary Differential Equations*, Wiley, New York, 1962.

41[65L05, 65L20].—RICHARD C. AIKEN (Editor), *Stiff Computation*, Oxford University Press, New York, 1985, xiv + 462 pp., 24 cm. Price $75.00.

The term *stiff differential systems* has been around for more than 30 years. It was introduced in 1952 by Curtiss and Hirschfelder [1]. In the intervening years, research on stiff systems has developed into different directions. We have learned to understand some of the mathematical properties of methods intended for such systems. A-stability is well known to all of us. In fact, several definitions of stability have been advanced. Around the same time as stiffness was born, the first code, based on the Kutta-Merson method, was written. From there on, we have witnessed a tremendous surge in the development of codes for stiff systems. These include codes intended both for numerical libraries and for use in larger simulation software, although the simulation researchers did not always adopt the best codes available.

The book under review contains the proceedings of a conference held April 12–14, 1982, in Park City, Utah. The purpose of this meeting was "to review the state of the art and practice of stiff computation, rather than to present latest research." Further, from the book's preface: "The speeches represented the spectrum of individuals involved in stiff computation from theoretical to software developer to end user." To me, this is important; researchers from all these aspects of stiff computation ought to be brought together to encourage maximum interaction.

In Chapter 1 Shampine describes stiffness. Most of us have a feeling for what stiffness is, but it is not easy to put it down in writing. This chapter clarifies the situation. However, there is still room for a precise definition, if that is possible.

Chapter 2 is devoted to application areas where stiff systems appear.

Newer methods for stiff systems are reviewed in Chapter 3, which is followed by a chapter on current software packages. Indeed, an interesting list of the most popular codes is found here. Naturally enough, a chapter on software tailored to specific applications is included.

It would be difficult to avoid a chapter on theoretical questions, presented by the field's "godfather", Germund Dahlquist. Let me cite Dahlquist: "Nothing is more practical than a good theory." The truth of this statement is deep. To write an efficient and robust code, we need theoretical insight.

The final chapter is most revealing. Cellier gives his opinion on where stiff computation is going. He points to very interesting open problems. Some of them are nearly solved, while others are wide open. Examples of the former are problems with discontinuities, examples of the latter, parallel methods. The chapter ends with a lively panel discussion. Everyone interested in stiff systems should read this part.

Let me close by citing Shampine from the last chapter: "... the theory is moving closer to practice...." To me, this is the ultimate goal of research. This book is a good contribution in that direction. Although a collection of individual papers, the book presents the material in a coherent way.

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1. C. F. CURTISS & J. O. HIRSCHFELDER, "Integration of stiff equations," *Proc. Nat. Acad. Sci. U.S.A.*, v. 38, 1952, pp. 235–243.

42[65L05, 65L20].—W. H. HUNDSDORFER, *The Numerical Solution of Nonlinear Stiff Initial Value Problems: An Analysis of One-Step Methods*, CWI Tract 12, Centre for Mathematics and Computer Science, Amsterdam, 1985, 138 pp., 24 cm. Price Dfl. 20.30.

This monograph is a reprint of the author's Ph.D. thesis written at the University of Leiden under the supervision of Professor M. N. Spijker. Broadly speaking, the topic considered is the use of one-step methods to solve nonlinear stiff initial-value problems of the form

$$y'(t) = f(y(t)), \qquad t > t_0,\ y(t_0) = y_0,$$

satisfying the one-sided Lipschitz inequality

$$\text{(1)} \qquad \operatorname{Re}\{(f(x) - f(y), x - y)\} \leqslant \beta(x - y, x - y),$$

where t, $\beta \in \mathbf{R}$ and $f, x, y \in \mathbf{C}^n$ (although sometimes restricted to $\mathbf{R}^n$).

However, as is the case for most good theses, this monograph examines in depth a much more narrowly defined topic. More specifically, the one-step methods that the author considers are restricted to implicit and semi-implicit Runge-Kutta methods, the latter being of the form

$$y_{n+1} = y_n + h \sum_{i=1}^{m} b_i(hJ(y_n))f(Y_i),$$

$$Y_i = y_n + h \sum_{j=1}^{i-1} a_{ij}(hJ(y_n))f(Y_j), \qquad 1 \leqslant i \leqslant m,$$

where b_i and a_{ij} are rational functions with real coefficients. Two classes of semi-implicit Runge-Kutta methods are examined in particular: the Rosenbrock methods for which $J(y_n) = f_y(y_n)$, and those for which J is constant.

The two principal questions that the author addresses are:

(i) Assuming only that f is continuous and satisfies (1), what conditions on the stepsize h and the coefficients of a Runge-Kutta method ensure that the algebraic equations associated with the method are well defined and have a unique solution?

(ii) To what extent do the conclusions about the numerical approximations which can be drawn for the simple test problem $y' = \lambda y$, $\lambda \in \mathbf{C}$, $\operatorname{Re}\lambda \leqslant \beta$, carry over to nonscalar nonlinear problems satisfying inequality (1)?

In addressing question (ii), the author pays particular attention to developing useful stability bounds for methods that are strongly A-stable but not B-contractive. This is done by suitably restricting the class of nonlinear problems to which the results apply, without limiting the stiffness of the problems under consideration.

In addition to developing several new results in answer to these questions, the author presents a concise summary of preliminary material needed to address these topics, as well as a review of results presented earlier by himself and others.

However, the prospective reader should note that this book does not contain a wide-ranging review of stability for stiff nonlinear initial-value problems. Nor is it a guide for the practicing scientist or engineer seeking to find advice on the numerical solution of stiff nonlinear problems. But it should not be faulted for failing to address these subjects: This book is exactly what it purports to be—a well-written research monograph on the narrowly focused topic that it addresses.

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43[41-02, 46-02].—ALLAN PINKUS, *n-Widths in Approximation Theory*, Springer-Verlag, Berlin, Heidelberg, New York, 1985, x + 291 pp., 25 cm. Price $39.00.

The n-width measures how well a subset of a normed linear space can be approximated by n-dimensional subspaces. A typical, and very important, example is the approximation of smooth functions. With this application in mind, Kolmogoroff [2] introduced the concept of n-width in 1936. For a long time, little progress has been made, except in a Hilbert space setting. But in the past 20 years, remarkable results have been obtained on the n-width of Sobolev spaces. Micchelli and Pinkus [3] were able to characterize the optimal subspaces in many cases. Kashin [1] showed the existence of approximation processes which converge at a substantially better rate than any of the standard approximation methods. To date, there are still many basic open problems; an example is the asymptotic order of the n-width of the Sobolev space W_1^1 in L_q for $2 < q < \infty$.

The book under review gives for the first time a comprehensive and up-to-date description of the theory of n-width and related s-numbers [4]. A major part of the book is devoted to the results on Sobolev spaces. In Chapter 5 and part of Chapter 4 the optimality of spline interpolation is shown as an application of a more general theory for the approximation of integral operators. A prerequisite is Chapter 3 which reviews the basic facts about total positivity and Chebyshev systems. As the diagrams on p. 233 (which do not yet represent the most complete description) indicate, the asymptotic results, which are described in Chapter 7, are fairly complicated. While not all proofs for the upper bounds are given, the basic techniques are covered and several illustrative special cases are discussed in detail. In addition to n-width for Sobolev spaces and related topics, the author considers n-width of matrices (Chapter 6), n-width in Hilbert spaces (Chapter 4), and n-width of algebraic functions (Chapter 9).

The book is well written and very systematically organized. It is an excellent text for the specialist. However, it might be difficult to read for anyone looking for an introduction to the subject.

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1. B. S. KASHIN, "Diameters of some finite-dimensional sets and classes of smooth functions," *Izv. Akad. Nauk SSSR*, v. 41, 1977, pp. 334–351.

2. A. KOLMOGOROFF, "Über die beste Annäherung von Funktionen einer gegebenen Funktionenklasse," *Ann. of Math.*, v. 37, 1936, pp. 107–110.

3. C. A. MICCHELLI & A. PINKUS, "Some problems in the approximation of functions of two variables and the n-widths of integral operators," *J. Approx. Theory*, v. 24, 1978, pp. 51–77.

4. A. PIETSCH, "s-numbers of operators in Banach spaces," *Studia Math.*, v. 51, 1974, pp. 201–223.

44[65D20].—C. G. VAN DER LAAN & N. M. TEMME, *Calculation of Special Functions: The Gamma Function, the Exponential Integrals and Error-Like Functions*, CWI Tract 10, Centrum voor Wiskunde en Informatica (Centre for Mathematics and Computer Science), 1984, iv + 231 pp., $23\frac{1}{2}$ cm. Price Dfl. 33.30.

This book is the first of a projected series intended to supplement AMS 55, the *Handbook of Mathematical Functions* [1], by providing detailed information for preparing and testing computer software for special functions. AMS 55 remains unsurpassed as a no-nonsense compilation of the properties of special functions, but it was prepared too early to contain the software information supplied here.

This book contains five chapters. The first is an introduction for the entire projected series. It contains an annotated bibliography on the computation of elementary and special functions, and a survey of major sources of function software. The latter includes a brief summary of the design criteria (e.g., whether portability is emphasized and how it is achieved) for a given collection whenever those criteria are known.

Chapter 2 is by far the longest in the book. It discusses topics that are fundamental to the following chapters. These include a brief discussion of error analysis, a particularly thorough discussion of linear recurrences, and quick overviews of continued fractions and hypergeometric functions.

Each of the remaining three chapters is dedicated to a different family of functions. Chapter 3 concerns the gamma family; Chapters 4 and 5 are dedicated, respectively, to the exponential integral family and the error function family. The overall plan of these three chapters is first to summarize important analytic properties, emphasizing those that are useful in numerical evaluation, and then to discuss algorithms and existing software in detail. The text is liberally sprinkled with references to recent work on practical convergence and utility of expansions, problems of range reduction, tables of coefficients, error analysis, etc. Thus, each chapter informs the potential designer or user of function software which methods might be useful, which have already been tried, which have succeeded, and where the software is to be found.

If there is a criticism of this book, it is in the binding. The paperback binding is not sturdy enough for a volume this useful. One of the two copies I own is already losing pages.

In summary, this book, together with those by the late Yudell Luke [2], [3], is an essential companion to AMS 55 and should be on the shelf of anyone concerned about the computation of special functions. We can all hope that Van der Laan and

Temme follow through on their intention to provide additional volumes in the series.

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1. M. ABRAMOWITZ & I. A. STEGUN (Editors), *Handbook of Mathematical Functions with Formulas, Graphs and Mathematical Tables*, Nat. Bur. Standards Appl. Math. Series, 55, U. S. Government Printing Office, Washington, D. C., 1964.

2. Y. L. LUKE, *The Special Functions and Their Approximations*. Vols. I and II, Academic Press, New York, 1969.

3. Y. L. LUKE, *Mathematical Functions and Their Approximations*, Academic Press, New York, 1975.

45[62Q05].—W. J. KENNEDY, R. E. ODEH & J. M. DAVENPORT (Editors), *Selected Tables in Mathematical Statistics*, Vol. VIII, Amer. Math. Soc., Providence, R.I., 1985, ix + 270 pp., 26 cm. Price $30.00.

This book is a collection of three sets of tables, each accompanied by a paper which presents the statistical problem being studied, summarizes the theory underlying the problem, discusses how the tables were calculated, explains how to use the tables, and provides several examples of their use. The first two sets of tables are of use in the design of experiments. The third set has applications to the construction of estimators and goodness of fit tests when the underlying distribution belongs to a family having structure similar to Student's t. Presumably, these tables have been constructed so as to make the associated statistical methodology more accessible to practitioners. In what follows I shall therefore try to describe situations for which the tables might be useful and comment on the readability of each paper.

The first set of tables by Benon J. Trawinski can be used to design and analyze balanced paired comparison experiments when the goal of the experimenter is to select from a set of T treatments a subset of size S containing the best. Balanced paired comparison experiments involve comparing all possible pairs of treatments equally often, specifying a preference each time. Such experiments arise, for example, in the food industry where food samples are to be compared for taste or visual preference, or in rehabilitation where treatments for the improvement of patient performance are to be compared.

The tables enable an experimenter to determine the number of replications of a balanced paired comparison experiment necessary to guarantee that the subset of size S selected contains the best treatment with probability no less than some prespecified value. They can be used to determine the rule for selecting the subset as well as the expected size of the subset. These tables would seem to be of particular value to researchers who conduct paired comparison experiments on a regular basis.

I found the explanatory material accompanying the tables a bit difficult to read. The examples in Sections 3 and 5 can be used to determine how to use the tables, but these examples are not as clear as they could be. This is especially true of the examples discussing how to carry out interpolations between values in the tables. In fact, Examples 5.1–5.4 refer to illustrations which have been omitted from the book. This lack of clarity means that potential users of these tables will have to do some extra work in order to correctly use the tables.

The second set of tables by Robert E. Bechhofer and Ajit C. Tamhane can be used to design experiments for comparing p test treatments with a control when the experiment is to be run in small (size 2 or 3) blocks. A matched pairs design would be an example of such an experiment with block size 2. The tables provide lists of so-called balanced treatment incomplete block designs for values of p up to 6 and block sizes 2 and 3. These designs are appropriate when the goal of the experimenter is to calculate simultaneous one- or two-sided confidence intervals for the p contrasts of test treatments with the control. One set of tables presents designs which are optimal in the sense of requiring the fewest number of blocks to achieve a prespecified confidence coefficient. Since it is possible that in certain situations an optimal design cannot be used (for example, a block in the optimal design contains a treatment combination which is unfeasible to employ), another set of tables is provided which lists so-called admissible (good) designs.

I found the explanatory material accompanying these tables to be well organized and fairly easy to read, although this may be partly due to my familiarity with this material. The examples are clear, and explanations of how to use the tables are easy to follow. Formulas for analyzing the data resulting from a balanced treatment incomplete block design are also provided. Any experimenter who has some familiarity with standard analysis of variance should have little difficulty employing these tables.

The final set of tables by Mofi Lai Tiku and S. Kumra present the expected values and covariances of order statistics of random samples from a family of symmetric distributions which contains Student's t. In fact, these distributions are reducible to Student's t by means of a linear transformation of the variable.

If one has reason to believe that the underlying distribution in a statistical problem belongs to this family, the tabulated means and covariances can be used to construct best linear unbiased estimators of the mean and variance of the distribution, goodness of fit tests, and robust estimators of the mean. The authors give examples of each of these applications. The examples are easy to follow and the use of the tables is presented clearly. I would, however, like to have seen some additional examples of situations for which these tables might be helpful.

As is the case with most sets of tables, this book will probably appeal to a fairly narrow audience. Experimenters who regularly conduct pairwise comparison experiments, who are involved in making treatment vs. a control comparison in block design settings, or who wish to do statistical inference when the underlying distribution is something like Student's t, should find this book helpful and may wish to have a copy on their shelf.

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46[62-04, 62H30].—Helmut Späth, *Cluster Dissection and Analysis; Theory, FORTRAN Programs, Examples* (Translated from German by Johannes Goldschmidt), Ellis Horwood Series in Computers and their Applications (Brian Meek, Series Editor), Ellis Horwood Limited, Chichester, and Wiley, New York, 1985, 226 pp., $24\frac{1}{2}$ cm. Price \$49.95.

This is the author's third book on clustering methods, and his second in the English language. (See the list of references below.) The book is quite different from any other existing text on cluster analysis: Only two pages are devoted to real-life applications, and no further motivation for the use of clustering methods is given. Thus, Späth's book cannot be recommended for readers with no previous knowledge in cluster analysis. For those who have some experience with clustering methods, however, this book may be quite useful and interesting.

The book is divided into three parts: Theory and methods (Part 1), Implementation of FORTRAN subroutines (Part 2), and Sample main programs, examples, suggestions for use (Part 3).

Part 1 concentrates on a mathematical treatment of cluster-analytic methods. This part is the main strength of the book, although the author's style is sometimes too formal, at least to my taste: Symbols are introduced where words would do as good a job—see, for example, the definitions of sets of partitions on page 37, some of which are never used in the sequel.

Thanks to the high mathematical level, various criteria can be presented in a unified way. For the mathematically trained reader this is a nice aspect, but for nonmathematicians it makes the book very hard to understand. I do not think that the readership indicated on the inside of the front cover (e.g., students from the biological and social sciences) will appreciate this text. They are well advised to read one of the standard textbooks instead, e.g., Späth [6], Everitt [2], Hartigan [3], or a chapter on cluster analysis in some text on multivariate statistics (Seber [5], Dillon and Goldstein [1], Johnson and Wichern [4]). However, I have enjoyed reading Part 1, except for its last chapter, which treats "clusterwise linear regression". I think that this technique is inappropriate in the context of clustering methods, since it involves a distinction between "dependent" and "independent" variables, which is most often inappropriate in applications of cluster analysis.

Part 2 lists FORTRAN subroutines and describes their implementation. Unfortunately, the print of the source listings is often quite poor, which may easily lead to errors. The documentation of the subroutines is rather disorganized. I would actually prefer a documentation according to some strict rules, for instance those used by NAG, or IMSL, or those used by *Applied Statistics* for contributions to its algorithms' section.

Part 3 is the weakest part of the book. No real data examples are presented. The examples have actually been constructed so as to fit the various criteria in an optimal way. I agree that using such examples is justified when checking whether an algorithm is doing what it is supposed to do. However, I think that one should not fill over fifty pages of a book with examples of this kind. Moreover, I got the impression that this part did not get as much attention from the author as the previous ones: The text is poorly organized, and there are no titles to the numerous figures and tables. The computer outputs are hardly structured, which makes them quite annoying to read and to interpret.

An appendix describes a magnetic tape containing all programs. The tape can be ordered from the publisher.

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47[03E72, 03H15, 60A05, 65G10].—ARNOLD KAUFMANN & MADAN M. GUPTA (Editors), *Introduction to Fuzzy Arithmetic, Theory and Applications*, Van Nostrand Reinhold, New York, 1985, xvii + 351 pp., $23\frac{1}{2}$ cm. Price $44.95.

Fuzzy numbers are one way to describe the vagueness and lack of precision of data. The theory of fuzzy numbers is based on the theory of fuzzy sets which was introduced in 1965 by L. Zadeh who wrote the foreword for this book. The concept of a fuzzy number was first used by Nahmias in the United States, and by Dubois and Prade in France in the late seventies. The present book by Arnold Kaufmann and Madan M. Gupta is the first introduction to the theory of fuzzy numbers; it is primarily aimed at the beginner who wants to learn these concepts from the start. But it contains numerous novel definitions and illustrative examples as well, and therefore it can be used as a collection of ideas for new research in the field of fuzzy numbers and for the application of these concepts. The authors are experienced researchers in the field of fuzzy sets and have written four books and numerous research papers on the subject before.

A fuzzy number represents an approximation of an unknown real or integer value. To every "level of presumption" between zero and one an "interval of confidence" is attributed which is believed to contain the true value with the corresponding degree of certainty. At least one value has to possess the highest level of presumption, i.e., one. The lower the level of presumption gets, the larger is the interval of confidence. Operations for fuzzy numbers are defined using the so-called max-min convolution. The authors illustrate the concept of a fuzzy number with the following example:

A certain job is known to be completed between May 15 and May 31; possibly it is completed on May 22. Then we can assign two levels of confidence in this situation, namely 1 for the interval [May 22, May 22] and 0 for [May 15, May 31]. Of course, an appropriate interval can be assigned for every level between 0 and 1.

In a certain way, this concept is a generalization of interval arithmetic, where every level of presumption could be assigned the same interval of confidence. An interval (as an approximation of a real number) contains the unknown true value with certainty and nothing is known about the precise location of the true value inside the interval. Fuzzy numbers are similar to random numbers as well, but they should not be confused: A random number is associated with the error in the measurement of a theoretically precise value, whereas a fuzzy number is a way to describe the uncertainty of human thought; it is a "subjective valuation assigned by one or more human operators."

A large number of generalizations of the concept of a fuzzy number are considered. Fuzzy numbers of higher dimensions can be introduced. Fuzzy complex numbers, fuzzy relative integers modulo n, fuzzy reals modulo one, and other concepts are considered. Fuzzy numbers of type two can be defined as fuzzy numbers where the intervals of confidence are not precisely known, being fuzzy numbers themselves. Numerous notions of statistics can be adapted to, or combined with the theory of fuzzy numbers; this, for example, leads to the novel concept of a hybrid number.

All of the concepts and operations are illustrated by a large number of examples which are in most cases given in the form of easy-to-understand figures. Many of these examples make use of triangular fuzzy numbers which are the easiest to compute with. In addition to these illustrative examples, the book contains some applications, including an optimization problem and an application of fuzzy numbers in catastrophe theory. The reviewer would like to make two recommendations about future research in the field of fuzzy numbers: First, fuzzy numbers are described by a function which tends to get more and more complicated after every operation; for example, the sum of triangular fuzzy numbers is a triangular fuzzy number again, but the product is not; is there a class of fuzzy numbers which can be easily represented in a computer and which is, in addition, closed, under all operations under consideration? Second, more applications should be investigated where fuzzy arithmetic is essential to obtain clear, reliable, detailed results that interval arithmetic, for example, could not deliver.

In summary, the present book is an excellent introduction for anybody who wants to get acquainted with the theory of fuzzy numbers. The numerous illustrative examples make it ideal for self-instruction as well as for courses on the subject. For the more advanced reader it contains a large number of novel ideas and incentives for application and research.

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48[68-02, 68P05, 68Q25, 68U05].—FRANCO P. PREPARATA & MICHAEL IAN SHAMOS, *Computational Geometry—An Introduction*, Texts and Monographs in Computer Science, Springer-Verlag, New York, 1985, xii + 390 pp., 24 cm. Price $45.00.

A book like this was badly needed by the scientific community, especially in view of the rapid growth and increasing popularity of the area of computational geometry, and its importance from both a theoretical and practical point of view. The book is very well written and is impressive in the breadth and depth of its coverage of the field. A major strength of the book is that it typically gives an excellent intuitive explanation of an algorithm or data structure before providing implementation details. The lower-bound proofs are given in the unified framework of Ben-Or's theorem; including this theorem was a particularly good decison by the authors. As a reference, the book is a must for the researcher in algorithm design and data structures, and for any programmer writing software that deals with geometric objects. As a textbook, the book is ideal for a graduate-level course taken by students who have already taken an algorithms course, or as a supplementary textbook for a senior or graduate-level course on algorithm design and analysis; in either case, the book's list of suggested exercises will be most valuable to the instructor.

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49[11T06].—HAROLD FREDRICKSEN & ROBERT WARD, *A Table of Irreducible Binary Pentanomials of Degrees* 4 *Through* 100, 4 pp. of introductory text, 1 fig., and 273 pp. of tables, deposited in the UMT file.

An irreducible binary pentanomial of degree n takes the form $f(x) = x^n + c_{n-1}x^{n-1} + c_{n-2}x^{n-2} + \cdots + c_1x + 1$. The c_i are 0's and 1's with exactly three c_i nonzero. $f(x)$ is irreducible means there are no polynomials $g(x)$ and $h(x)$, both of degrees less than n over the field of 2 numbers, for which $f(x) = g(x)h(x)$.

In the tables, all irreducible pentanomials over the field of 2 elements are given for each degree from $n = 4$ through $n = 100$. Each polynomial which appears is listed as a, b, c, where these are the exponents of x for which the coefficients of f are nonzero. If $x^n + x^c + x^b + x^a + 1$ is irreducible with $a < b < c$, then its reverse polynomial $x^n + x^{n-a} + x^{n-b} + x^{n-c} + 1$ is also irreducible. We do not list both of these polynomials in the interest of economy. For each degree we also list the total number of irreducible pentanomials of that degree. Here, of course, we count both a polynomial and its reverse polynomial.

There have not been extensive tables of irreducible binary pentanomials published. However, there have been a number of tables of irreducible binary trinomials published [1]–[2]. When one examines these tables of trinomials, it is clear that there are several degrees for which there are no irreducible trinomials of these degrees. In fact, for degree $8t$ there are no irreducible trinomials for any t and for degrees $8t \pm 3$ there are very few irreducible trinomials of those degrees. It has been conjectured that there is no degree above $n = 4$ which does not possess an irreducible pentanomial of that degree. Our table lends evidence to support that conjecture.

The number N_n, of pentanomials of degree n, is also plotted with our tables. At (n, N_n) we have plotted the value of n (mod 8). One of the main suggestions of our work is clear from this figure, that is, there is a distinct modulo 8 character to the number of irreducible pentanomials. This compares to the similar modulo 8 character of irreducible trinomials indicated in Swan [3] and Fredricksen et al. [4].

Curves drawn through the points with $n \equiv c \pmod 8$ and with $c \equiv 4$ and $\pm 1 \pmod 8$ are approximately the same curves and lie above the others. Next come the curves for $c \equiv \pm 2 \pmod 8$, then those for $c \equiv \pm 3 \pmod 8$ and finally that for $c \equiv 0 \pmod 8$. These curves reflect the distribution for the similar sums of irreducible trinomials of the same congruences. If $1/n$ of all pentanomials of degree n are irreducible then the curves should be roughly quadratic in growth rate, as there are $\binom{n-1}{3}$ pentanomials of degree n. The curves appear to fit parabolas quite well.

The factoring algorithm. The irreducible pentanomials were found by subjecting all pentanomials of degree n to the following algorithm. Only one of the pentanomials $P_{n,c,b,a}$ and $P_{n,n-a,n-b,n-c}$ need be tested as they are both either reducible or irreducible together. If $P_{n,a,b,c}$ passes a test it is subjected to the next test. Survivors of Test 5 are irreducible.

Test 1. Check that not all of n, a, b, c are even, for if all are even, $P_{n,c,b,a}$ is a square and hence reducible.

Test 2. $P_{n,c,b,a}$ is divisible by the polynomial $f(x)$ of the following table if any three of (n, a, b, c) are congruent to y and the other is congruent to z (modulo m) of the table.

m	$f(x)$	(y, z)
3	$x^2 + x + 1$	(2, 1)
7	$x^3 + x + 1$	(3, 1)(5, 4)(6, 2)
7	$x^3 + x^2 + 1$	(3, 2)(5, 1)(6, 4)
15	$x^4 + x + 1$	(4, 1)(8, 2)(9, 7)(10, 5)(12, 11)(13, 6)(14, 3)
15	$x^4 + x^3 + 1$	(4, 3)(8, 6)(9, 2)(10, 5)(12, 1)(13, 7)(14, 11)

Test 3. The GCD of $P_{n,a,b,c}$ and $x^{2^d} + x$ for $d = 5, 6, \ldots, 10$ is computed. A nontrivial GCD of $P_{n,a,b,c}$ and $x^{2^d} + x$ will identify $P_{n,a,b,c}$ as having an irreducible factor of degree d.

Test 4. Calculate $x^{2^n} \pmod{P_{n,a,b,c}}$. When $P_{n,a,b,c}$ is irreducible, it is necessary that $x^{2^n} \equiv x \pmod{P_{n,a,b,c}}$. Some pentanomials $P_{n,a,b,c}$ will pass this test but not be irreducible. The final test is required.

Test 5. The remaining pentanomials $P_{n,a,b,c}$ have $x^{2^n} \equiv x \pmod{P_{n,a,b,c}}$, so any irreducible factor $g(x)$ of $P_{n,a,b,c}$ will have degree m which divides n. If m is not equal to n, then $m = n/p$ for some prime p. The $g(x)$ divides $x^{2^{n/p}} + x$ and the GCD of $P_{n,a,b,c}$ and $x^{2^{n/p}} + x$ is not equal to 1. Thus, calculate the GCD of $P_{n,a,b,c}$ and $x^{2^{n/p}} + x$ for each prime factor p of n. If all these GCD's are 1, then $P_{n,a,b,c}$ is irreducible.

Conjectures. We gather together here the observations we have about the tables we present.

Conjecture 1. There is an irreducible binary pentanomial of each degree $d \geqslant 4$.

Conjecture 2. There is a distinct modulo 8 character to the number of irreducible binary pentanomials.

Conjecture 3. The congruence classes of degrees of irreducible binary pentanomials are ordered with classes $c \equiv \pm 1, 4$ being largest, then $c \equiv \pm 2$, followed by $c \equiv \pm 3$ and finally $c \equiv 0$ (modulo 8).

Conjecture 4. The congruence classes of degrees of irreducible binary pentanomials are quadratically generated so that approximately $1/n$ of all binary pentanomials are irreducible.

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