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# Automated Theorem Proving: After 25 Years 

W. W. Bledsoe and D. W. Loveland, Editors

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This volume contains papers based on a special session for automated theorem proving held at the annual meeting of the American Mathematical Society in Denver. January. 1983. At the meeting special awards were given to honor historically significant work (the Milestone Prize: Hao Wang. awardee) and to honor excellent current work (the Current Research prize: Lawrence Wos and Steven Winker. awardees). Roughly a dozen leading contributors to the field were invited to present papers: papers characterizing their research work or a broader perspective were encouraged. Papers range from a historical overview of twenty-five years of research in the automated theorem proving field to significant technical papers. including a reprint of a Scientia Sinica paper giving a new and elegant decision procedure for a portion of elementary geometry.
Most of the major efforts in building automated theorem provers (or theorem proving assistants) are covered by papers in this volume. a notable but less familiar example (to the ATP community) being the Suppes interactive theorem prover for teaching logic and axiomatic set theory. The well-known provers of Andrews. Bledsoe. Boyer and Moore. and Wos. et al. are represented as are term rewriting. combining decision procedures and automating mathematical discovery. The book is intended for every mathematician and computer scientist interested in the state-of-the-art in automated theorem proving. but in particular. it is intended to encourage active research mathematicians to contribute their insight to this field.
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