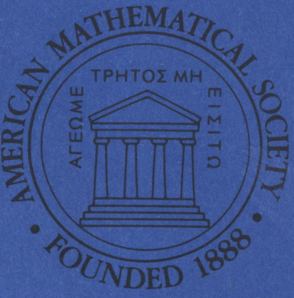
Mathematics of Computation



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Mathematics of Computation

Special Issue

Dedicated to

DANIEL SHANKS

January 1987

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Dedication

The editors of Mathematics of Computation are pleased to dedicate this issue to Daniel Shanks on the occasion of his seventieth birthday, January 17, 1987.

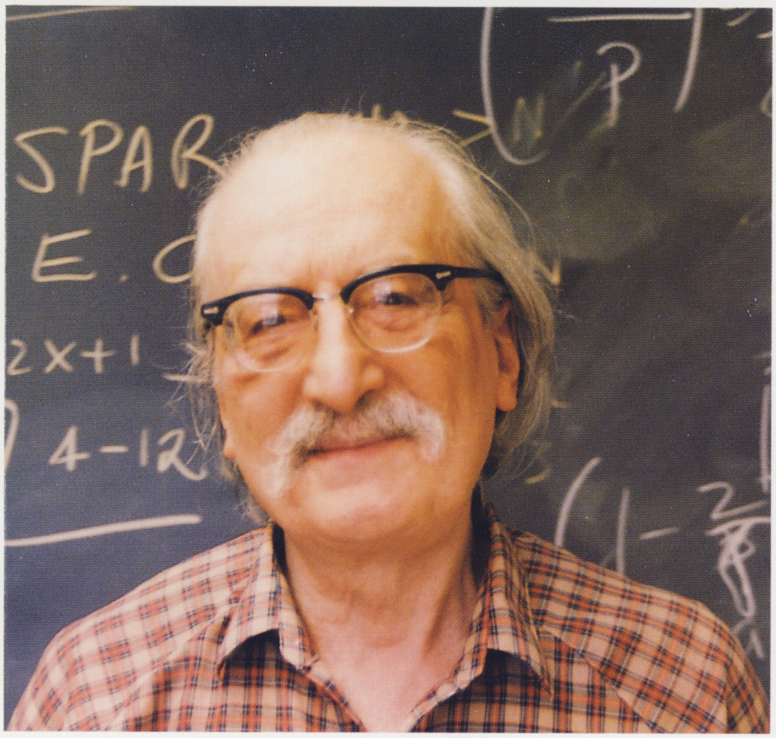
Dan joined the Editorial Committee in 1959 and has served, either on this committee or on The Board of Associate Editors, ever since. Indeed, when occasion has demanded, he has even acted in an ex officio capacity as the chairman of The Editorial Committee. He is also the custodian of the UMT File.

The extent of Dan's contribution to the computational number theory component of Mathematics of Computation is incalculable. Through his energy in finding new manuscripts, his enthusiasm in encouraging younger authors, and his innovative ideas, he is, to a very great degree, responsible for the flourishing state of this discipline. Some idea of his immense influence and wide range of interests can be gained by an examination of the table of contents of this issue.

In an effort to keep this dedication a surprise and to keep this issue from growing too large, the editors did not inform all of the people who would have liked to contribute articles to celebrate Dan's birthday. To those whose work does not appear here, we sincerely apologize.

The editors are pleased to acknowledge the tremendous amount of care and work put in by the authors of the articles contained in this volume. We would also like to thank the referees, whose patience, good humor, and professionalism, was a great help in putting up with what were frequently very close deadlines.

H. C. Williams
for the editors



DANIEL SHANKS

MATHEMATICS OF COMPUTATION
TABLE OF CONTENTS
January 1987

William W. Adams, Characterizing Pseudoprimes for Third-Order Linear Recurrences 1

Leonard M. Adleman, Dennis R. Estes, and Kevin S. McCurley, Solving Bivariate Quadratic Congruences in Random Polynomial Time 17

Richard Blecksmith, John Brillhart, and Irving Gerst, Parity Results for Certain Partition Functions and Identities Similar to Theta Function Identities 29

Johannes Buchmann, The Computation of the Fundamental Unit of Totally Complex Quartic Orders 39

Johannes Buchmann and H. C. Williams, On Principal Ideal Testing in Totally Complex Quartic Fields and the Determination of Certain Cyclotomic Constants 55

Nicholas Buck, Lones Smith, Blair K. Spearman, and Kenneth S. Williams, The Cyclotomic Numbers of Order Fifteen 67

Duncan A. Buell, Class Groups of Quadratic Fields. II 85

David G. Cantor, Computing in the Jacobian of a Hyperelliptic Curve 95

H. Cohen and A. K. Lenstra, Implementation of a New Primality Test 103

H. Cohen and J. Martinet, Class Groups of Number Fields: Numerical Heuristics 123

Harvey Cohn and Jesse Deutsch, Application of Symbolic Manipulation to Hecke Transformations of Modular Forms in Two Variables 139

T. W. Cusick and Lowell Schoenfeld, A Table of Fundamental Pairs of Units in Totally Real Cubic Fields 147

Daniel Gordon, Douglas Grenier, and Audrey Terras, Hecke Operators and the Fundamental Domain for $SL(3, \mathbf{Z})$ 159

Marie-Nicole Gras, Special Units in Real Cyclic Sextic Fields 179

R. K. Guy, C. B. Lacampagne, and J. L. Selfridge, Primes at a Glance 183

Neal Koblitz, Elliptic Curve Cryptosystems 203

D. H. Lehmer and Emma Lehmer, Cyclotomic Resultants 211

H. W. Lenstra, Jr. and R. J. Schoof, Primitive Normal Bases for Finite Fields 217

R. A. Mollin, Class Numbers of Quadratic Fields Determined by Solvability of Diophantine Equations 233

Peter L. Montgomery, Speeding the Pollard and Elliptic Curve Methods of Factorization 243

Morris Newman and Robert C. Thompson, Numerical Values of Goldberg's Coefficients in the Series for $\log(e^x e^y)$ 265

A. M. Odlyzko, On the Distribution of Spacings Between Zeros of the Zeta Function 273

M. Pohst, On Computing Isomorphisms of Equation Orders 309

Carl Pomerance, Very Short Primality Proofs 315

Herman J. J. te Riele, On the Sign of the Difference $\pi(x) - \mathrm{li}(x)$ 323

Robert D. Silverman, The Multiple Polynomial Quadratic Sieve 329

Jonathan W. Tanner and Samuel S. Wagstaff, Jr., New Congruences for the Bernoulli Numbers 341
Heinz M. Tschöpe and Horst G. Zimmer, Computation of the Néron-Tate Height on Elliptic Curves 351
Lawrence C. Washington, Class Numbers of the Simplest Cubic Fields 371
H. C. Williams, Effective Primality Tests for Some Integers of the Forms $A5^n - 1$ and $A7^n - 1$ 385
H. C. Williams and M. C. Wunderlich, On the Parallel Generation of the Residues for the Continued Fraction Factoring Algorithm 405
Don Zagier, Large Integral Points on Elliptic Curves 425
Reviews and Descriptions of Tables and Books 437
Shanks **1**, Rosen **2**, Riesel **3**, Conway, Curtis, Nortin, Parker, and Wilson **4**, Bach **5**, Muskat and Williams **6**
Corrigendum 447
Costa Pereira
Supplement to "Implementation of a New Primality Test" by **H. Cohen and A. K. Lenstra** S1
Microfiche Supplements
Nicholas Buck, Lones Smith, Blair K. Spearman, and Kenneth S. Williams, The Cyclotomic Numbers of Order Fifteen
Morris Newman and Robert C. Thompson, Numerical Values of Goldberg's Coefficients in the Series for $\log(e^x e^y)$

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(*Continued from back cover*)

Jonathan W. Tanner and Samuel S. Wagstaff, Jr., New Congruences for the Bernoulli Numbers 341

Heinz M. Tschöpe and Horst G. Zimmer, Computation of the Néron-Tate Height on Elliptic Curves 351

Lawrence C. Washington, Class Numbers of the Simplest Cubic Fields 371

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Don Zagier, Large Integral Points on Elliptic Curves 425

Reviews and Descriptions of Tables and Books 437

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TABLE OF CONTENTS

January 1987

William W. Adams, Characterizing Pseudoprimes for Third-Order Linear Recurrences 1

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(Continued on inside back cover)