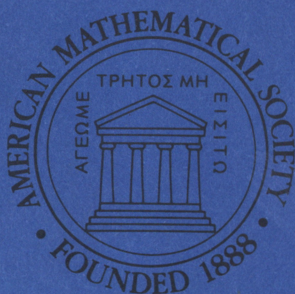


Mathematics of Computation



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Special Issue

Dedicated to

DANIEL SHANKS

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Dedication

The editors of Mathematics of Computation are pleased to dedicate this issue to Daniel Shanks on the occasion of his seventieth birthday, January 17, 1987.

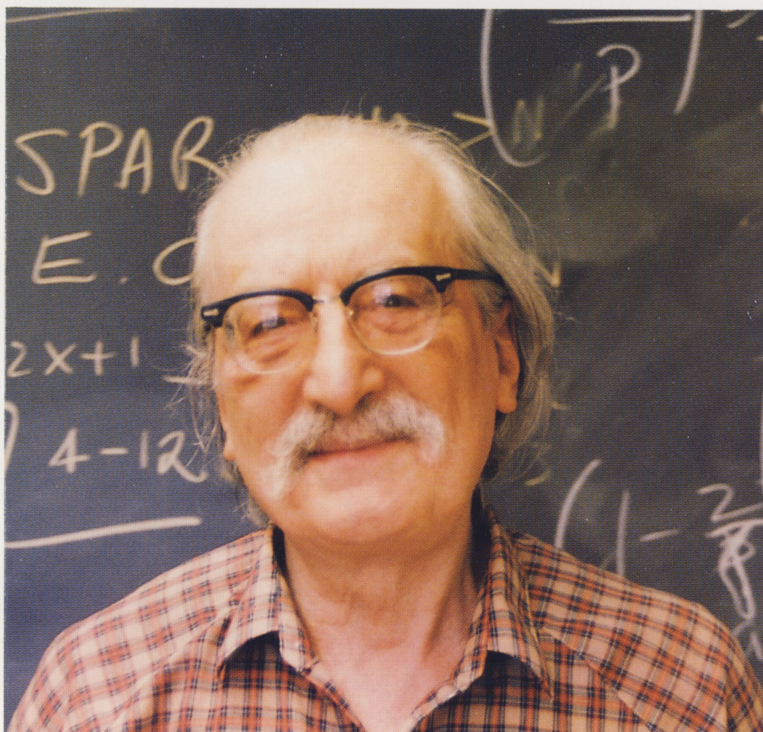
Dan joined the Editorial Committee in 1959 and has served, either on this committee or on The Board of Associate Editors, ever since. Indeed, when occasion has demanded, he has even acted in an ex officio capacity as the chairman of The Editorial Committee. He is also the custodian of the UMT File.

The extent of Dan's contribution to the computational number theory component of Mathematics of Computation is incalculable. Through his energy in finding new manuscripts, his enthusiasm in encouraging younger authors, and his innovative ideas, he is, to a very great degree, responsible for the flourishing state of this discipline. Some idea of his immense influence and wide range of interests can be gained by an examination of the table of contents of this issue.

In an effort to keep this dedication a surprise and to keep this issue from growing too large, the editors did not inform all of the people who would have liked to contribute articles to celebrate Dan's birthday. To those whose work does not appear here, we sincerely apologize.

The editors are pleased to acknowledge the tremendous amount of care and work put in by the authors of the articles contained in this volume. We would also like to thank the referees, whose patience, good humor, and professionalism, was a great help in putting up with what were frequently very close deadlines.

H. C. Williams
for the editors



DANIEL SHANKS

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