REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS

The numbers in brackets are assigned according to the American Mathematical Society classification scheme. The 1980 Mathematics Subject Classification (1985 Revision) can be found in the December index volumes of Mathematical Reviews.

1[65–01, 65N30].—R. E. WHITE, *An Introduction to the Finite Element Method with Application to Nonlinear Problems*, Wiley-Interscience, New York, 1985, x + 354 pp., $23\frac{1}{2}$ cm. Price \$39.50.

This text is intended for a semester course for graduate level engineers. It has three clear objectives, as stated in the Preface: (1) Giving the student the ability to modify existing finite element codes or to create new codes. (2) Giving the student some appreciation for the error estimates, and (3) Giving a summary and illustration of nonlinear algorithms. The author pursues these objectives without getting sidetracked, and he succeeds in reaching his first and third goal. (More about the second objective below.)

Apart from the introductory first chapter and a chapter on error estimates, the emphasis is on problems in two space dimensions. Applications, occupying Chapters 8 and 9, are beautifully selected. The reader of this review may judge for himself: Nonlinear heat conduction, Burgers' equation, incompressible viscous fluid flow, Stefan problem, obstacle problem for a string (for motivation), elliptic variational inequalities, fluid flow in a porous medium, and parabolic variational inequalities. Four appendices give computer programs for steady-state heat conduction, heat flow in a resistance transducer, solidification of water in a channel, and steady state flow in a porous medium.

Indeed, an interesting list!

The second objective stated above is to give the student some appreciation for error estimates. I proceed to elaborate on why I think the author has failed to do so. (I note in passing that in the exemplarily brief Introduction to this work, the second objective has disappeared. Only the first and third objectives from the Preface survive in explicitly or implicitly stated form.) To set the stage, I first describe some results from Chapter 5 concerned with error estimates for piecewise linear splines in a two-point boundary value problem. The main result, stated already in the first paragraph of the chapter and then in Theorem 5.6.1, says that the pointwise displacement error is of first order in the mesh-spacing parameter. On p. 138 a second-order displacement error estimate in L_2 is given, with the following remark: "Note that the norm $\|\cdot\|_{L_2}$ is weaker than absolute value, but the order is now 2 and not 1."

In my experience (I have taught similar courses), engineering students appreciate error estimates when they realize how valuable they are in practice, namely for testing that codes (including canned programs) are correct. Merely testing *convergence* on a model problem is not enough. *Rates of convergence* must be tested on smooth model problems, at least. If theory predicts fourth-order convergence and your program gives only first-order convergence, then there is an error in your program. (This is an example of what happened to one of my students; another student observed third-order rather than fourth. Both errors were rather subtle but could be found after further suitable experimentation. These students will, I hope, never use a program without testing it against theory on simple models.)

To repeatedly state, as this text does on three occasions, a first-order pointwise result for displacements is misleading. A student who does convergence rate tests on smooth problems and finds (correctly) second-order for piecewise linear approximations is likely to be confused.

(A second-order result for pointwise displacement error is stated, without proof, as Corollary 1 on p. 133. Apparently, in the five years that the author has taught this course prior to publishing the present text, no student got fascinated enough with error estimates to ask how this corollary would follow from Theorem 5.6.1. Well, it doesn't; completely different techniques are required to prove it.)

Another point where the theory as presented may mislead a beginner occurs in Chapter 6, devoted to time-dependent problems. Stability is discussed, mainly in the L_∞-norm, and then the Lax Equivalence Theorem is stated in its usual formulation, with the sole comment that it is an "important theorem". The student is likely to be confused when (if ever) he learns that, e.g., the widely used Lax-Wendroff scheme is unstable (in L_∞). At the introductory level, Lax's theorem is one of the most misunderstood results ever, and the present text continues the tradition.

In summary, why I think that the author has failed to impart any understanding of the role of error estimates: They are not put to any use, practical or otherwise. Engineering students are likely to be moderately enthusiastic about this approach.

It is traditional for a reviewer to present his minor differences of opinion with the author:

In the first and second sections of the first chapter, the author starts with a two-point boundary value problem and then considers an energy functional to be minimized. The same order of business occurs in the second section of the second chapter for a plane Poisson problem. Engineering students are likely to appreciate the reverse order: Minimizing the energy functional is the fundamental physical principle and the two-point boundary value problem or Poisson problem follows as a consequence. One may then point out that the finite element method is one step closer to the fundamental principle than, say, some finite difference method!

The remark on p. 251 on the Courant number condition (I prefer the term Courant-Friedrichs-Lewy number to honor all involved) ought to be amplified. Students immediately understand domain of dependence considerations.

In Section 8.4, (8.4.5), the viscous Navier-Stokes equations are presented with the wrong boundary conditions (inviscid). This is rectified six pages later (p. 261) with an explanation that I defy anyone to make sense of!

The method of lumping, described in Remark 1 on p. 265, should be given a reference, if not a brief explanation of why it works.

The relation between the approximate velocity space and the approximate pressure space alluded to on p. 267 also merits a reference. The explanation offered, namely "This is necessary because in (8.5.4)–(8.5.6) the approximation of the first derivatives of u and v and the approximation of P should have the same order", is nonsense. The situation is reasonably well understood by Numerical Analysts but often a mystery to engineering students.

The students may question the explanation on p. 276 of why test functions in the weak formulation of the enthalphy formulation of the Stefan problem suddenly need to depend on both space and time. After all, these test functions were time-independent in previous nonlinear parabolic problems!

Concluding this review, I congratulate Professor White on a fine text, written with clear perspectives which he sticks to throughout. Five years of classroom experience shows! The emphasis on interesting nonlinear problems in particular sets this book apart from the crowd of introductory texts on finite elements. Also, it is a handsome volume with typography that pleases the eye. I recommend it for its intended purpose without hesitation. If, in a next edition, the author elucidates the practical importance of knowing the correct rate of convergence, it may become the best introductory book on finite elements on the market for an engineering or physics audience.

And, for a second ending of my review: I had fun reading this book!

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2[65–01, 65N30].—R. WAIT & A. R. MITCHELL, *Finite Element Analysis and Applications*, Wiley, Chichester, 1985, xii + 260 pp., $23\frac{1}{2}$ cm. Price \$35.00.

This is a comprehensive text on finite elements. It is intended for "final year undergraduate or first year postgraduate students in mathematical sciences or engineering". Further, "no specialized mathematical knowledge beyond a familiarity with calculus and elementary differential equations is assumed".

I wish to add to that a general requirement of mathematical maturity: The Introduction breezes through function spaces, Hilbert spaces, linear operators, Riesz' representation theorem, Lax-Milgram's lemma, monotone operators, Sobolev spaces, trace theorems and other standard material in twelve pages. The second chapter covers extrema of integrals, Euler-Lagrange equations, constrained extrema, possibly with boundary conditions, Hamilton's principle, and dual variational principles in fourteen pages.

It is clear that the students referred to above are great students in Britain, not our typical students in a US university.

As already noted, the text is comprehensive, indeed almost encyclopedic. This leads to a lack of clear objectives (other than to "understand finite elements") that I suspect US undergraduate students or engineering graduate students will not be enthusiastic about.

The book succeeds in being comprehensive in 251 pages. It should serve well as a text for a graduate course in Mathematics or for self study for a mathematically mature person really interested in learning the subject. The style is readable.

The applications given in Chapter 7 are, to my mind, disappointing. Except for singular isoparametric elements in corner problems and first-order hyperbolic problems, the rest (seven) of the applications are in one space dimension. This hardly reflects the state of the art, either regarding applications or theory.

It is the privilege of a reviewer to quibble with minor details:

Exercise 1.15 (p. 20) is wrong as stated; rotation by 90° in the plane provides a counterexample.

On p. 23 the authors state: "It will be shown in later chapters that the most natural error bounds are defined in terms of Sobolev norms." Apart from the overall logic of this sentence, I quarrel with the word "natural". Most convenient error measure, yes; a lazy man's error measure, yes; but hardly the most natural error measure in general for a serious Numerical Analyst.

On p. 47 the complete quintic element is dismissed as being "of little practical use and will not be considered further". Fix and Strang in their 1973 book call the C^1 quintic "one of the most interesting and ingenious of all elements" (p. 82). Fashions change, or, should one take statements such as those seriously ...?

The corollary on p. 195, dealing with pointwise error estimates in piecewise linear finite elements for second-order elliptic problems, reproduces a well-known error for the exponent of the logarithmic factor. The examples of "sharpness" quoted are merely suggestive. A true example was constructed by Haverkamp in 1982.

The reference on p. 248 to Dupont's 1973 paper has nothing to do with "an alternative approach to the solution of hyperbolic equations". The article in question contains an extremely interesting counterexample.

In conclusion, this is a well-written and comprehensive text for a first course in finite elements for graduate students in Mathematics, or for self study for someone seriously interested in educating himself on the subject. It starts from scratch and quickly moves up to describe rather recent research.

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3[65–04].—WILLIAM H. PRESS, BRIAN P. FLANNERY, SAUL A. TEUKOLSKY & WILLIAM T. VETTERLING, *Numerical Recipes—The Art of Scientific Computing*, Cambridge Univ. Press, Cambridge, 1986, xx + 818 pp., 24 cm. Price $39.50.

The following quote from the Preface to *Numerical Recipes—The Art of Scientific Computing* (hereafter abbreviated *NR*) should help convey the spirit of the book: "... this book is indeed a 'cookbook' on numerical computation. However there is an important distinction between a cookbook and a restaurant menu. The latter presents choices among complete dishes in each of which the individual flavors are blended and disguised. The former—and this book—reveals the individual ingredients and explains how they are prepared and combined." To extend the analogy a bit, *NR* does not teach one to be a master chef, and it rarely recommends an occasional meal prepared by one.

The Preface claims the reader to need a "normal" undergraduate mathematics background and some computer programming experience, but no prior knowledge of numerical analysis. To indicate the scope of this book, a list of the chapter titles follows: 1. Preliminaries; 2. Solution of Linear Algebraic Equations; 3. Interpolation and Extrapolation; 4. Integration of Functions; 5. Evaluation of Functions; 6. Special Functions; 7. Random Numbers; 8. Sorting; 9. Root Finding and Nonlinear Sets of Equations; 10. Minimization or Maximization of Functions; 11. Eigensystems; 12. Fourier Transform Spectral Methods; 13. Statistical Description of Data; 14. Modeling of Data; 15. Integration of Ordinary Differential Equations; 16. Two Point Boundary Value Problems; 17. Partial Differenital Equations.

NR contains complete listings of approximately 200 Fortran subprograms implementing the numerical methods described in the text. There is a list of these subprograms by section after the table of contents and an alphabetized list at the end, with cross-references to other routines called. Pages 673–790 contain translations of those subprograms into Pascal. Machine-readable copies of these subprograms are available on DOS diskettes for the IBM PC and compatible computers at $19.95 per language. At first blush, this book appears to be merely a user's guide to the software, but this is not quite accurate. There are many instances in which the text contains valuable information on how and when to use the methods.

The scope of coverage is extremely ambitious. It is hard to know how to classify *NR*. This is not a mathematics book: It contains no proofs of the claimed properties of the methods under discussion. In its defense, however, *NR* does include numerous references to other literature where one may find out more about the methods. This is not a numerical analysis book: Although occasional lip service is paid to roundoff error and numerical stability, most of the programs are straightforward implementations of the formulas given in the text. That the authors are scientists or engineers (abbreviated s/e below), rather than mathematicians, is revealed by phrases like "down to the last possible epsilon of accuracy" (page 95), or "remains finite in some region where x is zero" (page 204).

Although *NR* exhibits moments of brilliance and includes a wealth of valuable information for the practicing s/e, I am afraid that a specialist in the area represented by almost every chapter will find something to find fault with. Several examples should help illustrate the unevenness in coverage in *NR*. In addition to the standard methods, Chapter 2 includes sections on solving Vandermonde and Toeplitz systems and sparse linear systems (one of the few times the reader is referred to existing quality software). It includes an algorithm for singular value decomposition, billed as "the method of choice for ... linear least squares problems", but not the less expensive QR factorization. (QR is mentioned as an alternative in Chapter 14, but there is no way the reader could figure out how to use it from the material in this book.) Although Chapter 4 includes a treatment of integrals with singularities not normally found at this level, there is no mention of adaptive quadrature, which is the basis of much current software in this area. Chapter 15 exhibits an obvious bias against predictor-corrector methods, ignoring the success of many respected software packages based on such methods and downplaying the shortcomings of Runge-Kutta methods. It is almost as though the authors are saying, "If you can't write it yourself, you shouldn't use it."

Some aspects of the programming style also make *NR* not a source of quality mathematical software. In most cases, encountering an erroneous input or reaching an unexpected case simply results in a program stop. Many of the routines depend heavily on saved internal variables, without even the benefit of an explicit SAVE declaration. Most routines use locally declared temporary storage, rather than input work arrays. (Notable exceptions are the routines in Chapter 8, which appear to have been taken largely from another source.) Further, the size of such arrays and the precision to which constants are given make it clear that the authors have in mind the solution of small problems on small machines. (However, the routines generally do not use double precision!) The authors would do well to read *Sources and Development of Mathematical Software*, Wayne R. Cowell, Ed., Prentice-Hall (Englewood Cliffs, NJ, 1984), and the references therein.

Despite comments to the contrary in the Preface, *NR* is not a textbook, for it contains no problems and very few examples. It is a reference book intended for the practicing s/e. In the Preface, the authors state: "Our purpose in this book is ... to open up a large number of computational black boxes to your scrutiny. We want to teach you to take apart these black boxes and to put them back together again, modifying to suit your specific needs." It is this aspect that makes this reviewer feel most uncomfortable. If one could be assured that the s/e would read all of the accompanying text, then in *most* cases he/she would be in a position to make intelligent use of these methods. Providing the software plus easy-to-modify example programs makes it easy to use them as "black boxes" (despite the authors' stated aversion to such) and/or transfer them to larger machines. However, *NR* contains virtually no information on how these methods will behave on large problems. *Caveat emptor*! Do not view *NR* as a bargain source of software.

There are a number of instances where one could take issue with the authors' choice of, or justification for methods. Unfortunately, however, there are also a number of instances of factual errors or misstatements. A list of them has been compiled by the reviewer and is available upon request.

This reviewer hereby acknowledges the assistance of his colleagues Mark Durst and Alan Hindmarsh in evaluating this book.

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4[65–04].—WILLIAM T. VETTERLING, SAUL A. TEUKOLSKY, WILLIAM H. PRESS & BRIAN P. FLANNERY, *Numerical Recipes Example Book* (*FORTRAN*), Cambridge Univ. Press, Cambridge, 1985, viii + 192 pp., $23\frac{1}{2}$ cm. Price $18.95. *Numerical Recipes Example Book* (*Pascal*), Cambridge Univ. Press, Cambridge, 1985, viii + 236 pp., $23\frac{1}{2}$ cm. Price $18.95.

As the title implies, *Numerical Recipes Example Book* is a set of examples to illustrate the use of the subprograms described in *Numerical Recipes* (hereafter abbreviated *NR*). The chapters are numbered and titled exactly the same as *NR*. Each chapter begins with a summary of the routines described in the corresponding chapter of *NR*. This is followed by a sequence of examples, in the same order as the routines in *NR* that they exercise. The nth example in chapter m has the name DmRn. An index shows which *NR* routines are demonstrated by which examples. There are two versions of the *Example Book*, one in Fortran and the other in Pascal. The text of the two appears to be identical, except that routine names are all capitals in the Fortran version and lower case in the Pascal version. The example programs listed in the Pascal version appear to be translations into Pascal of the Fortran examples.

As the title implies, the programs are intended more to serve as guides to usage than to be thorough test routines. It is a little annoying that in most cases in which comparison values are included in the program, they are merely printed for a visual check by the user. It would be better to have the *computer* do the comparison and print the difference or "ok". The book is also excessively repetitive. In those cases where essentially the same program is used to test several *NR* routines, each is still listed in complete detail.

The example programs are available on DOS diskettes for IBM PC and compatible computers at $19.95 each. It is this reviewer's opinion that the printed programs are worth very little without their machine-readable counterparts. Since the *Example Book* contains very little text besides the programs, the authors could have incorporated the text into the programs and saved the users $18.95!

We conclude by noting a few errors. In the text describing D3R12, the second derivative of the test function is $2x_1^2$, not $2x_1x_2$. (It is given correctly in the program itself.) The functions HEX2IN and IN2HEX, included in D7R13 to convert characters representing a hexidecimal number to/from its internal representation, are not quite as machine independent as the text claims. They assume that the characters '0' through '9' have internal representations that are consecutive integers; the same applies to 'A' through 'F'. The text before D12R1 is at least misleading. The program does not actually perform the four listed tests; it prints the results and expects the user to do a visual verification. The statement "if a data array is Fourier transformed twice in succession, the resulting array should be identical to the original" is false: the second transform must be the *inverse*, and one needs to include the factor $1/N$ that appears in (12.1.9) of *NR*, as in the program.

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5[65A05].—A. P. PRUDNIKOV, YU. A. BRYČKOV & O. I. MARIČEV, *Integrals and Series (Supplementary Chapters)* (in Russian), "Nauka", Moscow, 1986, 800 pp., 22 cm. Price 4 Rubles, 50 Kopecks.

Within the remarkably short period of five years, the authors have succeeded in the formidable task of preparing and presenting to the scientific community three volumes of integrals and series, each of about 800 pages. The volumes of integrals and series of elementary functions [1], reviewed in [2], and of integrals and series of special functions [3], reviewed in [4], have in the meantime been published in English [5], [6], and some errors or misprints have been corrected in [5]. The present table is the last volume of this collection. It consists of exactly 800 pages of formulas for integrals, sums, infinite series and functional relations of (mainly) "higher" special functions.

Perhaps even more than in [1] and [3], the formulas in this volume are characterized by their unusual complexity, often involving several functions of different type, and by the number of their parameters: in other words, by their generality. As is mentioned in the abstract of the table, many of these formulas are new, having been developed by the authors themselves and are here presented for the first time.

The main part of this volume is divided into eight chapters, with each chapter divided into many sections and subsections. As in [1] and [3], the notation is standard, and a knowledge of Russian (required only for the few short sections of text) is not essential. Chapter 1 (29 pages) deals with indefinite integrals, including integrands involving polylogarithms, generalized Fresnel integrals, Struve, Anger, Weber, Lommel, Kelvin and Airy functions, integral Bessel functions Ji_ν etc., elliptic integrals, Legendre functions, Whittaker functions, confluent, Gaussian, and generalized hypergeometric functions, Meijer's G-, MacRobert's E-, and Fox's H-functions, and the elliptic functions of Jacobi and Weierstrass. The long Chapter 2 (317 pages) consists of definite integrals. It is divided into sections for integrands containing the gamma function, the generalized zeta function, Bernoulli and Euler polynomials, polylogarithms, generalized Fresnel integrals, Struve, Anger, Weber, Lommel, Kelvin, Airy, integral Bessel, Laguerre and Bateman functions, elliptic integrals, Legendre functions, Whittaker functions, confluent, Gaussian, and generalized hypergeometric functions with one or more variables, G-, E-, and H-functions, theta functions and Mathieu functions. Many of the integrands contain combinations of these functions or combinations with other functions, e.g., elementary functions or Bessel functions.

The short Chapter 3 (7 pages) contains formulas for the Laplace transform of step functions or other piecewise continuous functions. Chapter 4 (6 pages) is composed of double integrals of Struve, Kelvin, Anger, Lommel functions, of confluent, Gaussian and generalized hypergeometric functions, with elementary functions as factors. It also contains multiple integrals of generalized hypergeometric functions and integrals on a sphere. Chapter 5 (13 pages) deals with finite sums of Bernoulli and Euler polynomials and numbers, Legendre functions, generalized hypergeometric functions and G-functions. Chapter 6 (34 pages) presents infinite series with terms composed of the generalized zeta function, Bernoulli and Euler polynomials and numbers, Legendre functions, confluent, Gaussian and generalized hypergeometric functions, E-, and G-functions. As in the case of definite integrals, many formulas contain several types of functions, including elementary functions.

Chapter 7 (186 pages) presents in a concise form properties, representations and—probably for the first time so extensively—tables of special cases for several kinds of hypergeometric functions. These tables have about 2800 entries for ${}_pF_q(a_1, \ldots, a_p; b_1, \ldots, b_q; z)$, where $p = q + 1$ $(q = 0, 1, 2, \ldots, 8)$, $p = 0$ $(q = 1, 2, 3)$, $p = q$ $(q = 1, 2)$, $p = q - 1$ $(q = 2, 3)$. The arguments a_i and b_j are special expressions or rational numbers. The results are sometimes given as functions of z, and sometimes for special values of this variable, e.g., $z = 1, -1, \frac{1}{2}$, and others. There are also formulas for general p and q.

The last chapter, Chapter 8 (117 pages), consists of two main sections. The first section (14 pages) consists of a concise description of the properties of Meijer's G- and of Fox's H-function. The much larger second section (103 pages) consists of tables of functions whose Mellin transform is (essentially) of the form

$$\prod_{j,k,l,m} \frac{\Gamma(a_j + A_j s)}{\Gamma(c_l + C_l s)} \frac{\Gamma(b_k - B_k s)}{\Gamma(d_m - D_m s)}$$

where $A_j, B_k, C_l, D_m > 0$. This table is characterized by the fact that it gives not only the image function of a given original function, but also expresses the latter as a special case of the G- or (in some cases) H-function. It can therefore also be used to find expressions which represent certain special cases of G or H. A similar table without the representation in terms of G or H, has been issued previously as part of a book [7] (under a somewhat misleading title) by one of the authors of the present table.

There are two appendices. Appendix I discusses general properties of integrals, series, products, and operations upon them, in particular, convergence criteria and the manipulation of formal power series. Appendix II lists definitions and properties of some special functions. A small dictionary of notation completes the table.

There is a bibliography of 65 items, but there are no references given with the formulas.

The printing and binding of this volume are good. The professional skill of the typographers deserves special mention: The formulas are set out with great care. As is the case with the two previous volumes, this table is an important reference book for mathematicians, physicists, theoretically interested engineers, and others working in fields where such formulas are likely to occur. Especially in view of its modest price, this table, as well as those for the elementary and special functions, ought to be available in the libraries of all relevant institutes and in the private libraries of those working with such formulas. Unfortunately, the number of copies printed (20 thousand) is even smaller than for the two other (Russian) tables. It is likely that there will be an English edition in due course.

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1. A. P. PRUDNIKOV, YU. A. BRYČKOV & O. I. MARIČEV, *Integrals and Series of Elementary Functions*, "Nauka", Moscow, 1981. (Russian)

2. Y. L. LUKE, Review **4**, *Math. Comp.*, v. 40, 1983, pp. 413–414.

3. A. P. PRUDNIKOV, YU. A. BRYČKOV & O. I. MARIČEV, *Integrals and Series of Special Functions*, "Nauka", Moscow, 1983. (Russian)

4. K. S. KÖLBIG, Review **2**, *Math. Comp.*, v. 44, 1985, pp. 573–574.

5. A. P. PRUDNIKOV, YU. A. BRYCHKOV & O. I. MARICHEV, *Integrals and Series*, v. 1: *Elementary Functions*, Gordon and Breach, New York, 1986.

6. A. P. PRUDNIKOV, YU. A. BRYCHKOV & O. I. MARICHEV, *Integrals and Series*, v. 2: *Special Functions*, Gordon and Breach, New York, 1986.

7. O. I. MARICHEV, *Handbook of Integral Transforms of Higher Transcendental Functions*, Ellis Horwood, Chichester, 1982.

6[62Hxx].—FIONN MURTAGH & ANDRÉ HECK, *Multivariate Data Analysis*, Astrophysics and Space Science Library, vol. 131, D. Reidel Publishing Co., Dordrecht, 1987, xvi + 210 pp., $24\frac{1}{2}$ cm. Price \$49.50/Dfl. 120.00.

This book gives a basic introduction to selected methods of multivariate statistical analysis. It is aimed at students and researchers in the astrophysical sciences, and its main strength is an extensive, carefully annotated bibliography of research papers in astronomy where multivariate methods have been applied. Because of its specialized audience and narrow coverage, the book is rather unlikely to appeal to statisticians or numerical analysts.

The topics covered include principal component analysis, cluster analysis and discriminant analysis. Some other techniques are briefly discussed. Most chapters are supplemented by illustrative examples and by listings of FORTRAN programs. Since some of the listings are fairly long and difficult to copy without error, the reviewer would have preferred appropriate references to subroutine libraries like NAG, IMSL and EISPACK.

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7[11F11, 11Y60, 33A25, 33A70, 65D15].—JONATHAN M. BORWEIN & PETER B. BORWEIN, *Pi and the AGM—A Study of Analytic Number Theory and Computational Complexity*, Canadian Mathematical Society Series of Monographs and Advanced Texts, Wiley, New York, 1987, xv + 414 pp., 24 cm. Price \$49.95.

When the reviewer was a teenager, he and three friends, after a high school basketball game, would frequently get in a car and drive over the labyrinth of country roads in the rural area in which they lived. The game we played was to guess the name of the first village we would enter. Since there were many meandering roads and countless small hamlets that dotted the rural landscape, since it was dark, and since we were not blessed with keen senses of direction, we were often surprised when the signpost identified for us the town that we were entering.

For one not too familiar with the seemingly disparate topics examined by the Borweins in their book, one might surmise that the authors were travelling along mathematical byways with the same naivete and lack of direction as the reviewer and his friends. However, the authors travel along well-lit roads that are marked by the road signs of elegance and usefulness and that lead to beautiful results. They do not take gravel-surfaced roads that lead to dead ends in cow pastures. But sometimes the destinations are surprising—at least to those not familiar with the landscape.

Instead of beginning at the high school gymnasium, the authors begin with the arithmetic-geometric mean and the contributions to it by Gauss. This leads to elliptic integrals, especially the complete elliptic integrals of the first and second kinds. Now we arrive at one of the main destinations, the calculation of π by the employment of the arithmetic-geometric mean and elliptic integrals. The authors return to the principal city of π several times, but they next take the road leading from elliptic integrals to their inversion and the big city of elliptic functions. The city's leading citizens are the theta functions, and the complete elliptic integral of the first kind makes another appearance when it is evaluated in terms of theta functions. The prominent theta functions then lead us to "squares." But we are not talking about unattractive lower class citizens, we have in mind beautiful formulas for the number of representations of positive integers as sums of certain numbers of squares. Another roadway from elliptic functions leads to the subject of partitions. In particular, Bressoud's elegant proof of the Rogers-Ramanujan identities is detailed.

We return now to elliptic integrals and their transformations, for which that of Landen is perhaps the paradigm. It is then a short journey to modular equations. The authors' discussion of modular equations is very enlightening and a focal point of the book. They begin with an algebraic approach before letting the principal townspeople, the theta functions, demonstrate their superiority. Modular equations then lead us again to algorithms for the calculation of π. These algorithms provide improvements in the rapidity of convergence.

A main interstate highway leads from elliptic functions to modular forms. The discussion of modular forms is very brief, but the absolute modular invariant is introduced, and we are led again to modular equations.

The authors next return to approximations to π. In fact, the subsequent work is more closely related to modular forms than is indicated by the authors. The approximations of π in question are due to Ramanujan and arise from formulas for Eisenstein series (not identified as such by the authors), which are modular forms. However, rightly so, the requisite formulas and the subsequent approximations to π are derived within the context of modular equations. These ideas naturally lead to work of Ramanujan and G. N. Watson on singular moduli. The authors are to be commended for their careful presentation of much of the content of Ramanujan's famous paper, "Modular Equations and Approximations to π". This material has not heretofore appeared in book form. However, more importantly, Ramanujan provided no proofs for many of the claims that he made, and so the authors provided many of the missing details.

We now return to the town from which we started our journey. Various variants and generalizations of the arithmetic-geometric mean are presented. If the arithmetic-geometric mean were useful for the computation of only π, its reputation would dwindle. The authors show how it can be used to rapidly calculate elementary functions. Other methods to approximate elementary functions are also explored. There is also a lengthy discussion of computational complexity and the fast Fourier transform.

At the end of their journey, the authors return to π. In an unusual feature for a book of this type, they provide an interesting historical account of attempts to calculate π. Then they conclude the tour with a discussion of the transcendence of π and irrationality measures.

As we have indicated, our visits with the theta functions were the highlights of this picturesque excursion. Frobenius echoed the thoughts of many mathematicians when he declared that

> "In der Theorie der Thetafunctionen ist es leicht, eine beliebig grosse Menge von Relationen aufzustellen, aber die Schwierigkeit beginnt da, wo es sich darum handelt, aus diesem Labyrinth von Formeln einen Ausweg zu finden."

The Borweins, indeed, have helped us to find the right roads.

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8[11A51, 68P25, 11K45].—EVANGELOS KRANAKIS, *Primality and Cryptography*, Wiley Teubner Series in Computer Science, Wiley, Chichester, 1986, xv + 235 pp., $23\frac{1}{2}$ cm. Price \$41.95.

Some number theorists consider number theory as the exclusive domain of a select group of scholars, and they regard the appearance of strangers as an unwelcome intrusion. These strangers, they say, have a tendency to walk outside the marked pathways, to give different names to the roses that they encounter, and to introduce a lot of noise that disturbs the pure and quiet atmosphere; moreover, if they are caught in one of the many traps that the Queen of Mathematics has set, they do not seem to notice.

Other number theorists acclaim the arrival of modern times in their underpopulated area. They welcome visitors from outside as bringing in fresh air and financial resources. They had always believed that oil could be found in their lot of land, and they are happy to make their visitors believe this as well.

The present tour bus visiting the community is not likely to cause major excitement. It contains a collection of quiet *theoretical computer scientists* that came to see the *primality testing* plant, and among themselves they carry on a cultured conversation on abstract properties of *pseudorandom generators* and *public key cryptosystems.* They appear to be more profoundly interested in primality testing than is justified by the application that they have in mind. Should they not spend some time at the *factorization* facility as well? Their experienced driver, who was in the area before, determined otherwise. But he has an enthusiastic and original way of explaining to his passengers what they do see, and it is sure that at the end of the trip they will be better and wiser computer scientists.

Unfortunately, the zealous driver does in a technical sense not obey the rules as carefully as is traditional in this region. The eulogy on *mathematical rigor* that he recited at the border inspired confidence. But then, the very first exercise: *Let G be a finite abelian group. Show that all equations of the form $x^2 = a$, where $a \in G$, have exactly the same number of solutions in G.* If this is one of numerous typographical sins, here is another exercise (Section 2.10): *Show that every finite abelian group* *can be embedded into the multiplicative group* $\mathbf{C}^*$ *of complex numbers*. And why simplify the true state of affairs and assert that *any* odd prime has a representation as a sum of two squares (Theorem 2.7)? Why, conversely, make something simple as *Pratt's test* (Section 2.6) so complicated that it actually becomes wrong? One can only admire the originality of the mistakes that are made.

In conclusion, this tour of *Primality and Cryptography* should not be taken by number theorists that wish to be informed about the many connections that exist between number theory and cryptography; the primality excursion is somewhat *adventurous*; and what is said about cryptography is not likely to be of interest outside the theoretical computer science community. But who knows, one day it may become just as useful as number theory itself.

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9[68-01, 68Q25].—LYDIA KRONSJÖ, *Algorithms: Their Complexity and Efficiency*, 2nd ed., Wiley, Chichester, 1987, xiii + 363 pp., $23\frac{1}{2}$ cm. Price $49.95.

The first edition of this book was reviewed in [1]. At that time, the book provided a welcome contrast to other books on algorithms by concentrating on the analysis of basic techniques used in Scientific Computing. The majority of these texts still remains centered around problems from Graph Theory, Combinatorics, Operations Research, and Logic. So it is nice that this different approach continues to be a viable alternative.

The overall structure of the book remains the same: about two thirds devoted to numerical techniques, and one third to sorting and searching. The apparent deficiency of not addressing NP-completeness has not been remedied by incorporating this subject into the text. Rather, the author opted to write a companion book devoted to treating NP-completeness in detail.

On the whole, this is a book one should consider using in a seminar on a modern approach to numerical analysis or on a more diversified view of algorithms.

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1. C. M. HOFFMANN, Review **5**, *Math. Comp.*, v. 38, 1982, pp. 651–652.

10[65-06].—EDUARDO L. ORTIZ (Editor), *Numerical Approximation of Partial Differential Equations*, North-Holland Mathematics Studies, vol. 133, North-Holland, Amsterdam, 1987, xii + 433 pp., 24 cm. Price $77.75/Dfl. 175.00.

This volume contains selected papers from the International Symposium on Numerical Analysis held at the Polytechnic University of Madrid on September 17–19,

1985. In addition to 9 invited papers there are 28 contributed papers organized in five parts: Results on Computational Linear Algebra; Discrete Variable Methods; Polynomial and Rational Approximation Methods; Variational Methods and Special Techniques; Applications. The titles and authors of the invited papers are: "Recent progress in the two-dimensional approximation of three-dimensional plate models in nonlinear elasticity" by Philippe G. Ciarlet; "Formulation of alternating-direction iterative methods for mixed methods in three space" by Jim Douglas, Jr., Ricardo Durán and Paola Pietra; "Iterative methods for singular systems" by Ivo Marek; "On different numerical methods to solve singular boundary problems" by Francisco Michavila; "Some numerical techniques for the solution of problems related to semiconductor devices" by John J. H. Miller; "Recent progress in the numerical treatment of singular problems for partial differential equations with techniques based on the tau method" by Eduardo L. Ortiz; "Present state and new trends in parallel computation" by Rafael Portaencasa and Carlos Vega; "Finite element methods for treating problems involving singularities, with applications to linear elastic fracture" by J. R. Whiteman; "Finite element solution of the fundamental equations of semiconductor devices" by Miloš Zlámal.

W. G.

11[41–02].—C. K. CHUI, L. L. SCHUMAKER & J. D. WARD (Editors), *Approximation Theory V*, Academic Press, Orlando, Fla., 1986, xviii + 654 pp., $23\frac{1}{2}$ cm. Price $65.00.

These are the proceedings of the Fifth International Symposium on Approximation Theory held at Texas A&M University in College Station, Texas, January 13–17, 1986. They contain nine survey papers (229 pages) and 98 short research papers (390 pages). The titles of the survey papers, and their authors, are: "Positive quadrature methods and positive polynomial sums" by Richard Askey; "Bases in function spaces" by Z. Ciesielski; "Some recent convergence results on diagonal Padé approximants" by A. A. Gonchar; "Box splines" by Klaus Höllig; "Polynomial approximation numbers, capacities and extended Green functions for $\mathbf{C}$ and $\mathbf{C}^N$" by J. Korevaar; "Group theoretical methods in approximation theory, elementary number theory, and computational signal geometry" by Walter Schempp; "Some recent results on Walsh theory of equiconvergence" by A. Sharma; "Scientific computation on some mathematical conjectures" by Richard S. Varga; "Some constrained approximation problems" by Joseph D. Ward. The volume concludes with an update of some 450 additional items to the bibliography on Bernstein type operators [1] published in the proceedings of the previous conference.

W. G.

1. HEINZ H. GONSKA & JUTTA MEIER, "A bibliography on approximation of functions by Bernstein type operators (1955–1982)," *Approximation Theory IV* (C. K. Chui, L. L. Schumaker & J. D. Ward, Editors), Academic Press, New York, 1983, pp. 739–785.

12[65D30, 65D32].—M. MORI & R. PIESSENS (Editors), *Numerical Quadrature*, North-Holland, Amsterdam, 1987, 236 pp., 27 cm. Price $55.50.

This is a reissue in book form of *Journal of Computational and Applied Mathematics*, v. 17, 1987, nos. 1 & 2. All papers are devoted to numerical integration, two thirds to univariate, the remaining third to multivariate integration.

W. G.