## REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS

The numbers in brackets are assigned according to the American Mathematical Society classification scheme. The 1980 Mathematics Subject Classification (1985 Revision) can be found in the December index volumes of Mathematical Reviews.

**13[65N30].**—J. TINSLEY ODEN & GRAHAM F. CAREY, *Finite Elements: Special Problems in Solid Mechanics*, Vol. 5, Prentice-Hall, Englewood Cliffs, N. J., 1984, xi+273 pp., $23\frac{1}{2}$ cm. Price $38.95.

This book contains five chapters written by specialists in finite element methods applied to problems arising in solid mechanics.

The first one is by M. Bernadou, and gives a detailed study of the numerical analysis of thin shell problems. After a brief review of thin shell theories—in both linear and geometrically nonlinear frameworks—the author presents a conforming finite element method with a rigorous error analysis. Several elements are discussed, and the appropriate choice for numerical integration is pointed out. Finally, an application to an arch dam is presented.

The second chapter concerns finite element methods in nonlinear incompressible elasticity and was written by R. Glowinski and P. Le Tallec. There are five sections. The first two are devoted to the formulation of the problem and a description of the notation that is used. The modeling chosen for the numerical solution in the third section includes two unknown fields: the displacement and the pressure (in Lagrange coordinates). The discrete compatibility condition, which *should be* satisfied by the approximate spaces, is pointed out. Then, several examples are exhibited. As a matter of fact, they are the same as those used in fluid mechanics (pressure-velocity formulation). The effective solution of the nonlinear discrete problem via a Newton algorithm is discussed in the fifth section. Finally, an augmented Lagrangian method, based on a three fields approach (displacement, pressure and change of volume ratio), is suggested. This is a tricky procedure which permits one to "localize" the nonlinear constitutive relation for discrete fields. Finally, attractive numerical illustrations are given in the sixth section.

Finite element methods applied to the mechanical study of plastic buckling is the topic of the fourth chapter, which was written by A. Needleman and V. Ivergaard. As a matter of fact, finite element methods are used as a tool, and numerical analysis is very limited in this chapter, in contrast with the other chapters; but a very interesting mechanical discussion, including mechanics of materials, is presented. The numerical tests are very new and presented in an attractive way. In comparison with the other chapters, this chapter contains a lot of numerical information connected with a difficult and important problem.

Chapter 4 by N. Kikuchi and J. T. Oden is devoted to classical contact problems in elastostatics. After a brief recall of Signorini's model, the authors introduce simplifications in order to avoid difficulties connected with the existence of a solution. A finite element method is presented and analyzed for these approximate models. The numerical solution, based on a penalty technique, is also discussed. Finally, a computational test (cylindrical punch on a body) is given.

A new friction law is introduced in Chapter 5. It is due to J. T. Oden and E. B. Pires, who are the authors of this last chapter. In order to avoid difficulties connected with existence of a solution in Signorini's model, a nonlocal law—which can be understood as a regularization of Coulomb's law—is suggested. The local value of the normal stress is approximated by "an average" around the concerned point on the boundary. The variational formulation leads to an existence and uniqueness result for this new model. The final section is devoted to a finite element approximation of the model, for which the same numerical test, as in Chapter 4, is checked.

This book appears to be interesting for a reader who is concerned with mathematical aspects of finite element methods applied to some problems arising in mechanics. Furthermore, its presentation is very good and a homogeneity between the different chapters has successfully been obtained by the editors.

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**14[76S05, 76T05, 65N30].**—GUY CHAVENT & JEROME JAFFRE, *Mathematical Models and Finite Elements for Reservoir Simulation—Single Phase, Multiphase and Multicomponent Flows Through Porous Media*, Studies in Mathematics and its Applications, vol. 17, North-Holland, Amsterdam, 1986, xi+376 pp., 23 cm. Price $80.00/Dfl. 200.00.

The aim of the book is threefold. First, there is a presentation of several mathematical models related to the reservoir simulation problem. Being unable myself to judge the accuracy and the practical relevance of each model, I can nevertheless say that the exposition is simple, very clear, and accessible, even to people with a rather weak physical background. The various models are presented in a synthetic way, using the new feature of the "global pressure". Clear hints on various practical situations in which one or another model comes into the game gives one the feeling of being "in contact with the real world".

A second scope of the book is to provide a rigorous mathematical study of some of the simpler problems. In reading this part, I was rather happy to have spent less time in my life on the study of physics and more on the study of functional analysis. The treatment is indeed very well done, clean, precise and reasonably understandable, provided one has some background in functional analysis.

A third aim of the book is to present some of the new finite element techniques in order to deal with some of these problems. The range of methods that are analyzed is not very wide, but at least the choice falls on methods and techniques which are very recent and effective.

Therefore, in a sense, all three aims of the book have been successfully achieved. Although the field itself is, I think, much wider than what is covered in the 376 pages of this book, I believe that the book can be very useful, both to experts and to beginners in the field.

The contents of the book are organized as follows: Chapter I: Basic laws and models for flow in porous media (Generalities, The geometry of the field, The basic laws for one- and two-phase flow, The basic models, Qualitative behavior of the solution in the no-diffusion and no-capillary pressure case). Chapter II: Slightly compressible monophasic fields (Construction of the pressure equation, Existence and uniqueness theorems, An alternative model of monophasic wells). Chapter III: Incompressible two-phase reservoirs (Introduction, Construction of the state equations, Summary of equations of two-phase flows for incompressible fluids and rock, An alternative model for diphasic wells, Mathematical study of the incompressible two-phase flow problems, The case of fields with different rock types). Chapter IV: Generalization to compressible, three-phase, black oil or compositional models (The two-phase compressible model, The three-phase compressible model, The black oil model, A compositional model). Chapter V: A finite element method for incompressible two-phase flow (Introduction, Approximation of the pressure-velocity equations, Resolution of the algebraic system for pressure-velocity, Approximation of the one-dimensional saturation equation: the case with neither capillary pressure nor gravity, Approximation of the one-dimensional saturation equation in the general case, Approximation of the saturation equation in two dimensions, Notes and remarks).

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**15[65L60, 65M60, 65R20].**—H.-J. REINHARDT, *Analysis of Approximation Methods for Differential and Integral Equations*, Applied Mathematical Sciences, Vol. 57, Springer-Verlag, New York, 1985, xi+390 pp., $23\frac{1}{2}$ cm. Price $45.00.

This is a major contribution to the literature on the approximate solution of differential and integral equations. Most of the material comes from the research of the author and colleagues during recent years. A unified theory yields general convergence results and error estimates for approximate solutions of linear and nonlinear problems. The theory is applied to finite difference approximations for initial and boundary value problems, projection methods for differential and integral equations, and quadrature methods for integral equations.

The book is divided into four parts: numerical methods and examples, general convergence theory; applications to boundary value problems and integral equations; inverse stability, consistency and convergence for initial value problems.

The general theory relates solutions of equations

$$Au = w, \quad A_n u_n = w_n, \qquad n = 1, 2, \ldots,$$

where $A$ and $A_n$ are maps between normed linear spaces. Thus,

$$A\colon E \to F, \qquad A_n\colon E_n \to F_n.$$

These spaces are connected by means of abstract restriction maps:

$$R_n^E\colon E \to E_n, \qquad R_n^F\colon F \to F_n,$$

which, for example, could be ordinary restrictions or projections onto subspaces.

Solutions of $Au = w$ and $A_n u_n = w_n$ are related by means of discrete convergence. This general concept was formulated primarily by F. Stummel, and developed further by R. D. Grigorieff and the author, H.-J. Reinhardt. Discrete convergence is a map, denoted by lim, from a set of sequences $u_n \in E_n$ to elements $u \in E$. It satisfies

$$\lim u_n = u, \qquad \|u_n - v_n\|_n \to 0 \Leftrightarrow \lim u_n = \lim v_n.$$

For example,

$$\lim u_n = u \Leftrightarrow \|u_n - R_n^E u\|_n \to 0.$$

Particular cases of discrete convergence are provided by continuous functions, $L^p$ spaces, and weak convergence of measures.

Discrete convergence of mappings, $A_n \to A$, is defined by

$$u_n \to u \Rightarrow A_n u_n \to Au.$$

This is equivalent to stability and consistency. Discrete convergence $A_n \to A$ is used to obtain the convergence $u_n \to u$ of solutions of equations $A_n u_n = w_n$ and $Au = w$. The maps $A$ and $A_n$ are assumed to be equidifferentiable, or have discrete compactness properties, or have approximation regularity properties.

Applications include difference methods for boundary value problems via maximum principles or variational principles. Inverse stability, consistency, and convergence are obtained for initial value problems, using difference approximations or Galerkin methods.

This monograph presents an impressive array of theoretical results and a wealth of significant examples.

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**16[34–02, 35–02, 47G05, 65N30].**—JOHANNES ELSCHNER, *Singular Ordinary Differential Operators and Pseudodifferential Equations*, Lecture Notes in Math., vol. 1128, Springer-Verlag, Berlin, 1985, 200 pp., 24 cm. Price $14.40.

A differential operator is called degenerate if its leading coefficient vanishes at some point. The first three chapters of the book deal with various properties, such as normal solvability, Fredholm property, and index, of ordinary degenerate differential operators in spaces of type $C^\infty$, $L_p$, Sobolev (also weighted) and distributions. In Chapter 4 the results are extended to degenerate partial differential operators. Chapter 5 gives a very nice introduction (based on Fourier series) into the classical theory of pseudodifferential operators on closed curves, concentrating again on the degenerate case and in particular on the degenerate oblique derivative problem. Chapter 6 extends the well-known convergence analysis of the finite element method to the operators considered in Chapter 5.

In spite of its heavy mathematical content, the book is extremely readable, at least for readers who are familiar with the work of authors such as Triebel, Hörmander and Lions-Magenes. It should be useful to mathematicians who need a thorough treatment of the operator theory of degenerate operators, as well as to numerical analysts interested in numerical methods for degenerate operators.

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**17[42–01, 42A16, 65T05].**—VÁCLAV ČÍŽEK, *Discrete Fourier Transforms and Their Applications*, Adam Hilger, Bristol, Boston, 1986, 141 pp., 24 cm. Price $28.00.

*Preliminary Remarks.* The advent of high-speed electronic computers and fast analog-to-digital converters have created not only an increased need for familiarity with Fourier methods but, just as important, it has shifted emphasis to different parts of Fourier theory.

To Fourier himself and many generations of mathematicians and engineers, Fourier analysis meant the use of an expansion of a piecewise continuous function on a finite interval into a series of discrete sines and cosines or, equivalently, complex exponentials, where, hopefully, the series converged rapidly. An important generalization of this has an integral instead of a series. Mathematicians have put the theory on a firm footing and practitioners have become skilled in the use of Fourier methods in analyzing and solving equations of electrical circuits, mechanical systems and analog devices of all kinds.

Owing to the great success of analytic methods and the great labor involved in the numerical application of Fourier methods, there was little emphasis on the latter until the advent of electronic computers and fast new algorithms. Since then, there has been a rapid shift from analytic to numerical methods.

While the object of interest, in analytic methods, is a piecewise continuous function having properties which ensure the convergence of the series or the existence of the integral, the computer must work with sequences of discrete sample values of the function. Consequently, the digital process must work with a Fourier transform which maps a discrete sequence into another discrete sequence. This transform has been called a "discrete" transform or a "finite" Fourier transform. In a sense these are equivalent since finiteness in one domain means discreteness in the other domain.

The theory of the finite Fourier transform has theorems which are counterparts of almost all of the theorems in the Fourier theory of integral transforms. Of course, continuity, convergence, and analyticity, are absent in the discrete Fourier transform. However, beyond that, the finite transform has theorems and properties which are strictly dependent on discreteness. Thus, theorems of number theory and group theory become involved. The notable consequences of these are the fast Fourier transform algorithm, the prime factor algorithms, and many other algorithms which are used in numerical processes.

The present book was originally published in the Czech language as a textbook for graduate students and practicing engineers. It treats the very important transition between the older analytic methods to the new digital processes and describes the theorems, properties and algorithms with a rather uneven degree of success.

1. *Introduction.* The first problem in the book is in the first line of the introduction, which says "Discrete Fourier transforms, better known to the general public as fast Fourier transforms, represent one of the computational methods,.... Discrete Fourier transforms as a means of computing Fourier transforms...". To most of us, the discrete Fourier transform (DFT) is a mathematically defined object: the discrete sequence of coefficients of sines and cosines or complex exponentials. (A third form, also defined in the book is in terms of amplitude and phase.) The name "DFT" says nothing about an algorithm for computing it. On the other hand, common usage is to drop the word "algorithm" from the name "fast Fourier transform" (FFT) when we mean a particular class of algorithms for computing the DFT. This as well as the problems of translation of the book may have contributed to the confusion of terms.

2. *Fourier Series.* The start of the second chapter states "Discrete Fourier transforms are related to Fourier series and Fourier transforms." This is still a little confusing but is closer to the truth. The remainder of Chapter 2 is a good description of Fourier series, convergence, Gibbs phenomena, and the minimum mean square error property of the truncated Fourier series. One section heading says "Fourier series of distribution" but the section says very little about the theory of distributions. The only distribution it treats is the delta function which is given in an acceptable fashion, so the only complaint is about the section heading. Fourier integral transforms and their properties are then described.

3. *Practical Methods of Computing Fourier Transforms.* This chapter starts by describing the calculation of Fourier integral transforms. It is hard to see why the author started with the integral transform. It would seem more logical to start with the finite integral for the coefficients of the Fourier series. In any case, he suggests a kind of semianalytic method for approximating the integrals of the tails. Then, for the integral over a finite domain, he describes various methods, such as integrating by parts, in order to express the integral in terms of derivatives, and later, differences. Examples are given for which this is a good method. Of course, where there are discontinuities, one will get better results by doing this, but, where the function is continuous, it gains nothing. Furthermore, this puts the frequency variable in the denominator which, for low frequencies will amplify errors. There is not enough emphasis on the usual or common situations and too much on special methods.

4. *Discrete Fourier Transforms.* The first sentence says "Discrete Fourier transforms are transforms of finite sequences of complex or real numbers." It may be mentioned here that some prefer the terminology "finite Fourier transform" since finite in one domain implies discrete in the other. On the other hand, the use of "DFT" may be better since the abbreviation of "finite Fourier transform" is already in use.

Chapter 4 proceeds with a very good and complete description of properties of the finite (discrete) Fourier transform, including the inversion theorem, relations involving symmetry, the effects of shifting, stretching, padding with zeros, repeating a sequence within a period, and of course, the most important theorem of all, the convolution theorem. This chapter covers an important part of Fourier theory which, as mentioned before, was much neglected before computers and the FFT algorithm.

5. *Other Properties of Discrete Fourier Transforms and Their Use in Computing Fourier Transforms.* Here, the book describes the next aspect of Fourier theory which requires more emphasis in numerical Fourier theory. When programmers were faced with the task of computing Fourier integrals, they often evaluated the integral with Simpson's rule or Newton-Cotes formulas, using the error estimates of those formulas. Chapter 5 gives a proper treatment which shows that sampling a function produces aliasing and that errors in Fourier integrals should really be expressed in terms of this aliasing. This not only yields more accurate error estimates but, to the engineer, it is more intuitive and suggestive of ways to reduce or avoid the errors.

The only problem in this chapter is the one cited earlier with respect to Chapter 3 where the author suggests converting the integral to a sum of differences. The implementation is described here with no word of caution about the frequency variable in the denominator.

6. *Methods of Computation of Discrete Fourier Transforms.* In a brief survey in the introduction to this chapter, a number of methods are mentioned without relations to each other or chronology. For example, the Goertzel algorithm is mentioned after the FFT and others. Actually it came long before. However, a good description of this important algorithm is given later in the chapter.

The FFT algorithms for the radix 2 are described very well with a good and liberal use of the signal flow graphs which have always been popular with engineers. This is followed by the mixed radix algorithms with an effective presentation and good flow graphs. Results of error analysis are stated concisely as they should be in a short book such as this.

A section titled "Winograd algorithm" gives a very brief summary of the prime factor algorithms and the Winograd transform. However, the distinction is not quite made clear. It is unfortunate that the author did not continue the type and quality of presentation of the previous section. Even engineering students who do not plan to program FFT's would find prime factor and Winograd algorithms interesting. Furthermore, there will be occasions when they will want to know what considerations go into making good choices of algorithms and subroutines.

The basic FFT algorithm is for complex to complex transforms. For data having special symmetries, such as being real, conjugate even, symmetric, etc., special algorithms are given. There are many good and important papers on these special algorithms which are not mentioned or referred to.

7. *Some Applications of Discrete Fourier Transforms.* A very brief but useful sketch of the application of the FFT algorithm to convolution calculations is given. This can also go under the names of correlation, covariance, digital filtering, and so on. The author fails to point out how his treatment can handle all of these and what alterations must be made in the basic techniques in order to handle the special cases. There is very little said about the power spectrum except to state that it is the magnitude of the Fourier transform. The two-dimensional transform is simply described as a row-column iteration of the FFT algorithm. Nothing about special considerations such as how to treat real data is given.

8. *Discrete Hilbert Transforms.* Although this topic is important in systems and signal processing, it has a disproportionate amount of attention in this book. A student who has to study at the level presented here will not only fail to see the significance but will have difficulty going through the intricate derivations. Furthermore, it does not seem pedagogically wise to present, in the first full-scale application, a case where the integrals do not even exist in the sense in which the reader has understood them throughout the book. He should have some understanding of principle value definitions of integrals and of the theory of distributions. He should also know something about the problems in treating such integrals numerically. Of course, it goes without saying that he should have a better understanding of why the Hilbert transform is important.

*Summary.* The book has a good introduction to the Fourier theory needed for understanding its numerical applications. Either as a textbook or as source of information for a practicing engineer, the book is somewhat uneven in quality. The FFT sections are very good, but as mentioned above, they should contain more about prime factor algorithms. It is a rather short book of 141 pages. The bibliography is extensive and well referenced in the text. However, as one may expect from a book written in Eastern Europe, one is often disappointed to find references, which one would like to read, published in Russian.

The book probably served its purpose very well when written and used where access to books on the subject were perhaps limited and where one had the author to teach and explain. The English language book could probably be used effectively with supporting material. It has very few examples, it has no problems or exercises for the student and surprisingly, it gives no program listings. Among the many books on the subject in the English language, there are far more useful books for teaching or for reference.

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**18[33–00, 65A05].**—MILTON ABRAMOWITZ & IRENE A. STEGUN (Editors), *Pocketbook of Mathematical Functions—Abridged edition of Handbook of Mathematical Functions, Milton Abramowitz and Irene A. Stegun (eds.)*, Material selected by Michael Danos and Johann Rafelski, Verlag Harri Deutsch, Thun, Frankfurt/Main, 1984, 468 pp., 24 cm. Price $20.00.

As noted in the Preface, the need for numerical tables, particularly those of the elementary mathematical functions, has been largely obviated by the advent of microelectronics in the interim of more than two decades since the original Handbook first appeared (see the review in [1]). Accordingly, in this abridged edition only one-third of the original numerical tables have been retained, and further reduction has been achieved through the omission of the first and final two chapters as well as, regrettably, the lists of references at the ends of successive chapters.

Otherwise, the body of the original text, including the numbering of the formulas, has been preserved to permit direct cross reference to the original. An improvement has resulted from the correction of most of the known typographical errors and the slight enlargement of the original second chapter, including the updating of tabulated physical constants.

For many users of the original, bulky volume this portable abridgment should be a convenient, adequate substitute.

J.W.W.

1. RMT 1, *Math. Comp.*, v. 19, 1965, pp. 147–149.

**19a[33–04, 33A45, 65A05].**—H. F. BAUER & W. EIDEL, *Tables of Roots with Respect to the Degree of a Cross Product of Associated Legendre Functions of First and Second Kind*, Forschungsbericht der Universität der Bundeswehr München, Institut für Raumfahrttechnik, LRT-WE-9-FB-9, 1986, 81 pp.

**b[33–04, 33A45, 65A05].**—HELMUT F. BAUER & W. EIDEL, *Tables of Roots with Respect to the Degree of a Cross Product of Associated Legendre Functions and Derivative of First and Second Kind*, Forschungsbericht der Universität der Bundeswehr München, Institut für Raumfahrttechnik, LRT-WE-9-FB-14, 1986, 183 pp.

**c[33–04, 33A45, 65A05].**—H. F. BAUER & W. EIDEL, *Tables of Roots with Respect to the Degree of a Cross Product at the First Derivative of Associated Legendre Functions of First and Second Kind*, Forschungsbericht der Universität der Bundeswehr München, Institut für Raumfahrttechnik, LRT-WE-9-FB-15, 1986, 83 pp.

Helmut F. Bauer has previously written about Legendre functions [1] and has published tables of zeros of the associated Legendre function of the first kind, $P_\lambda^m(\cos\alpha)$, and its derivative [2], [3]. The three tables reviewed here are an extension of that work. The tables supply five-decimal values of the first ten $\lambda$-zeros of the following cross products:

(FB-9) $\quad P_\lambda^m(\cos\alpha)Q_\lambda^m(\cos\beta) - P_\lambda^m(\cos\beta)Q_\lambda^m(\cos\alpha),$

(FB-14) $\quad P_\lambda^{m'}(\cos\alpha)Q_\lambda^m(\cos\beta) - P_\lambda^m(\cos\beta)Q_\lambda^{m'}(\cos\alpha),$

(FB-15) $\quad P_\lambda^{m'}(\cos\alpha)Q_\lambda^{m'}(\cos\beta) - P_\lambda^{m'}(\cos\beta)Q_\lambda^{m'}(\cos\alpha).$

Here, $P_\lambda^m$ and $Q_\lambda^m$ are the associated Legendre functions in the usual notation and primes indicate derivative with respect to the argument $\cos\alpha$ or $\cos\beta$. In each table the ranges of the parameters are $\alpha = 20°(10°)170°$, $\beta = 10°(10°)\alpha - 10°$, $m = 0(1)9$.

Using software developed at the National Bureau of Standards [4], [5], [6] and run on a CDC 180/855 computer, values in the tables were checked by calculating the value of the appropriate cross product for the given $\lambda$, and also for $\lambda \pm .00001$. In every case tested, the absolute value of the cross product at $\lambda$ was the smallest of the three, and there was a change of sign from $\lambda - .00001$ to $\lambda + .00001$, confirming that the given zero was correct. FB-9 was most fully tested. For $\alpha \leq 80°$, at least four values of $\lambda$ were tested for each pair of $\alpha$ and $\beta$. For $\alpha \geq 90°$, at least two values of $\lambda$ were tested for each $\alpha$ and $\beta$ pair. In FB-14 and FB-15, at least one $\lambda$ for each pair of $\alpha$ and $\beta$ was tested. Overall, 626 of 40800 entries, approximately 1.5%, were tested and all were correct. The introductory pages of each table have a number of typographical errors and inconsistencies. For example, $P_\lambda^{m\prime}(\cos\alpha)$ also appears as $P_\lambda^{\prime m}(\cos\alpha)$ and reference [3] of this review (Bauer's reference [24]) is listed as being on pages 601–602 and 529–541 of this journal instead of on pages 601–602 and S29–S41. However, these misprints do not affect the accuracy of the tables.

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1. H. F. BAUER, "Flüssigkeitsschwingungen in Kegelbehälterformen," *Acta Mech.*, v. 43, 1982, pp. 185–200.

2. H. F. BAUER, "On the numerical value of the roots of the associated Legendre function with respect to the order*," *Z. Angew. Math. Mech.*, v. 61, 1981, pp. 525–527.

3. HELMUT F. BAUER, "Tables of the roots of the associated Legendre function with respect to the degree," *Math. Comp.*, v. 46, 1986, pp. 601–602, S29–S41.

4. J. M. SMITH, F. W. J. OLVER & D. W. LOZIER, "Extended-range arithmetic and normalized Legendre polynomials," *ACM Trans. Math. Software*, v. 7, 1981, pp. 93–105.

5. D. W. LOZIER & J. M. SMITH, "Algorithm 567. Extended-range arithmetic and normalized Legendre polynomials," *ACM Trans. Math. Software*, v. 7, 1981, pp. 141–146.

6. F. W. J. OLVER & J. M. SMITH, "Associated Legendre functions on the cut," *J. Comput. Phys.*, v. 51, 1983, pp. 502–518.

* "Order" here should be "degree".

**20[01A35, 11D09].**—LEONARDO PISANO FIBONACCI, *The Book of Squares*, An Annotated Translation into Modern English by L. E. Sigler, Academic Press, Orlando, Fla., 1987, xx+124 pp., $23\frac{1}{2}$ cm. Price $19.95.

Leonardo Pisano, generally referred to as Fibonacci for the past century and a half, has been acclaimed the greatest European mathematician of the Middle Ages. His renown is largely due to his authorship of several mathematical classics, of which the most advanced is *Liber quadratorum* (*The Book of Squares*). Therein he ingeniously used geometrical algebra, as exemplified in Book II of Euclid's *Elements*, to explore the relation of integer squares to sums of sequences of odd integers.

He thereby solved a number of indeterminate number-theoretic problems which included several solved earlier in a different manner by Diophantus.

The origin of this book is ascribed in the Prologue to a problem posed to Leonardo by John of Palermo, involving a special case of what are now called congruent numbers. These numbers have been discussed in detail by several modern writers, notably Dickson [1] and Ore [2], and are still being investigated.

Professor Sigler has supplemented this careful translation of *Liber quadratorum* into modern English with detailed comments in contemporary mathematical notation and terminology as well as with a brief biography of Leonardo Pisano, which includes an outline of his works. The sources drawn upon for this version are contained in an appended list of 20 references.

Although the original book was written without numbering of theorems, this translation presents the text conveniently in the form of 24 numbered propositions with proofs and the aforementioned subjoined comments.

Regrettably, a number of typographical errors appear in the comments. For convenient reference these are listed with corrections in the Errata section of this issue.

This book clearly reveals Leonardo Pisano as a highly original, ingenious mathematician, unquestionably the greatest number theorist in the period from Diophantus to Fermat. It should be of special interest to all those interested in the history of the theory of numbers.

J. W. W.

1. L. E. DICKSON, *History of the Theory of Numbers*, Vol. II, Carnegie Institute, Washington, D. C., 1920, reprinted by Chelsea, New York, 1952.

2. O. ORE, *Number Theory and Its History*, McGraw-Hill, New York, 1948.

**21[65–06].**—A. H. P. VAN DER BURGH & R. M. M. MATTHEIJ (Editors), *Proceedings of the First International Conference on Industrial and Applied Mathematics (ICIAM 87): Contributions from the Netherlands*, CWI Tract, Vol. 36, Centre for Mathematics and Computer Science, Amsterdam, 1987, 433 pp., 24 cm. Price Dfl.56.80.

Four applied mathematics organizations, GAMM, IMA, SIAM and SMAI, from Germany, England, the United States and France, joined in organizing the First International Conference on Industrial and Applied Mathematics, which took place in Paris on June 29–July 3, 1987. While no official proceedings of this major event are going to be published, a national committee in the Netherlands decided to invite the Dutch contributors to prepare their manuscripts for publication in this volume. The volume contains 29 contributions, which are presented in seven categories entitled Applied Mathematical Analysis, Scientific Computing, Control Theory and Signal Processing, Computational Geometry, Applied Probability and Statistics, Mathematics of Natural Sciences, Software and Hardware Aspects.

W. G.

**22[65–06, 41–06].**—L. COLLATZ, G. MEINARDUS & G. NÜRNBERGER (Editors), *Numerical Methods of Approximation Theory*, International Series of Numerical Mathematics, Vol. 81, Birkhäuser, Basel and Boston, 1987, 261 pp., 24 cm. Price $47.00.

These are the proceedings of an international workshop held in Oberwolfach, September 28–October 4, 1986. They contain 21 papers on a variety of topics in pure and applied approximation theory.

W. G.

**23[65–06].**—P. KEAST & G. FAIRWEATHER (Editors), *Numerical Integration—Recent Developments, Software and Applications*, NATO Advanced Science Institute Series, Series C: Mathematical and Physical Sciences, Vol. 203, Reidel, Dordrecht, 1987, xiii+394 pp., $24\frac{1}{2}$ cm. Price $89.00.

These are the proceedings of the NATO Advanced Research Workshop on Numerical Integration held at Dalhousie University, Halifax, Canada, August 11–15, 1986. They contain 20 full-length papers and 10 abstracts organized in four parts: Theoretical aspects of one-dimensional quadrature; Theoretical aspects of multiple quadrature; Algorithms, software and applications; Software classification and testing. Several papers deal with the implementation of quadrature algorithms on vector and parallel machines. (The reviewer takes exception to the publication of referee's reports, apparently without the consent of the referees involved, in one of the contributions.)

W. G.

**24[65–02, 68–02].**—LEAH H. JAMIESON, DENNIS GANNON & ROBERT J. DOUGLASS (Editors), *The Characteristics of Parallel Algorithms*, MIT Press Series in Scientific Computation, The MIT Press, Cambridge, Mass., 1987, xi+440 pp., $23\frac{1}{2}$ cm. Price $27.50.

This book evolved from a workshop held in Santa Fe on November 30–December 2, 1983, that was to lay the groundwork for a Taxonomy of Parallel Algorithms. It contains 16 articles written by specialists in the respective areas and arranged here in three sections: General characteristics of parallel computation models; Application domain characterizations of parallelism; Software tools. While the emphasis is on architectures, programming and software, there are three contributions in the second section that discuss parallelism in Partial Differential Equations, Matrix Computation and Fast Fourier Transform.

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**25[65–06, 68–06].**—P. DEUFLHARD & B. ENGQUIST (Editors), *Large Scale Scientific Computing*, Progress in Scientific Computing, Vol. 7, Birkhäuser, Boston, 1987, xii+388 pp., $23\frac{1}{2}$ cm. Price $30.00.

These are the proceedings of a meeting held at the Oberwolfach Mathematical Institute July 14–19, 1985. The 22 papers are grouped into six parts entitled: Initial value problems for ODE's and parabolic PDE's; Boundary value problems for ODE's and elliptic PDE's; Hyperbolic PDE's; Inverse problems; Optimization and optimal control problems; Algorithm adaptation on supercomputers. Among the large-scale applications covered are semiconductor design, chemical combustion, flow through porous media, climatology, seismology, fluid dynamics, tomography, rheology, hydro power plant optimization, subway control and space technology.

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