

REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS

The numbers in brackets are assigned according to the American Mathematical Society classification scheme. The 1980 Mathematics Subject Classification (1985 Revision) can be found in the December index volumes of Mathematical Reviews.

34[65-04].—J. L. MOHAMED & J. E. WALSH (Editors), *Numerical Algorithms*, Manchester/Liverpool Summer Schools in Numerical Analysis, Clarendon Press, Oxford, 1986, xii + 356 pp., 24 cm. Price \$60.00.

This is a pleasing book and I am going to praise it. The impetus for producing it came from a summer school held in England in July, 1984 but please do not dismiss it on that account. This is not the Proceedings of a standard summer research conference.

The book describes algorithms which give cheap and reliable solutions to a diverse set of numerical tasks. Moreover, each chapter points to easily obtained software that implements the algorithms. There is a list of most of the organizations that purvey mathematical software, but it should be mentioned that the NAG library is the dominant reference. The editors have evidently exercised strong control in giving coherence to the sixteen chapters. Not the least of their contributions is the provision of a single set of references for the whole book, thereby saving many pages.

I think it is fair to say that this book is not for beginners, not for those who have had no previous exposure to numerical analysis, and this pleases me because I am not a beginner and so can profit from a taut, but not too terse, expository style. Although I know a lot about one or two of the topics, and something about those which I have covered when teaching introductory courses, I am almost totally ignorant of some of the contents. So, in order to be fair I have read with care one chapter from each category. It was from this exercise that my enthusiasm arose. I could find no fault with the contributions in my own domain, I was pleased to be brought up to date on material that I teach from time to time, and, last but not least, I could follow the chapter that was new to me. In other words the authors and, I suspect, the editors have taken great pains to expound clearly to someone who has at least some exposure to scientific calculation.

It should be mentioned that the book covers a huge amount of material in a mere 350 pages of sticking rigidly to the goal of simply describing algorithms. There are very few numerical comparisons or worked examples. This is not the place to learn what a Givens rotation is. The reward of such discipline is that what the reader gets is a review of the state of mathematical software in 1984. That is something that many who crunch numbers will wish to possess.

How complete is the coverage? From where I sit the company of numerical analysts may be divided into two big groups with only a small overlap: people in

computational fluid dynamics (CFD) and the rest. The book under review is not the place to find out about random choice methods in two or more space dimensions nor to find out how vortex methods are implemented. However, the method of lines is mentioned in Chapter 9. By and large the CFD community is not much concerned with general purpose numerical libraries nor with portable software.

Since the summer school that launched this book was inspired by the high degree of sophistication attained in some libraries, it is quite appropriate that the book should reflect that achievement. One can now find well-tested software covering most recognized special tasks in the areas of matrix computations, ordinary differential equations, integral equations, definite integrals, elliptic and parabolic partial differential equations, approximation and optimization. It is pleasant to work in a mathematical field that leads to tangible products.

It is strange that many engineers are happy to use FORTRAN and the elementary functions it provides, yet still, in the 1980s, will write their own programs to integrate a system of differential equations. If the day comes when the working engineer routinely builds his (or her) programs using high-level building blocks from libraries such as NAG, then it will be due, in part, to the availability of books such as 'Numerical Algorithms'.

B. N. P.

35[76B15].—I. KINMARK, *The Shallow Water Wave Equations: Formulation, Analysis and Application*, Lecture Notes in Engineering, Vol. 15, Springer-Verlag, New York, 1985, xxv + 187 pp., 24 cm. Price \$18.00.

The modelling of wave propagation in shallow water has a long history, especially with the need to understand and predict tides. While much of the tidal work since the time of Laplace is based on the hydrostatic approximation and a linear approximation to the conditions at the free surface, it was already appreciated by Boussinesq in 1886 that small nonlinear effects at the free surface could also play a decisive role in the evolution of long waves—our understanding of the propagation of tsunamis is based on such models. Thus, the modelling of longwave phenomena in practical situations, trying to balance the effects of dispersion, nonlinearity, dissipation, irregular bottom topography, the influence of the earth's rotation, and the role of atmospheric interaction in the generation of waves, is known to be an intricate process. It follows that a book purporting to discuss the formulation, analysis and applications of the shallow water wave equations will have lots of interesting fluid dynamics to discuss. Not so. In fact, the present volume provides a discussion of the pros and cons of various methods for obtaining numerical approximations to the solution of Laplace's tidal equations. So the term 'formulation' here refers to considerations of whether or not it is better to take the equations in conservation form for numerical computation, and 'analysis' refers to stability considerations for numerical approximations to the model equations.

When obtaining numerical solutions to the tidal equations, the problems of aliasing, associated with grid-scale discretization error, are known to be a major source of difficulty and so the main focus of these lecture notes is, in fact, directed towards the development of methods that suppress grid-scale oscillations. Thus, the central

theme and the main thrust of the book concerns linear stability analyses of various formulations of the equations together with some examples, based on model problems, of the relative efficacy of the various methods. In a final brief chapter, entitled 'Applications', a specific example associated with the southern region of the North Sea is considered in the light of the preceding discussions.

The red-covered Springer series of lecture notes aims to provide rapid, refereed publication of topical items, longer than ordinary journal articles, but shorter and less formal than most monographs and textbooks. This relative informality probably accounts for the narrow theme of the book, but even then I found the book to be unappealing on several counts. Too much of the discussion revolves around detailed elaboration of highly specific issues, utilizing quite standard methods of analysis and presenting detail of little interest to the general reader: to wit, the twenty-five pages of text and tables concerning the character of roots of low-degree polynomials. I felt that with a little extra mathematical sophistication the same material and concepts could have been presented far more succinctly, opening up the possibility of greater elaboration of the modelling procedures. As an example, there is no discussion in the book of how boundary conditions are to be incorporated into the models and yet, in the chapter on applications, an arbitrary truncation of the natural flow domain is made on which so-called 'open boundary conditions' are applied, but never defined. Also I would dearly have liked the presentation of quantitative convergence studies to enable a more direct comparison of the relative merits of the various methods under discussion. And, as implied above, there is no discussion whatsoever of the physics underlying the models or of their application to the practical world.

At a more mundane level, the English expression is sometimes a little awkward and punctuation is often lacking, especially with regard to displayed formulae. The text is aimed at a very narrow engineering audience. I do not see it as being of general interest to readers of *Mathematics of Computation*.

W. G. PRITCHARD

Department of Mathematics
The Pennsylvania State University
University Park, Pennsylvania 16802

36[35Q20, 35-02].—P. L. SACHDEV, *Nonlinear Diffusive Waves*, Cambridge Univ. Press, Cambridge, 1987, vii + 246 pp., 23½ cm. Price \$49.50.

The nonlinear diffusive waves of the title are waves satisfying Burgers' equation

$$(1) \quad u_t + uu_x = \frac{\delta}{2} u_{xx}$$

or one of its generalizations. Burgers' equation is often used as a simplified model of turbulence and is remarkable because it may be linearized using the Cole-Hopf transformation. For contrast, the author also discusses some nonlinear waves that are not diffusive and some diffusion processes that are not waves. The Korteweg-deVries soliton, solving an equation like (1) with the term uu_x replaced by $-u_{xxx}$, is the nondiffusive nonlinear wave discussed here. The discussion is however exceedingly brief, presumably because of the extensive treatments to be found in many

other texts. The nonlinear diffusion processes chosen for contrast are those satisfying the porous media equations, i.e., those having a diffusion coefficient proportional to a power of the diffusing quantity. These diffusion equations are discussed at some length, and include some recent topics not to be found in other texts. On the whole, about 90% of the text is devoted to Burgers' equation. The analysis is formal with references to the literature for proofs and details. The level of presentation is suitable for advanced undergraduate or beginning graduate courses in applied or engineering mathematics.

The fifth and last chapter of the book is devoted to a compendium of numerical methods for solving nonlinear diffusion equations. Pseudo-spectral numerical methods are emphasized, but the introduction to these methods is cursory. A novice will need to study some other treatment such as that of Gottlieb, Hussaini, and Orszag [3] to obtain a thorough grounding in the subject. The emphasis throughout is placed on summarizing the known results for nonlinear diffusion and nonlinear diffusive waves.

The shortcomings of the text that I noted were these: (i) Although there are many references given to pertinent literature, I found that the author's choices did not always include the first or most significant citation for the topic being discussed. For example, when discussing a class of nonlinear equations that can be solved exactly, Sachdev references the work of Fokas and Yortsos [2] but not the earlier work of Bluman and Kumei [1]. The choices made in such cases seemed arbitrary to me. (ii) The discussion of the Korteweg-deVries equation seemed too brief to be useful, as if the author assumed substantial prior knowledge of solitons and soliton equations. This lack of completeness makes the text of less value than it might have been as a supplementary text in a course on nonlinear partial differential equations.

The book is certainly a useful guide to much of the recent literature on nonlinear diffusion and diffusive waves, providing a very thorough summary of current knowledge of the behavior of solutions to Burgers' equation and its generalizations. As such, it will no doubt become a widely used reference for researchers beginning to explore this part of the fascinating field of nonlinear applied mathematics.

JAMES G. BERRYMAN

Courant Institute of Mathematical Sciences
251 Mercer Street
New York University
New York, New York 10012

1. G. BLUMAN & S. KUMEI, "On the remarkable nonlinear diffusion equation $(\partial/\partial x) \cdot [a(u+b)^{-2}(\partial u/\partial x)] - (\partial u/\partial t) = 0$," *J. Math. Phys.*, v. 21, 1980, pp. 1019-1023.

2. A. S. FOKAS & Y. C. YORTSOS, "On the exactly solvable equation $S_t = [(\beta S + \gamma)^{-2} S_x]_x + \alpha(\beta S + \gamma)^{-2} S_x$ occurring in two-phase flow in porous media," *SIAM J. Appl. Math.*, v. 42, 1982, pp. 318-332.

3. D. GOTTLIEB, M. Y. HUSSAINI & S. A. ORSZAG, "Theory and application of spectral methods," in *Spectral Methods for Partial Differential Equations* (R. G. Voigt, D. Gottlieb, and M. Y. Hussaini, eds.), SIAM, Philadelphia, Pa., 1984, pp. 1-54.