REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS

The numbers in brackets are assigned according to the American Mathematical Society classification scheme. The 1980 Mathematics Subject Classification (1985 Revision) can be found in the December index volumes of Mathematical Reviews.

1[65–01, 65M60, 65N30].—CLAES JOHNSON, *Numerical Solution of Partial Differential Equations by the Finite Element Method*, Cambridge Univ. Press, Cambridge, 1987, 278 pp., 23 cm. Price \$69.50 hardcover, \$24.95 paperback.

This text is an introduction to the finite element method and is classroom tested by its author in upperclass engineering courses at his home institution, Chalmers. The prerequisites are given as follows: "Basic courses in advanced calculus and linear algebra and preferably some acquaintance with the most well-known linear partial differential equations in mechanics and physics ...". Actually, at times, quite sophisticated tools are used, such as a priori estimates for elliptic problems, for the equation $\operatorname{div} v = q$, and for the behavior of downstream and parabolic layers in convection-dominated problems. The basic framework is that of bilinear forms on Hilbert spaces. Although the basic concepts are clearly explained and the more sophisticated ones referenced, I consider the present book as somewhat more demanding, mathematically, than, say, the excellent very elementary introduction to the subject by White [1]. Considering the situation in the US, by no means is the present text accessible only to the mathematics majors, but merely a sophomore study of calculus and linear algebra is insufficient preparation to really appreciate the presentation.

And appreciated it should be! In my opinion it is a masterful introduction to the subject. Its two main distinguishing features are: a thoroughly modern and up-to-date treatment, and the inclusion of some topics of current research interest. This sets it apart from most introductory texts, where developments within the last five years are seldom treated. (There is always a price tag: a revised edition of this work will probably be called for within a few years.) The writing is brisk and lively while still rigorous; it is a distinct pleasure to read. It will give the student a sound and clean conceptual framework for the methods, modern practical recipes for their implementation, and, to tease his imagination, a study of some methods which have not yet been canonized. Also, it is organized in such a way that an instructor who wishes to expand on certain topics beyond the presentation given, while cutting down on other parts, will have little problem in doing so.

I proceed to describe the contents and note in passing that the author does not aim at covering everything. For example, eigenvalue problems are not even mentioned.

The first part of the book, Chapters 1–7, covers standard material. The author provides the usual modern cleancut conceptual framework for the method in elliptic problems, giving plenty of examples, goes into the actual construction of some finite elements, gives some simple error estimates and describes direct and iterative methods for the solution of the resulting systems of linear equations. There is no deadwood, e.g., the chapter on iterative methods goes more or less directly (treating first general gradient methods, for later purposes) to the conjugate gradient method, with preconditioning, and then briefly treats multigrid methods (SSOR and ADI are not even mentioned). The short Section 4.6 on the use of error estimates for adaptive mesh refinement is noteworthy.

Chapter 8 is devoted to parabolic problems. Most of the material is traditional, but the discontinuous Galerkin method, and the use of error estimates for time-step control, are treated, again showing the up-to-date nature of this book.

Chapter 9 is thoroughly unusual for an introductory text. Here recent developments in applications of the finite element method to hyperbolic, and convection-dominated parabolic, problems are given. The streamline diffusion and discontinuous Galerkin methods are treated. It should be thought-provoking for students to come in contact with this recent research material, which has not yet made it into the canon of practical methods.

Chapters 10–12 then treat more standard material, boundary element methods, mixed methods, and curved elements and numerical integration, respectively.

A chapter on nonlinear problems such as obstacle problems, minimal surfaces, the incompressible Euler and Navier-Stokes equations, and Burgers' equation, concludes the book. The treatment in this chapter is very sketchy, often just giving the method. In my opinion this chapter is weak; however, twelve good chapters out of thirteen is a fine batting average.

It is traditional for a reviewer to show that he has read the book by picking quarrels with the author: The section on preconditioning, 7.4, is unnecessarily brief. As examples, only incomplete factorizations are mentioned, and since these methods do not lead to condition numbers bounded independently of the meshsize, they hardly do justice to the idea of preconditioning. A more convincing elementary example is that of a variable coefficient problem on a logically rectangular mesh, which may, after a piecewise mapping argument, be preconditioned by a Poisson problem on a square with a uniform mesh. The latter problem can be solved fast, e.g., by first applying the FFT in one direction and then tridiagonal solvers over each line in the other direction. The resulting effective condition numbers are bounded independently of the meshsize. I suggest expansion of this section in future editions (or, by the instructor), and also of the section on multigrid methods, which does little more than say that there exists a marvellous method called multigrid.

While error estimates are used to motivate adaptive procedures, it is not elucidated how useful they are for checking correctness of programs. A student may well wonder why one bothers to derive error estimates of the form $Ch^r|u|_s$, where C and u are not known, until it is pointed out to him that knowledge of r *alone* presents an invaluable debugging tool.

On p. 237, in connection with mixed methods for the Stokes problem, the author remarks that it is not clear how to solve the resulting equations iteratively in an efficient way. However, in the notation of (11.14), the pressure θ satisfies the equation $B^TA^{-1}B\theta = -B^TA^{-1}F$. The matrices $B^TA^{-1}B$ have condition numbers bounded independently of the mesh size, and thus, e.g., the conjugate gradient method will converge rapidly. Each step involves solving a standard Poisson problem. After that, one easily solves for the velocities. Thus, an efficient iterative method is easily found.

On p. 254, in connection with an obstacle problem, the choice $K_h = \{v \in V_h : v \geq \psi \text{ in } \Omega\}$ for the approximate constraint set is given. This choice is hard to implement. Imposing the inequality only at nodes, say, is easier to implement (but somewhat harder to analyze).

In conclusion, this is an impeccable introduction to the subject for an audience with some mathematical maturity beyond sophomore calculus and linear algebra. I predict that, for many purposes, it will replace the well-known book by Strang and Fix, [2], which is, very naturally, out of date in many respects.

L. B. W.

1. R. E. WHITE, *An Introduction to the Finite Element Method with Applications to Nonlinear Problems*, Wiley-Interscience, New York, 1985. (Review **1**, *Math. Comp.*, v. 50, 1988, pp. 343–345.)

2. G. STRANG & G. FIX, *An Analysis of the Finite Element Method*, Prentice-Hall, Englewood Cliffs, N.J., 1973. (Review **35**, *Math. Comp.*, v. 28, 1974, pp. 870–871.)

2[65N30, 65M60, 76D05, 76D07].—GRAHAM F. CAREY & J. TINSLEY ODEN, *Finite Elements: Fluid Mechanics*, The Texas Finite Element Series, Vol. VI, Prentice-Hall, Englewood Cliffs, N.J., 1986, x+323 pp., $23\frac{1}{2}$ cm. Price \$38.95.

This is the sixth in the series devoted to Finite Element methods for the numerical solution of problems in Mechanics governed by partial differential equations. The first four volumes were concerned with the general exposition of the method while the fifth volume specifically concentrated on problems in Solid Mechanics. As the authors point out, Finite Elements were originally used to solve problems in Structural Mechanics, and their application to Fluid Dynamics is comparatively recent, a prerequisite for this being the formulation of basic problems in Fluid Dynamics in variational form.

The volume is self-contained since the authors include a brief but complete description of the general Finite Element method in the first chapter, giving a lucid explanation in terms of problems associated with Laplace's equation. The second chapter, dealing with compressible flow, is concerned mostly with transonic flow, and makes a strong case for applying Finite Element methods to such challenging problems as supercritical flow past airfoils, previously treated almost exclusively by finite difference methods. The technique for shock fitting is particularly appealing.

The third chapter is probably the most important in the volume, since it contains a thorough derivation of the Navier-Stokes equations, including the variational formulation of associated viscous flow problems. The latter leads to a detailed description of Finite Element methods, developed in turn for slow flows governed by the Stokes' equation, and for higher-speed steady flows governed by the full Navier-Stokes equations, including unsteady and compressibility effects. Applications to cavity flows, corner flows and wakes are particularly instructive. The subsequent chapter, concerned with stream function—vorticity formulation, is more limited since it is confined to two-dimensional flows. The final chapter on transport processes appears to be of more academic interest than the earlier three chapters, although it does contain interesting remarks on numerical techniques.

The main text concludes with an impressive list of references. As a whole, the book covers new ground in Computational Techniques for Fluid Mechanics. It is clearly written and aids the understanding of a valuable approach only partially appreciated by those presently working in Computational Fluid Dynamics. The inclusion of a set of examples at the end of each important section enhances the value of the volume as a graduate course text.

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3[15–02, 65F50].—I. S. DUFF, A. M. ERISMAN & J. K. REID, *Direct Methods for Sparse Matrices*, Monographs on Numerical Analysis, Clarendon Press, Oxford, 1986, xiii+341 pp., 24 cm. Price $42.50.

Iain Duff, Al Erisman, and John Reid have made an outstanding contribution to the literature on sparse matrices. *Direct Methods for Sparse Matrices* contains a wealth of information which is extremely well organized and is presented with exceptional clarity. The book will be a valuable addition to the libraries of practitioners whose problems involve working with sparse matrices; it also includes many fine exercises and is suitable for use as a textbook at the graduate or upper division undergraduate level. In addition, selected topics addressed in this book could profitably be included in courses not dedicated exclusively to sparse matrices, such as courses in data structures, algorithms, or numerical linear algebra.

Readers of *Direct Methods for Sparse Matrices* are assumed to be familiar with elementary linear algebra and to have some computing background. Other background material is included in the first four chapters. Students and general readers will appreciate finding that the concepts and techniques presented are illustrated with examples throughout. Practitioners whose primary goal in consulting this book is selection of library subroutines to solve their problems will find practical direction in the choice of library routines for specific problems. The authors draw on their extensive computational experience in making recommendations of one procedure over another for particular applications. Researchers will find a very well-organized survey of sparse matrix techniques along with abundant references to appropriate literature for more extensive exposition.

The first four chapters contain introductory material. In Chapter one, sparsity patterns are related to elementary concepts in graph theory, and issues associated with the efficient use of advanced computer architectures are introduced. Chapter two introduces data structures that are suitable for storing, accessing, and performing operations on sparse matrices and vectors. A summary of computational issues associated with Gaussian elimination performed on dense matrices constitutes Chapters three and four.

Means of exploiting sparsity in the solution of large linear systems are introduced in Chapter five, with emphasis on Gaussian elimination. This chapter contains an outline of the organization and objectives of available library routines. Chapter six contains explanations of informally stated algorithms for reducing general sparse matrices to block triangular form, along with a description of the class of matrices for which such reduction is likely to be possible and useful.

In Chapter seven, several heuristic strategies for selecting orderings of equations are presented in an attempt to maintain sparsity in matrix factorization without sacrificing stability of the factorization. Strategies discussed in this chapter are *local*, meaning that the selection process consists of choosing the ordering one equation at a time with the aim of keeping the amount of fill-in that occurs as a result of the current selection small; these strategies do not consider effects on subsequent steps. The Markowitz criterion, the minimum-degree strategy, and extensions and modifications to them are among the strategies discussed. Balance between maintaining sparsity and stability is also addressed, and specific recommendations are made. Chapter eight addresses preservation of sparsity through the *global* strategy of a priori permutation to desirable forms, such as band and variable-band matrices, block tridiagonal, bordered block diagonal, bordered block triangular, or spiked matrices. Several algorithms for ordering matrices to desirable forms are given, explained, and illustrated with examples. They include Cuthill-McKee, reverse Cuthill-McKee, Gibbs-Poole-Stockmeyer, one-way dissection, nested dissection, the Hellerman-Rarick procedures P^3 and P^4, and a variant due to Erisman, Grimes, Lewis, and Poole, termed P^5.

Implementation details of ordering and solution techniques are presented in Chapters nine and ten. The focus of Chapter nine is analysis of sparsity pattern along with numerical values, so that a matrix factorization is actually carried out as the analysis proceeds. Chapter ten considers analysis of sparsity patterns independent of the numerical factorization phase. Chapter nine details implementation of the Markowitz criterion combined with threshold pivoting for balance between preservation of sparsity and stability of the factorization, the Doolittle decomposition, and the solution phase for sparse linear systems. The authors also include in Chapter nine a discussion of the use of drop tolerances (dropping entries smaller than a specified absolute or relative tolerance) to preserve sparsity. Implementation details are given in Chapter ten for a solution process by phases of: ordering to preserve sparsity, symbolic factorization, numerical factorization, and solution. Special techniques for band and variable-band matrices are included here, along with a discussion of their ability to exploit vector and parallel architectures. A variation of the variable-band technique called the frontal method, and a generalization called the multifrontal method, are explained in detail.

In Chapter eleven, the solution of huge systems is approached through partitioning, tearing, and perturbation to a more easily solved system. Efficient implementation of the Sherman-Morrison-Woodbury formula to adjust the solution of the perturbed system is outlined. A cautionary note concerning the stability of such procedures is included.

The concluding chapter, Chapter twelve, is devoted to a collection of sparsity issues aside from the solution of sparse linear systems. Notable inclusions in this chapter are the Curtis-Powell-Reid algorithm for efficient calculation of sparse Jacobian estimates and an algorithm of Toint for updating sparse Hessian approximations for quasi-Newton calculations. (Unfortunately, positive definiteness of the approximate Hessian is not retained.) The open question of sparsity-constrained backward error analysis is also discussed.

Direct Methods for Sparse Matrices will be a valuable addition to the bookshelf of every reader interested in the solution of large sparse problems.

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4[62–04].—PETER LANE, NICK GALWAY & NORMAN ALVEY, *Genstat* 5—*An Introduction*, Clarendon Press, Oxford, 1987, xii+163 pp., $25\frac{1}{4}$ cm. Price \$45.00.

GENSTAT is a general statistics program designed to analyze data with the help of a computer. It combines the advantages of a programming language like FORTRAN with those of specialized "canned packages" like SAS or SPSS.

The Genstat 5 introduction by Lane, Galway & Alvey is designed to help the beginner getting started. It covers only the basic features and a few selected statistical methods like plots of data, linear regression, tabulation of data, and analysis of designed experiments. The reader is carefully guided from the first steps of reading and writing data to the actual statistical analyses and to the writing of more complicated Genstat programs. The numerous examples and exercises provide ample opportunity to gain experience with Genstat. I liked particularly the refreshing, nontechnical style in which the book has been written, and I am sure that students will find pleasure in learning to analyze data using this introductory guide. My only criticism of the book is its relatively high price.

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5[62Q05, 62E15, 62F07, 62J15, 62H10].—R. E. ODEH, J. M. DAVENPORT & N. S. PEARSON (Editors), *Selected Tables in Mathematical Statistics*, Vol. 11, Amer. Math. Soc., Providence, R.I., 1988, xi+371 pp., 26 cm. Price \$46.00.

This volume includes tables constructed by R. E. Bechhofer and C. W. Dunnett of selected percentage points of the central multivariate Student t distribution in which there is a common variance estimate on ν degrees of freedom in the denominators of the variates, and the numerators either are equicorrelated (Tables A and B) or have a certain block correlation structure (Tables C and D).

Tables A and B (which practically cover the volume) provide in the equicorrelated (ρ) case one-sided and two-sided equicoordinate 80, 90, 95, and 99 percentage points with 5 decimal place accuracy for p (the number of variates) $= 2(1)16(2)20$; $\nu = 2(1)30(5)50, 60(20)120, 200, \infty$; $\rho = 0.0(0.1)0.9, 1/(1+\sqrt{p})$.

The other two tables deal with a block correlation structure with p_1 variates in the first block and p_2 in the second block; the variates in each block have $\rho = 0.5$ and the variates in different blocks have $\rho = 0$. They provide one-sided 80, 90, and 95 percentage points with 5 decimal place accuracy for $p_1 = 1(1)4$; $p_2 = p_1(1)6, 9$; $\nu = 5(1)30(5)50, 60(20)120, 200, \infty$. Table C gives equicoordinate percentage points, while Table D gives percentage points of a particular form which is not equicoordinate unless $p_1 = p_2$.

These tables have many statistical applications. They are typically needed in procedures devised for selection among normal means, using either the indifference-zone or subset approach, and for multiple comparisons involving contrasts among means. These and other applications are described in detail. Examples are given illustrating applications of the tables.

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6[65–04].—WILLIAM H. PRESS, BRIAN P. FLANNERY, SAUL A. TEUKOLSKY & WILLIAM T. VETTERLING, *Numerical Recipes in C—The Art of Scientific Computing*, Cambridge Univ. Press, Cambridge, 1988, xxii+735 pp., 24 cm. Price \$44.50.

This is an edition in the C computer language of the original FORTRAN and Pascal version of [1]. A subsection has been added discussing some of the C conventions for scientific computing. Also, errors in the original volume that have come to the authors' attention have been corrected in this edition.

W. G.

1. WILLIAM H. PRESS, BRIAN P. FLANNERY, SAUL A. TEUKOLSKY & WILLIAM T. VETTERLING, *Numerical Recipes—The Art of Scientific Computing*, Cambridge Univ. Press, Cambridge, 1986. (Review **3**, *Math. Comp.*, v. 50, 1988, pp. 346–348.)

7[65–04].—WILLIAM T. VETTERLING, SAUL A. TEUKOLSKY, WILLIAM H. PRESS & BRIAN P. FLANNERY, *Numerical Recipes Example Book (C)*, Cambridge Univ. Press, Cambridge, 1988, ix+239 pp., $23\frac{1}{2}$ cm. Price \$19.95.

This is an edition in the C computer language of the original FORTRAN and Pascal versions [1].

W. G.

1. WILLIAM T. VETTERLING, SAUL A. TEUKOLSKY, WILLIAM H. PRESS & BRIAN P. FLANNERY, *Numerical Recipes Example Book (FORTRAN)*; *Numerical Recipes Example Book (Pascal)*, Cambridge Univ. Press, Cambridge, 1985. (Review **4**, *Math. Comp.*, v. 50, 1988, pp. 348–349.)

8[65–06, 65K05, 65R20, 86–06, 86–08, 86A20].—ANDREAS VOGEL (Editor), *Model Optimization in Exploration Geophysics*, Theory and Practice of Applied Geophysics, Vol. 1, Vieweg, Braunschweig/Wiesbaden, 1987, vii+396 pp., $24\frac{1}{2}$ cm. Price DM 108.00.

The proceedings of the 4th International Mathematical Geophysics Seminar held at the Free University of Berlin, February 6–8, 1986, this volume deals with mathematical aspects and geophysical applications of ill-posed inverse problems. The papers with predominantly mathematical and computational content are: "A general numerical method for ill-posed inverse problems using cubic splines" by G. Eriksson; "Iterative solution of ill-posed problems—a survey" by M. Brill & E. Schock; "Regularization of incorrectly posed problems by linear inequalities and quadratic programming" by U. Eckhardt; "On the adaptive solution of inconsistent systems of linear equations" by J. Baumeister; "On projection methods for solving linear ill-posed problems" by H. W. Engel & A. Neubauer.

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