## Mathematics of Computation



EDITED BY
James H. Bramble
Bille C. Carlson
Walter Gautschi, Managing Editor
Donald Goldfarb
Eugene Isaacson
Heinz-Otto Kreiss
James N. Lyness
Harald Niederreiter
Syvert P. Nørsett
Andrew M. Odlyzko
Frank W. J. Olver
John E. Osborn
Stanley Osher
Carl Pomerance
René Schoof
Larry L. Schumaker
Ridgway Scott
Daniel Shanks
Frank Stenger
Hans J. Stetter
G. W. Stewart

Vidar Thomée
Lars B. Wahlbin
Hugh C. Williams
John W. Wrench, Jr.

January 1989
Volume 52, Number 185, Pages 1-254
Published by the American Mathematical Society Providence, Rhode Island USA
ISSN 0025-5718

## Editorial Committee

WALTER GAUTSCHI, Chairman. Dept. of Computer Sciences, Purdue Univ., West Lafayette, IN 47907
DONALD GOLDFARB, Dept. of Industrial Engineering and Operations Research, Seely W. Mudd Building, Columbia Univ. in the City of New York, New York, NY 10027
JOHN E. OSBORN, Dept. of Mathematics, Univ. of Maryland, College Park, MD 20742
HUGH C. WILLIAMS, Dept. of Computer Science, Univ. of Manitoba, Winnipeg, Manitoba, Canada R3T 2N2

Technical Editor
ERIKA GAUTSCHI, Dept. of Computer Sciences, Purdue Univ., West Lafayette, IN 47907

## Board of Associate Editors

JAMES H. BRAMBLE, Dept. of Mathematics, Cornell Univ., Ithaca, NY 14853
BILLE C. CARLSON, Dept. of Mathematics, Iowa State Univ., Ames, IA 50011
EUGENE ISAACSON, Courant Institute of Mathematical Sciences, New York Univ., 251 Mercer Street, New York, NY 10012
HEINZ-OTTO KREISS, Dept. of Applied Mathematics, California Institute of Technology, Pasadena, CA 91125
JAMES N. LYNESS, Argonne National Laboratory, 9700 South Cass Avenue, Argonne, IL 60439
HARALD NIEDERREITER, Mathematical Institute, Austrian Academy of Sciences, Dr.-Ignaz-SeipelPlatz 2, A-1010 Vienna, Austria
SYVERT P. NØRSETT, Div. of Numerical Mathematics, The University of Trondheim and The Norwegian Institute of Technology, Alfred Getz vei 1, N-7034 Trondheim-NTH, Norway
ANDREW M. ODLYZKO, AT\&T Bell Laboratories, 600 Mountain Avenue, Murray Hill, NJ 07974
FRANK W. J. OLVER, Inst. for Physical Science and Technology, Univ. of Maryland, College Park, MD 20742
STANLEY OSHER, Dept. of Mathematics, Univ. of California, Los Angeles, CA 90024
CARL POMERANCE, Dept. of Mathematics, The Univ. of Georgia, Athens, GA 30602
RENÉ SCHOOF, Mathematisch Instituut, Rijksuniversiteit Utrecht, Budapestlaan 6, Postbus 80.010 , 3508 TA Utrecht, The Netherlands
LARRY L. SCHUMAKER, Dept. of Mathematics, Vanderbilt Univ., Nashville, TN 37235
RIDGWAY SCOTT, Dept. of Mathematics, Pennsylvania State Univ., University Park, PA 16802
DANIEL SHANKS, Dept. of Mathematics, Univ. of Maryland, College Park, MD 20742
FRANK STENGER, Dept. of Mathematics, Univ. of Utah, Salt Lake City, UT 84112
HANS J. STETTER, Institut für Numerische Mathematik, Technische Universität Wien, Wiedner Hauptstrasse 6-10, A-1040, Wien, Austria
G. W. STEWART, Dept. of Computer Science, Univ. of Maryland, College Park, MD 20742

VIDAR THOMÉE, Mathematics Dept., Chalmers Univ. of Technology, S-412 96 Göteborg, Sweden
LARS B. WAHLBIN, Dept. of Mathematics, Cornell Univ., Ithaca, NY 14853
JOHN W. WRENCH, JR., 102 Mt. Olivet Blvd., Frederick, MD 21701
SUBSCRIPTION INFORMATION: MATHEMATICS OF COMPUTATION is published quarterly, with issues numbered serially since Volume 1, Number 1. Subscription prices for Volumes 52 and 53 (1989) are $\$ 183.00$ list; $\$ 146.00$ institutional member; $\$ 119.00$ member of CBMS organizations; $\$ 110.00$ individual AMS member. A late charge of $10 \%$ of the subscription price will be imposed upon orders received from nonmember institutions and organizations after January 1 of the subscription year. Subscribers outside the United States and India must pay a postage surcharge of $\$ 8.00$; subscribers in India must pay a postage surcharge of $\$ 18.00$. Combination paper and microfiche subscription prices are $\$ 243.00$ list; $\$ 195.00$ institutional member. Microfiche of each issue will be mailed the fastest way before the issue is mailed by the printer.

BACK NUMBER INFORMATION: Back number prices per volume are for Volumes 1-21, $\$ 82.00$ list, $\$ 66.00$ member; for Volumes $22-33, \$ 124.00$ list, $\$ 99.00$ institutional member; for Volumes $34-43, \$ 82.00$ list, $\$ 64.00$ institutional member; Volumes $44-45, \$ 113.00$ list, $\$ 90.00$ institutional member; Volumes 46-47, \$127.00 list, \$102.00 institutional member; Volumes 48-49, \$135.00 list, \$108.00 institutional member; Volumes $50-51, \$ 136.00$ list, $\$ 109.00$ institutional member. Back volumes may be purchased on microfilm or microfiche from University Microfilms International, 300 North Zeeb Road, Ann Arbor, MI 48106.

UNPUBLISHED MATHEMATICAL TABLES: The editorial office of the journal maintains a repository of Unpublished Mathematical Tables (UMT). When a table is deposited in the UMT repository a brief summary of its contents is published in the section Reviews and Descriptions of Tables and Books. Upon request, the chairman of the editorial committee will supply copies of any table for a nominal cost per page. All tables and correspondence concerning the UMT should be sent to Walter Gautschi, Chairman, Editorial Committee, Mathematics of Computation, Department of Computer Sciences, Purdue University, West Lafayette, IN 47907.

Orders for subscriptions and publications of the American Mathematical Society should be addressed to the AMS, P.O. Box 1571, Annex Station, Providence, RI 02901-1571. All orders must be accompanied by payment. Other correspondence should be addressed to P.O. Box 6248, Providence, RI 02940.

MATHEMATICS OF COMPUTATION is published quarterly by the American Mathematical Society,
201 Charles Street, Providence, RI 02940. Second-class postage is paid at Providence, Rhode Island,
and additional mailing offices. Postmaster: Send address changes to Mathematics of Computation,
American Mathematical Society, P.O. Box 6248, Providence, RI 02940.

# Mathematics of Computation 

EDITED BY<br>James H. Bramble<br>Bille C. Carlson<br>Walter Gautschi, Managing Editor<br>Donald Goldfarb<br>Eugene Isaacson<br>Heinz-Otto Kreiss<br>James N. Lyness<br>Harald Niederreiter<br>Syvert P. Nørsett<br>Andrew M. Odlyzko<br>Frank W. J. Olver<br>John E. Osborn<br>Stanley Osher<br>Carl Pomerance<br>René Schoof<br>Larry L. Schumaker<br>Ridgway Scott<br>Daniel Shanks<br>Frank Stenger<br>Hans J. Stetter<br>G. W. Stewart<br>Vidar Thoméé<br>Lars B. Wahlbin<br>Hugh C. Williams<br>John W. Wrench, Jr.

VOLUME 52•1989 • NUMBERS 185-186
Providence, Rhode Island, USA
ISSN 0025-5718

## Editorial Committee

WALTER GAUTSCHI, Chairman. Dept. of Computer Sciences, Purdue Univ., West Lafayette, IN 47907
DONALD GOLDFARB, Dept. of Industrial Engineering and Operations Research, Seely W. Mudd Building, Columbia Univ. in the City of New York, New York, NY 10027
JOHN E. OSBORN, Dept. of Mathematics, Univ. of Maryland, College Park, MD 20742
HUGH C. WILLIAMS, Dept. of Computer Science, Univ. of Manitoba, Winnipeg, Manitoba, Canada R3T 2N2

## Technical Editor

ERIKA GAUTSCHI, Dept. of Computer Sciences, Purdue Univ., West Lafayette, IN 47907

## Board of Associate Editors

JAMES H. BRAMBLE, Dept. of Mathematics, Cornell Univ., Ithaca, NY 14853
BILLE C. CARLSON, Dept. of Mathematics, Iowa State Univ., Ames, IA 50011
EUGENE ISAACSON, Courant Institute of Mathematical Sciences, New York Univ., 251 Mercer Street, New York, NY 10012
HEINZ-OTTO KREISS, Dept. of Applied Mathematics, California Institute of Technology, Pasadena, CA 91125
JAMES N. LYNESS, Argonne National Laboratory, 9700 South Cass Avenue, Argonne, IL 60439
HARALD NIEDERREITER, Mathematical Institute, Austrian Academy of Sciences, Dr.-Ignaz-SeipelPlatz 2, A-1010 Vienna, Austria
SYVERT P. NØRSETT, Div. of Numerical Mathematics, The University of Trondheim and The Norwegian Institute of Technology, Alfred Getz vei 1, N-7034 Trondheim-NTH, Norway
ANDREW M. ODLYZKO, AT\&T Bell Laboratories, 600 Mountain Avenue, Murray Hill, NJ 07974
FRANK W. J. OLVER, Inst. for Physical Science and Technology, Univ. of Maryland, College Park, MD 20742
STANLEY OSHER, Dept. of Mathematics, Univ. of California, Los Angeles, CA 90024
CARL POMERANCE, Dept. of Mathematics, The Univ. of Georgia, Athens, GA 30602
RENÉ SCHOOF, Mathematisch Instituut, Rijksuniversiteit Utrecht, Budapestlaan 6, Postbus 80.010, 3508 TA Utrecht, The Netherlands
LARRY L. SCHUMAKER, Dept. of Mathematics, Vanderbilt Univ., Nashville, TN 37235
RIDGWAY SCOTT, Dept. of Mathematics, Pennsylvania State Univ., University Park, PA 16802
DANIEL SHANKS, Dept. of Mathematics, Univ. of Maryland, College Park, MD 20742
FRANK STENGER, Dept. of Mathematics, Univ. of Utah, Salt Lake City, UT 84112
HANS J. STETTER, Institut für Numerische Mathematik, Technische Universität Wien, Wiedner Hauptstrasse 6-10, A-1040, Wien, Austria
G. W. STEWART, Dept. of Computer Science, Univ. of Maryland, College Park, MD 20742

VIDAR THOMÉE, Mathematics Dept., Chalmers Univ. of Technology, S-412 96 Göteborg, Sweden LARS B. WAHLBIN, Dept. of Mathematics, Cornell Univ., Ithaca, NY 14853
JOHN W. WRENCH, JR., 102 Mt. Olivet Blvd., Frederick, MD 21701
SUBSCRIPTION INFORMATION: MATHEMATICS OF COMPUTATION is published quarterly, with issues numbered serially since Volume 1, Number 1. Subscription prices for Volumes 52 and 53 (1989) are $\$ 183.00$ list; $\$ 146.00$ institutional member; $\$ 119.00$ member of CBMS organizations; $\$ 110.00$ individual AMS member. A late charge of $10 \%$ of the subscription price will be imposed upon orders received from nonmember institutions and organizations after January 1 of the subscription year. Subscribers outside the United States and India must pay a postage surcharge of $\$ 8.00$; subscribers in India must pay a postage surcharge of $\$ 18.00$. Combination paper and microfiche subscription prices are $\$ 243.00$ list; $\$ 195.00$ institutional member. Microfiche of each issue will be mailed the fastest way before the issue is mailed by the printer.

BACK NUMBER INFORMATION: Back number prices per volume are for Volumes 1-21, $\$ 82.00$ list, $\$ 66.00$ member; for Volumes $22-33, \$ 124.00$ list, $\$ 99.00$ institutional member; for Volumes $34-43, \$ 82.00$ list, $\$ 64.00$ institutional member; Volumes $44-45, \$ 113.00$ list, $\$ 90.00$ institutional member; Volumes 46-47, $\$ 127.00$ list, $\$ 102.00$ institutional member; Volumes 48-49, $\$ 135.00$ list, $\$ 108.00$ institutional member; Volumes $50-51, \$ 136.00$ list, $\$ 109.00$ institutional member. Back volumes may be purchased on microfilm or microfiche from University Microfilms International, 300 Nortli Zeeb Road, Ann Arbor, MI 48106

UNPUBLISHED MATHEMATICAL TABLES: The editorial office of the journal maintains a repository of Unpublished Mathematical Tables (UMT). When a table is deposited in the UMT repository a brief summary of its contents is published in the section Reviews and Descriptions of Tables and Books. Upon request, the chairman of the editorial committee will supply copies of any table for a nominal cost per page. All tables and correspondence concerning the UMT should be sent to Walter Gautschi, Chairman, Editorial Committee, Mathematics of Computation, Department of Computer Sciences, Purdue University, West Lafayette, IN 47907.

Orders for subscriptions and publications of the American Mathematical Society should be addressed to the AMS, P.O. Box 1571, Annex Station, Providence, RI 02901-1571. All orders must be accompanied by payment. Other correspondence should be addressed to P.O. Box 6248, Providence, RI 02940.

MATHEMATICS OF COMPUTATION is published quarterly by the American Mathematical Society,
201 Charles Street, Providence, RI 02940. Second-class postage is paid at Providence, Rhode Island,
and additional mailing offices. Postmaster: Send address changes to Mathematics of Computation,
American Mathematical Society, P.O. Box 6248, Providence, RI 02940
Copyright © 1989, American Mathematical Society. All rights reserved. Printed in the United States of America.
The paper used in this journal is acid-free and falls within the guidelines
established to ensure permanence and durability. ()

# MATHEMATICS OF COMPUTATION <br> TABLE OF CONTENTS 

## January 1989

Susanne C. Brenner, An Optimal-Order Multigrid Method for P1 Nonconform- ing Finite Elements ..... 1
M. I. Comodi, The Hellan-Herrmann-Johnson Method: Some New Error Esti- mates and Postprocessing ..... 17
E. P. Stephan and M. Suri, On the Convergence of the $p$-Version of the Boundary Element Galerkin Method ..... 31
A. Bellen, Z. Jackiewicz, R. Vermiglio, and M. Zennaro, Natural Continuous Extensions of Runge-Kutta Methods for Volterra Integral Equations of the Second Kind and Their Applications ..... 49
Ivar Lie and Syvert P. Nørsett, Superconvergence for Multistep Collocation ..... 65
Ian H. Sloan and James N. Lyness, The Representation of Lattice Quadrature Rules as Multiple Sums ..... 81
Giuseppe Mastroianni, On the Convegence of Product Formulas for the Evalua- tion of Certain Two-Dimensional Cauchy Principal Value Integrals ..... 95
Giovanni Sacchi Landriani and Hervé Vandeven, Polynomial Approximation of Divergence-Free Functions ..... 103
David M. Smith, Efficient Multiple-Precision Evaluation of Elementary Func- tions ..... 131
Harald Niederreiter, The Serial Test for Congruential Pseudorandom Numbers Generated by Inversions ..... 135
Jürgen Eichenauer-Herrmann, Holger Grothe, and Jürgen Lehn, On the Period Length of Pseudorandom Vector Sequences Generated by Matrix Generators ..... 145
Johannes Buchmann and Attila Pethö, Computation of Independent Units in Number Fields by Dirichlet's Method ..... 149
Johannes Buchmann and David Ford, On the Computation of Totally Real Quartic Fields of Small Discriminant ..... 161
R. J. Stroeker, On Quartic Thue Equations with Trivial Solutions ..... 175
R. Creutzburg and M. Tasche, Parameter Determination for Complex Number- Theoretic Transforms Using Cyclotomic Polynomials ..... 189
Eric Bach and Jeffrey Shallit, Factoring with Cyclotomic Polynomials ..... 201
Jeff Young and Aaron Potler, First Occurrence Prime Gaps ..... 221
François Morain, On the lcm of the Differences of Eight Primes ..... 225
Daniel M. Gordon, On the Number of Elliptic Pseudoprimes ..... 231

# Reviews and Descriptions of Tables and Books <br> Johnson 1, Carey and Oden 2, Duff, Erisman and Reid 3, Lane, Galway and Alvey 4, Odeh, Davenport and Pearson, Editors 5, Press, Flannery, Teukolsky and Vetterling 6, Vetterling, Teukolsky, Press and Flannery 7, Vogel, Editor 8 <br> Supplement to "Computation of Independent Units in Number Fields by Dirichlet's Method" by Johannes Buchmann and Attila Pethö 

## Information for Contributors

Authors are encouraged to prepare articles electronically with the $\mathcal{A}_{M} S-T_{\mathrm{E}} \mathrm{X}$ software package in the AMS pre-print style and to provide the article in this electronic form for typesetting. While this procedure may not reduce the interval between submission and publication of an article, generally much more accurate copy will be returned for proofreading. Production time for manuscripts prepared with other systems, even $\mathrm{T}_{\mathrm{E}} \mathrm{X}$ itself without $\mathscr{A}_{\mathcal{M}} S-\mathrm{T}_{\mathrm{E}} \mathrm{X}$, currently prevents cost-effective use of the existing electronic form. Before sending an $A_{M} S-T_{E} X$ manuscript for typesetting, contact the AMS Composition department for details.

Manuscripts prepared by some means other than $A_{M S} S-T_{E X}$ should be doubled-spaced and produced in the format used by the journal. For journal abbreviations, see the latest Mathematical Reviews volume index. An author should submit the original and two copies of the manuscript and retain one copy. The author may suggest an appropriate editor for his paper. It is recommended that the author acquaint himself with the pertinent material contained in "A Manual for Authors of Mathematical Papers," which is available from the American Mathematical Society. All contributions intended for publication and all books for review should be addressed to Walter Gautschi, Chairman, Editorial Committee, Mathematics of Computation, Department of Computer Sciences, Purdue University, West Lafayette, Indiana 47907. The date received, which is published with the final version of an accepted paper, is the date received in the office of the Chairman of the Editorial Committee, and it is the responsibility of the author to submit manuscripts directly to this office.

Each article submitted for publication must be accompanied by a brief and reasonably selfcontained abstract, and by 1980 Mathematics Subject Classification ( 1985 Revision) numbers. If a list of key words and phrases is included, it will be printed as a footnote on the first page. A list of the classification numbers may be found in the 1984 Subject Index to Mathematical Reviews. Authors are also encouraged to supply electronic addresses when available. These will be printed after the postal address at the end of each article.

## Copying and Reprinting

Individual readers of this publication, and nonprofit libraries acting for them, are permitted to make fair use of the material, such as to copy an article for use in teaching or research. Permission is granted to quote brief passages from this publication in reviews provided the customary acknowledgement of the source is given.

Republication, systematic copying, or multiple reproduction of any material in this publication (including abstracts) is permitted only under license from the American Mathematical Society. Requests for such permission should be addessed to the Executive Director, American Mathematical Society, P.O. Box 6248, Providence, Rhode Island 02940.

The appearance of the code on the first page of an article in this journal indicates the copyright owner's consent for copying beyond that permitted by Sections 107 or 108 of the U.S. Copyright Law, provided that the fee of $\$ 1.00$ plus $\$ .25$ per page for each copy be paid directly to Copyright Clearance Center, Inc., 21 Congress Street, Salem, Massachusetts 01970. This consent does not extend to other kinds of copying, such as copying for general distribution, for advertising or promotion purposes, for creating new collective works, or for resale.

# Reviews and Descriptions of Tables and Books <br> 247 <br> Johnson 1, Carey and Oden 2, Duff, Erisman and Reid 3, Lane, Galwayand Alvey 4, Odeh, Davenport and Pearson, Editors 5, Press, Flannery,Teukolsky and Vetterling 6, Vetterling, Teukolsky, Press and Flannery7, Vogel, Editor 8 <br> Supplement to "Computation of Independent Units in Number Fields by Dirichlet's Method" by Johannes Buchmann and Attila Pethö <br> Sl 

No microfiche supplement in this issue

# MATHEMATICS OF COMPUTATION <br> TABLE OF CONTENTS <br> January 1989 

Susanne C. Brenner, An Optimal-Order Multigrid Method for P1 Nonconform- ing Finite Elements ..... 1
M. I. Comodi, The Hellan-Herrmann-Johnson Method: Some New Error Esti- mates and Postprocessing ..... 17
E. P. Stephan and M. Suri, On the Convergence of the $p$-Version of the Boundary Element Galerkin Method ..... 31
A. Bellen, Z. Jackiewicz, R. Vermiglio, and M. Zennaro, Natural Continuous Extensions of Runge-Kutta Methods for Volterra Integral Equations of the Second Kind and Their Applications ..... 49
Ivar Lie and Syvert P. Nørsett, Superconvergence for Multistep Collocation ..... 65
Ian H. Sloan and James N. Lyness, The Representation of Lattice Quadrature Rules as Multiple Sums ..... 81
Giuseppe Mastroianni, On the Convegence of Product Formulas for the Evalua- tion of Certain Two-Dimensional Cauchy Principal Value Integrals ..... 95
Giovanni Sacchi Landriani and Hervé Vandeven, Polynomial Approximation of Divergence-Free Functions ..... 103
David M. Smith, Efficient Multiple-Precision Evaluation of Elementary Func- tions ..... 131
Harald Niederreiter, The Serial Test for Congruential Pseudorandom Numbers Generated by Inversions ..... 135
Jürgen Eichenauer-Herrmann, Holger Grothe, and Jürgen Lehn, On the Period Length of Pseudorandom Vector Sequences Generated by Matrix Generators ..... 145
Johannes Buchmann and Attila Pethö, Computation of Independent Units in Number Fields by Dirichlet's Method ..... 149
Johannes Buchmann and David Ford, On the Computation of Totally Real Quartic Fields of Small Discriminant ..... 161
R. J. Stroeker, On Quartic Thue Equations with Trivial Solutions ..... 175
R. Creutzburg and M. Tasche, Parameter Determination for Complex Number- Theoretic Transforms Using Cyclotomic Polynomials ..... 189
Eric Bach and Jeffrey Shallit, Factoring with Cyclotomic Polynomials ..... 201
Jeff Young and Aaron Potler, First Occurrence Prime Gaps ..... 221
François Morain, On the lcm of the Differences of Eight Primes ..... 225
Daniel M. Gordon, On the Number of Elliptic Pseudoprimes ..... 231

