# REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS

The numbers in brackets are assigned according to the American Mathematical Society classification scheme. The 1980 Mathematics Subject Classification (1985 Revision) can be found in the December index volumes of Mathematical Reviews.

**1[65–01, 65N30].**—H. R. SCHWARZ, *Finite Element Methods*, Computational Mathematics and Applications (translated from the German by Caroline M. Whiteman), Academic Press, London, 1988, x+386 pp., 23 cm. Price $16.50 paperback.

This is an English translation of a book originally published in German in 1984. It is a descriptive introduction to finite element methods, starting with mathematical foundations, going through various elements, how to formulate the global equations, and how to solve them. Eigenvalue problems are then taken up, and the book ends with applications. Theory is given very little treatment. To quote from the Introduction: "It is intended for mathematicians, physicists, engineers and natural scientists who are interested in an elementary presentation of the methods of finite elements that has both an introductory character and gives some practical hints for efficient implementation on a computer".

Within its aims, the book succeeds admirably. The writing is both careful and lucid. The choice of material is standard but, in contrast to many other introductory books, pleasantly up to date; material from the seventies is included (and even some references to the eighties).

The book is recommended for its purpose.

L. B. W.

**2[34–01, 34C35, 65L05, 65L07].**—THOMAS S. PARKER & LEON O. CHUA, *Practical Numerical Algorithms For Chaotic Systems*, Springer, 1989, xiv + 348 pp., 24 cm. Price $49.50.

This is an unusual book: it combines components from the theory of dynamical systems, from numerical analysis, and from software engineering to achieve its purpose, which is "to present robust, reliable algorithms for simulating nonlinear dynamics". In none of the above areas, the reader is assumed to have more than a superficial knowledge; in consequence, the style remains introductory throughout. On the other hand, it is surprising and often delightful how the authors manage to acquaint the reader with important results from all these areas by viewing them in the context of their overall objective.

The use of the word "chaotic" in the book's title is a gross overstatement: no algorithms which would be particularly suitable for the analysis of chaotic systems have been presented, and within the algorithmic analysis of general dynamical systems, only two pages have been devoted to special aspects of chaotic systems. This is the more disappointing as the authors have succeeded well in introducing the reader to the mathematical and phenomenological aspects of chaotic solutions of dynamical systems. Hence, the title of the book should simply have been "Practical Numerical Algorithms for Dynamical Systems".

The first three chapters of the book are an introduction to the theory of dynamical systems, with a good deal of intuitive appeal, but sufficient mathematics to make the development quite rigorous. Autonomous and nonautonomous time-periodic systems are considered in parallel, and discrete-time systems are introduced as a preparation to Poincaré maps, which naturally form a major tool of analysis throughout. Chaos appears as a special kind of bounded steady-state behavior which becomes more distinctive in its Poincaré orbits and is finally characterized by the sign distribution of the Lyapunov exponents of its strange attractors. There are only two allusions to algorithmic tasks within this part: the location of hyperplane crossings (in Poincaré maps) and the determination of eigenvalues, for which reference is made to the QR algorithm in EISPACK or a similar subroutine library.

The next chapter is a short course on the numerical solution of systems of ODE's, with a very nice and up-to-date section dealing with the many modules, besides the integrator, which make up an ODE integration routine. This leads to the chapter on "Locating Limit Sets", which is fully in the spirit of the objectives of the book, with applications of the generalized Newton's method in various contexts.

Now the authors introduce the stable and unstable manifolds of equilibrium points, homoclinic and heteroclinic trajectories, and identify the presence of Smale horseshoes as a characteristic of unpredictive systems. Algorithms for the location of stable manifolds are discussed. A chapter on various dimension concepts is an interlude to the introduction, analysis and algorithmic determination of bifurcation diagrams. Here, too, the introductory, yet intuitive and rather rigorous level, is maintained. Specialists in the computation of bifurcation diagrams should not expect to find new material; but the book has clearly not been written for them.

The next chapter acquaints the reader with (numerical) software engineering; it is full of important remarks and observations. This is also an introduction to the package INSITE (Interactive Nonlinear Systems Investigative Toolkit for Everyone) developed by the authors. Its application to the generation of phase portraits, with limit sets and boundaries of bases of attraction, is the topic of the last chapter. Several appendices give short introductions to mathematical concepts which have been used offhand in the text.

It is difficult to characterize the audience which is likely to profit most from this experimentally-minded introduction to dynamical systems. Like the package's name indicates, it seems to be meant for "everyone". I do think that everyone interested in the subject (except the specialist) will actually find a good deal of stimulating material in some parts of the book, but may perhaps be indifferent to others. In any case, it should guide both mathematicians and application scientists to a hands-on experience with dynamical systems, which would be a very desirable effect.

H. J. S.

**3[45–01, 45A05, 65R20].**—RAINER KRESS, *Linear Integral Equations*, Applied Mathematical Sciences, Vol. 82, Springer, 1989, xi + 299 pp., 24 cm. Price $49.00.

This is truly an exciting little textbook on the functional analysis treatment of linear integral equations. In writing this text, the author was careful to select a relatively broad range of topics from the area of linear integral equations which are important to applications and whose numerical solutions are currently sought after and studied. The principles studied in the text are precisely those needed to study the error and convergence of numerical methods for approximating solutions to these problems. Understanding the principles of the text would therefore also assist the reader in selecting a good numerical method for approximating the solution to a linear integral equation problem. It is a pleasure to see these topics treated in a text. His pretty presentation of the material demonstrates the author's love for this type of mathematics.

I believe this would be an excellent contender as a text for the first two quarters of a three-quarter graduate course on the numerical solution of integral equations. The third quarter could be spent illustrating the principles covered in the book on specific problems from applications. I look forward to using this text in a class.

The following list of chapter headings gives a fairly good idea about the topics covered: 1. Normed Spaces; 2. Bounded and Compact Operators; 3. The Riesz Theory; 4. Dual Systems and Fredholm Theory; 5. Regularization in Dual Systems; 6. Potential Theory; 7. Singular Integral Equations; 8. Sobolev Spaces; 9. The Heat Equation; 10. Operator Approximations; 11. Degenerate Kernel Approximations; 12. Quadrature Methods; 13. Projection Methods; 14. Iterative Solution and Stability; 15. Equations of the First Kind; 16. Tikhonov Regularization; 17. Regularization by Discretization; 18. Inverse Scattering Theory.

F. S.

**4[65–01, 65Fxx].**—GENE H. GOLUB & CHARLES F. VAN LOAN, *Matrix Computations*, Johns Hopkins Series in the Mathematical Sciences, Vol. 3, The Johns Hopkins University Press, 2nd ed., 1989, xix + 642 pp., 24 cm. Price $59.50 hardcover, $29.95 paperback.

I was somewhat apprehensive when I hefted the second edition of *Matrix Computation* and found it heavier than the first by about one hundred and fifty pages. Too often a good book is ruined in its second edition. It grows in size, but not in substance, as the author adds extraneous topics. Lively passages are replaced by ponderously correct ones. The very structure of the book seems to weaken so that it diffuses knowledge rather than channeling it.

The second edition of *Matrix Computations*, I am happy to say, avoids these pitfalls. The first edition had quickly established itself as the definitive reference on numerical linear algebra. It was necessarily incomplete, owing to the size of the field; but it was judicious in its selection of topics, and its exercises and extensive bibliographical notes pointed the reader to areas for further study. Moreover, it was well written. All this and more can be said of the second edition.

Since the first edition was reviewed extensively when it appeared (for a review in this journal, see [1]), I will only sketch the contents of the second and then focus on the differences between the editions.

The book begins with the basics: a chapter that uses matrix multiplication to explain the issues in implementing matrix algorithms and a chapter on matrix analysis, including norms and elementary perturbation theory. The next three chapters treat linear systems—general and special—and linear least squares problems. Then follows a new chapter on parallel matrix computations. The next two chapters treat dense eigenvalue problems, unsymmetric and symmetric. The authors make their bow to sparse matrix computations with two chapters on the Lanczos and conjugate gradient algorithms and conclude with two chapters on special topics.

The changes from the first to the second edition may be arranged in four overlapping categories: new notation, new orientation, new algorithms, and a new chapter on parallel matrix computations. The notational change is the language used to describe algorithms, which is now loosely based on MATLAB. The advantage is that it allows algorithms to be written in terms of submatrices rather than scalars and fits hand in glove with the new orientation, to which I now turn.

One of the most exciting developments in matrix computations over the last decade has been the increasing abstraction with which matrix algorithms are expressed. The trend started with the basic linear algebra subprograms (BLAS) for vector operations, which were used with success in LINPACK. However, the vector BLAS did not solve the problems associated with virtual memories; and, surprisingly, they did not perform optimally on some vector computers. The second step was to introduce new subprograms, the BLAS2, that operate at the matrix-vector level; e.g., programs to multiply a matrix by a vector or add a rank-one matrix to another matrix. More recently, the emphasis has been on block algorithms that permit matrix-matrix operations as the unit of computation.

The spirit of this development completely informs the new edition. Many old algorithms have been written in a form suitable for implementation with the BLAS2, and new block algorithms are presented. In addition, there is a revised treatment of the Jacobi method and new material on the Arnoldi algorithm and the method of preconditioned conjugate gradients. If I had to select the one most significant improvement in the book, it would be this thorough reworking of the algorithms.

Only time will tell whether the authors' decision to include a long chapter on parallel matrix computations was courageous or foolhardy. It is a timely topic, and the authors serve the research community well with their clear exposition of the issues and techniques. They treat algorithms on global and distributed memory systems, with particular attention paid to the problems of synchronization and communication. However, the area is in a state of flux, and there is a chance that the chapter will have to be entirely rewritten in later editions. An unfortunate side effect of the inclusion of this chapter is that the authors' treatment of pipelined vector computing is sketchy and inconclusive. To treat both topics in detail would have overloaded the book, and the authors' choice is easily defended; but it slants the book away from the practitioner and toward the researcher.

In their exercises and bibliographical notes the authors maintain the high standard they set for themselves in the first edition. They continue the useful practice of giving full references on the spot, while also providing a complete bibliography at the end of the book. The bibliography itself has been expanded and includes references dating to just before publication. It is *the* place to begin a search for references on the topics treated in the book.

The publishers, unfortunately, have not taken the same pains as the authors. The layout is vanilla LaTeX—a serviceable format, but too much in evidence nowadays. More disturbing, the publishers use a low resolution printing device to prepare the book, and the effect of the blurry chapter and section heads may be likened to fingernails grating over a blackboard. The authors and their readers deserve better of The Johns Hopkins University Press.

But to judge by content rather than appearance, this is in every way an excellent book. It belongs on the desk of anyone with an interest in numerical linear algebra. The authors have clearly worked hard to incorporate the most recent advances in the area into the new addition, and we are greatly in their debt.

G. W. S.

1. D. F. Mayers, Review **24**, Math. Comp. **47** (1986), 376–377.

**5[65-02, 65F10, 65H10].**—JOZSEF ABAFFY & EMILIO SPEDICATO, *ABS Projection Algorithms: Mathematical Techniques for Linear and Nonlinear Equations*, Ellis Horwood Series in Mathematics and its Applications, Wiley, New York, 1989, 220 pp., $24\frac{1}{2}$ cm. Price \$59.95.

The name ABS is composed of the initials of Abaffy, Broyden, and Spedicato, the three main contributors to the development of a unified theory for a rather large class of numerical algorithms for solving systems of $m$ (linear and/or nonlinear) equations in $n$ unknowns, where $m$ is less than or equal to $n$. In the linear case these algorithms can be viewed as iterative procedures having the property that the vector produced at the $i$th iterate is a solution of the first $i$ equations. Hence a solution of the original linear system is obtained after at most $m$ iterations. The idea to solve the first $i$ equations in $i$ iterations is clearly not new. In the linear case this was used by the so-called escalator method a long time ago, and in the nonlinear case by Brown's algorithm which precedes by ten years the first paper of Abaffy from 1979. The escalator method was never recommended because of its higher computational complexity and numerical instability and, in fact, the above-mentioned paper of Abaffy was motivated by Huang's algorithm from 1975. There are other algorithms belonging to the ABS class that have been developed prior to Abaffy's paper. Among the most relevant we mention the method's of Pyle (1964) and Sloboda (1978) for linear systems, and the methods of Brent (1973), Cosnard (1975) and Gay (1975) for nonlinear systems.

The generality of the ABS class is due to a number of free parameters that can be chosen by different implementations in order to achieve different purposes. Typically, at step $i$ of an ABS algorithm, one has a current approximation of the solution and a current value of some "projection matrix". The new approximation of the solution is obtained through line search along a certain search direction such that the $i$th equation is satisfied. The search direction itself can be arbitrarily chosen from the range of the transpose of the "projection matrix", as long as it is not perpendicular on the projection of the vector formed by the coefficients of the $i$th equation. Hence the first "free parameter". Finally, the "projection matrix" itself is updated via a rank-one update which involves a second "free parameter". The whole construction resembles the one employed for quasi-Newton methods in nonlinear optimization. A further generalization is obtained by considering at each iteration a new parameter vector, called the scaling vector. The scaled ABS algorithm is equivalent to the generalized conjugate direction algorithm for linear systems of Stewart (1973) in the sense that the two algorithms generate the same search directions and approximations to the solution, but they differ in their algebraic formulation, so that their numerical behavior may differ.

The book, overall, is very well written, the exposition is clear, the proofs are elegant, and a number of carefully worded bibliographical remarks clarify the relationship between the algorithms of the ABS class and related algorithms.

In the preface the authors express their hope that the monograph justifies their opinion that "not only the ABS approach is a theoretical tool unifying many algorithms which are scattered in the literature but also that it provides promising and, in some cases, proven effective techniques for computationally better algorithms for a number of problems". While being in complete agreement with the authors on the first statement, we have some reservation concerning the second one. This does not mean that no competitive software for linear and nonlinear systems could be based on the ABS approach. We only want to say that there is still a lot of work to be done.

FLORIAN A. POTRA

Department of Mathematics
University of Iowa
Iowa City, Iowa 52242

**6[65H10, 65L10, 65M60, 58C27].**—HANS D. MITTELMANN & DIRK ROOSE (Editors), *Continuation Techniques and Bifurcation Problems*, International Series of Numerical Mathematics, Vol. 92, Birkhäuser, Basel, 1990, 218 pp., 24 cm. Price $52.00.

This volume of invited articles addresses aspects of the computational analysis of parameter-dependent nonlinear equations. Twenty-seven authors, all active in the field, have contributed thirteen papers which may be loosely categorized into four groups.

The first group concerns numerical continuation techniques for problems with one real parameter. E. L. Allgower, C. S. Chie, and K. Georg discuss continuation in the case of large sparse systems. Continuation methods for variational inequalities are treated in papers by E. Miersemann and H. D. Mittelmann, and—using multigrid approaches—by R. H. W. Hoppe and H. D. Mittelmann. The applications of continuation procedures to partial differential equations modeling semiconductor devices are studied by W. M. Coughran, Jr., M. R. Pinto, and R. K. Smith. A continuation process involving damped Newton methods is analyzed by R. E. Bank and H. D. Mittelmann.

The papers in the second group involve symmetry in the study of bifurcation phenomena. The computation of symmetry-breaking bifurcation points for a class of semilinear elliptic problems is discussed by C. Budd. M. Dellnitz and B. Werner show how group-theoretic methods can be employed in the detection of bifurcation points and the computation of (multiple) Hopf points. For a two-parameter problem, symmetry is used by A. Spence, K. A. Cliffe, and A. D. Jepson in the computational determination of branches of Hopf points.

The third group consists of papers on the computational analysis of higher singularities. A. Griewank and G. W. Reddien develop a method for the computation of cusp catastrophes for steady-state operator equations and their discretizations. Numerical experiments on the interaction between fold points and

Hopf points in certain two- and three-parameter problems are presented by B. DeDier, D. Roose, and P. VanRompay. C. Kaas-Petersen examines the Gray-Scott model of isothermal autocatalytic processes when the standard symmetry is broken by unequal boundary conditions and events with higher codimension occur.

Finally, the fourth group of papers concerns parameter-dependent time-dependent systems. The computation of heteroclinic orbits connecting two saddle points is discussed by E. J. Doedel and M. J. Friedman, and E. Lindtner, A. Steindl, and H. Troger study the loss of stability of the basic periodic motion of a robot.

The results in this volume certainly provide an interesting contribution to this very active area.

W. C. Rheinboldt

Department of Mathematics and Statistics
University of Pittsburgh
Pittsburgh, Pennsylvania 15260

**7[65-01, 65D07, 65L60, 65N30].**—P. M. Prenter, *Splines and Variational Methods*, Wiley Classics Edition, Wiley, New York, 1989, xi + 323 pp., 23 cm. Price: Softcover $24.95.

This is a reprint of a book first published in 1975 and now elevated to the status of a "classic", in good company with other books in the Wiley Classics Library such as Courant-Hilbert, Curtis-Reiner and Dunford-Schwartz. It is intended as an introduction to the subject of its title, aimed at first-year graduate students in engineering and mathematics. To quote from the Preface: "... to introduce them gradually to the mathematician's way of thinking ... "; "... a book on the subject that could be read by ordinary mortals ... ". Basic concepts of one-dimensional and multivariate polynomial and piecewise polynomial interpolation are covered and then finite element and collocation methods for differential equations. Concepts, e.g., from elementary functional analysis, are introduced as needed.

The book is much in the spirit of Strang and Fix's influential 1973 book [3], although it does not try to cover the then research frontier as [3] did. Prenter's book includes more of "classical" approximation theory.

It is a very pleasant and well-written book. However, some parts of it have not aged well in the decade and a half since its publication, and it cannot now be used as a textbook. I proceed to give two examples of why.

First, although there is a "guest" reference to Bramble and Hilbert 1970 on p. 271, the Bramble-Hilbert lemma is not stated or used. Consequently, error estimates for multivariate approximation are restricted to the maximum (Tchebycheff) norm, although they are later applied to energy and $L_2$ estimates. Excessive, in many applications fatal, smoothness demands result. Also, as was common before the Bramble-Hilbert lemma, the estimates in approximation theory are slugged out on a tedious case by case basis (which sometimes may give more explicit constants than use of the Bramble-Hilbert lemma). As the author remarks, p. 127: "Waning sadism forbids us to go further", which can be taken as a nice comment on the effort-, ink-, and tree-saving role of the Bramble-Hilbert lemma. (That role was duly appreciated at the time, e.g., in [3, p. 146].)

Secondly, in the analysis of the finite element method in one space dimension, the author derives maximum norm error estimates that are off by one full order of accuracy, pp. 215 and 221–222. The case on pp. 221–222 was actually solved by Wheeler [4] in 1973, while the general case, including that on p. 215, came later [1].

Finally, reprinting good books is an idea that deserves strong applause from the mathematical community. In numerical analysis, will Richtmyer and Morton's 1967 book [2] be next?

L. B. W.

1. J. Douglas, Jr., T. Dupont, and L. B. Wahlbin, *Optimal $L_\infty$ error estimates for Galerkin approximations to solutions of two-point boundary value problems*, Math. Comp. **29** (1975), 475–483.

2. R. D. Richtmyer and K. W. Morton, *Difference methods for initial-value problems*, 2nd ed. Interscience, New York, 1967.

3. G. Strang and G. J. Fix, *An analysis of the finite element method*, Prentice-Hall, Englewood Cliffs, N.J., 1973.

4. M. F. Wheeler, *An optimal $L_\infty$ error estimate for Galerkin approximations to solutions of two-point boundary value problems*, SIAM J. Numer. Anal. **10** (1973), 914–917.

**8[41-02, 41A29, 41A50, 41A52, 41A65, 65D15].**—Allan M. Pinkus, *On $L^1$-approximation*, Cambridge Tracts in Mathematics, Vol. 93, Cambridge Univ. Press, Cambridge, 1989, x + 239 pp., $23\frac{1}{2}$ cm. Price $44.50.

We welcome this book as the first comprehensive monograph on approximation in the mean. It merits much praise for being all that such a work should be: it takes a global viewpoint and proceeds systematically and efficiently through the entire subject. All the classical results are here—often in generalized form and with improved proofs. Fully half the book is devoted to the progress made in the last ten years. The author has played a leading role in all this recent activity and is uniquely qualified to be its chronicler and interpreter.

Mean approximation (or $L^1$-approximation) is the problem of minimizing the expression $\int |f - u|$ by choosing $u$ from some given class of functions. Here, $f$ is a function to be approximated, and the integral is over a fixed measure space. A typical example occurs when $f$ is a continuous function on a closed and bounded interval of the real line, and $u$ is chosen from the family of cubic spline functions with a prescribed set of knots. In this case the integral could be the usual Lebesgue or Riemann integral on the interval, but much more general measures are admitted in the theory.

In the general theory, if the approximating function $u$ is further constrained to satisfy (for all $x$) $f(x) \leq u(x)$, the problem is, of course, changed, but the theory for such one-sided $L^1$-approximations is well developed.

The first Chapter ("Preliminaries") gives a succinct account of basic approximation theory. It includes characterization theorems for best approximations in normed linear spaces, allowing arbitrary convex sets of approximants. These theorems are couched in two different forms—either with separating linear functionals or with one-sided Gateaux derivatives. The author includes the recent theory of "strong unicity".

Chapter 2 concerns approximation from finite-dimensional subspaces in a space $L^1(B, \Sigma, \nu)$, where $(B, \Sigma, \nu)$ is a $\sigma$-finite measure space. This includes as a special case the space $l^1$, and particular results for that space are mentioned. In these general spaces, best approximations are usually *not* unique, and continuous best-approximation maps usually do not exist.

The bad features of the problems discussed in Chapter 2 are largely avoided in Chapter 3, by moving the setting of the problems to the normed linear space $C_1(K, \mu)$. This space consists of continuous functions on a compact space $K$, but the norm is the $L^1$ norm induced by a nonatomic measure, $\mu$. The topology and the measure are related by requiring that all nonempty open sets be measurable and have positive measure. Characterization and unicity of best approximations are dealt with, as are continuous best-approximation maps.

In Chapter 4, the unicity question is investigated in greater detail in order to discover subspaces that have the unicity property for large classes of measures. Splines and their generalizations provide interesting examples of the theory developed here.

Chapter 5 is devoted to one-sided approximation, and Chapter 6 to discrete $L^1$-approximation, i.e., approximation in $l_1^m$.

Chapter 7, of 40 pages, is devoted to algorithms for $L^1$-approximation. Here the discussion is aimed at strategies and procedures, not computer programs. Methods of descent, linear programming, discretization, and the Newton procedure are all discussed.

The book closes with two lengthy appendices, dealing principally with Chebyshev systems and weak Chebyshev systems of functions. At the end of each chapter there are copious notes and references to the literature. A bibliography of eight pages is included.

Altogether, this monograph is an important addition to the literature on approximation theory.

E. W. Cheney

Department of Mathematics
University of Texas
Austin, Texas 78712

**9[41-06, 41A63].**—C. K. Chui, W. Schempp & K. Zeller (Editors), *Multivariate Approximation Theory*, International Series of Numerical Mathematics, Vol. 90, Birkhäuser, Basel, 1989, ix + 342 pp., 24 cm. Price $63.00.

Interest in Multivariate Approximation Theory has increased considerably in the past few years because of its important applications in diverse areas of science and engineering. Multivariate approximation methods have always been important for data interpolation and approximation, cubature, and for the numerical solution of boundary value problems (the finite element method). More recently, multivariate approximation methods have assumed an important role in CAD/CAM (computer-aided design and manufacture), robotics, image processing, pattern recognition, signal processing, and elsewhere.

The keen interest in the field has resulted in a considerable increase in both the number of publications in the area (several new journals have been started in the past few years), and in the number of conferences being held on the subject. The book under review is the proceedings of one such recent conference held at Oberwolfach, Germany from February 12 to 18, 1989.

The book contains 37 complete research papers. Eight of the papers deal with multivariate splines of one sort or another (for example, the dimension and construction of local bases, box splines, blending, vector spherical splines, polyharmonic splines, and exponential eigensplines). Another group of seven papers deals with periodic interpolation and approximation (including sampling theorems, cardinal interpolation, Fourier integrals and transforms, trigonometric operators, convolution methods, Hermite-Birkhoff interpolation, and Bochner-Riesz means). Several papers deal with polynomials and rational functions ( $L_p$ approximation, Bernstein methods, quasi-interpolation). Two papers deal with radial basis functions, and two with band-limited functions. There are also papers on cubature, wavelets, and on holograms and neural networks. The book will be of interest to approximation theorists and to all parties who make use of approximation methods.

L. L. S.

**10[41-01, 41A15, 68U05].**—Su Bu-Qing & Liu Ding-Yuan, *Computational Geometry: Curve and Surface Modeling*, Academic Press, Boston, 1989, x + 295 pp., $23\frac{1}{2}$ cm. Price $39.95.

Mathematical methods for dealing with curves and surfaces are important tools in a number of traditional areas (such as data interpolation and fitting, quadrature and cubature, and numerical solution of operator equations such as ordinary and partial differential equations, integral equations, etc.). More recently, they have become increasingly important in several newer, rapidly developing areas such as CAD/CAM (computer-aided design and manufacture), robotics, and image processing. This increased interest has spawned numerous publications, several new journals, a plethora of conferences, and several books.

The book under review treats mathematical properties of some function classes which are particularly useful for curve and surface *modelling*. It is a translation of a book originally published in Chinese some ten years earlier. The book can be read by anyone acquainted with calculus and linear algebra (but of course will be better appreciated by those with some knowledge of numerical analysis and approximation theory). It should be of special interest to both researchers and practitioners working with CAD.

The first chapter of the book is an introduction to the field. Except for one short chapter on surfaces, the remainder of the book deals with planar curves. The second chapter introduces splines, concentrating on the cubic and quartic cases. Chapters 3 and 4 discuss parametric and Bézier methods for fitting and modelling curves in the plane. Bicubic splines and Coon's and Bézier patch methods are treated in Chapter 5 on surfaces. The remaining three chapters deal with nonlinear splines (including the mechanical spline), with curve and net fairing, and with affine invariants of parametric curves.

Despite the fact that the book was written more than ten years ago, I think it will be a very useful supplement to more recent books on splines, curve and surface fitting, and computer-aided geometric design. In particular, there is a great deal of information about affine invariants and the characterization of inflection points and singular points (leading to loops and cusps) which is not treated in other books. In general, the authors have adhered to modern notation and terminology (referring to the control polygon as a characteristic polygon is one exception), and the translator has done an excellent job of rendering the text in very readable smooth English. The book has very few obvious misprints, and benefits from a large number of well-drawn figures.

L. L. S.

**11[41–02, 41A60].**—R. Wong, *Asymptotic Approximation of Integrals*, Computer Science and Scientific Computing, Academic Press, Boston, 1989, xiii+543 pp., $23\frac{1}{2}$ cm. Price \$69.95.

J. E. Littlewood once remarked on the fading—with the growing emphasis on rigor—of the "aroma of paradox and audacity" that had pervaded the subject of divergent series—those curious expansions that were usually divergent but nevertheless from which information could be obtained. It is true that the subject now resides on a firm theoretical foundation, but for most of us the scent still clings to what for Abel was "the invention of the devil." Some of this mystery is shared by asymptotic expansions—typically a kind of divergent series—and probably constitutes a large part of their appeal: they represent a let-it-all-hang-out approach to mathematics, a swift uppercut to the fatiguing demands of rigor. Asymptotics constitutes a collection of powerful tools, and if we are sometimes a little careless in devising and using them, perhaps they provide us with what we secretly want: to be both naughty and effective.

There are still vital matters to be resolved in asymptotic analysis. At least one widely quoted theory, the asymptotic theory of irregular difference equations expounded by G. D. Birkhoff and W. R. Trjitzinsky [5, 6] in the early 1930's, is vast in scope; but there is now substantial doubt that the theory is correct in all its particulars. The computations involved in the algebraic theory alone (that is, the theory that purports to show there are a sufficient number of solutions which formally satisfy the difference equation in question) are truly mindboggling. (For some modern comments on this venerable work, see [26].) A large part of the literature dealing with the uses of asymptotic expansions, particularly the literature from the physical sciences, dispenses with rigor altogether. The mere fact that one or two terms of the expansion can somehow be produced is to be accepted as sufficient justification that the rest of the expansion exists and represents asymptotically the quantity in question. This approach, and the results it displayed, is sometimes called *formal*, although mathematicians whose philosophical view of mathematics puts them in the camp of formalism would strenuously object to the term. A more accurate description would be: lacking in rigor. Suffice it to say that very much of what we know about the physical world comes to us through the use or maybe even misuse of asymptotics. Asymptotics are—to paraphrase Bismarck's observation about laws and sausages—always in demand, but we'll sometimes sleep better if we don't know how they're made.

The expression *asymptotic analysis* often implies the process of obtaining a single term describing the behavior of the quantity in question. Obtaining an expression for the coefficients in the complete asymptotic expansion if, indeed, one exists, is in most physical problems simply too difficult.

Wong's book provides ingenious examples to remind us that a totally casual approach to asymptotics will not work as a general policy. Strange, even illogical, results may be a consequence of the most straightforward methods. Quixotic results can occur when one is dealing with asymptotic scales more general than the Poincaré scale, for instance. Sometime ago I gave the following example [53]. Let

$$\Gamma(1+t) = \sum_{n=0}^{\infty} (-1)^n a_n t^n ,$$

the Taylor series converging for $|t| < 1$. It can be shown that

$$a_n - \sum_{j=0}^{k-1} \frac{(-1)^j}{j!(j+1)^{n+1}} = O[(k+1)^{-n}], \qquad n \to \infty,$$

in other words,

$$a_n \sim \sum_{k=0}^{\infty} \frac{(-1)^k}{k!(k+1)^{n+1}}, \qquad n \to \infty.$$

This is a perfectly valid asymptotic series; but not with the ordinary Poincaré scale $\{n^{-k}\}$, rather with scale $\{(k+1)^{-n}\}$. It is also a convergent series. *But it does not converge to* $a_n$.[1]

[1] This phenomenon almost never occurs with the Poincaré scale, since a sequence having a convergent asymptotic series in this scale may usually be considered a restriction to integer arguments of a function analytic at $\infty$.

Nonsense can result when one applies standard methods in a facile way, neglecting the error term. In his book Wong gives an example where two different widely used methods give incorrect results, both applied to the function

$$S(x) = \int_0^\infty \frac{1}{(x+t)(1+t)^{1/3}}\,dt, \qquad x > 0.$$

Repeated integration by parts gives us

$$S(x) \sim -\sum_{n=1}^{\infty} \frac{3^n (n-1)!}{2 \cdot 5 \cdot \cdots \cdot (3n-1)} x^{-n}, \qquad x \to \infty,$$

which is obviously incorrect, since $S(x)$ is positive. And termwise integration using the series

$$(1+t)^{-1/3} = \sum_{s=0}^{\infty} \binom{-1/3}{s} t^{-s-1/3},$$

also gives an incorrect (though less obviously so) answer

$$S(x) \sim \frac{2\pi}{\sqrt{3}} \sum_{s=0}^{\infty} (-1)^s \binom{-1/3}{s} x^{-s-1/3}.$$

The correct expansion is the *sum* of the two expansions above, and can be obtained by the use of distribution theory. (Another way of dealing with this integral is to let $t = u/(1-u)$ to make the identification

$$S(x) = \frac{3}{x} {}_2F_1\left( \begin{matrix} 1, 1 \\ 4/3 \end{matrix} ; 1 - \frac{1}{x} \right),$$

and then use connecting formulas for the Gaussian hypergeometric function.)

Asymptotics are as invaluable in analysis as in the physical sciences. Almost all of what we know about the theory of polynomials orthogonal on the real line is due to our detailed asymptotic knowledge of the three-term recurrence that such polynomials satisfy, see [37, 51]. Analytic number theory is indebted to and has the debt of asymptotics; one of the most elegant methods—the circle method of Rademacher [2], for instance—was introduced to study the growth of arithmetical functions. The asymptotics of nonlinear recurrences are crucial in the resolution of important questions in orthogonal polynomials originally raised by G. Freud. (See [3] and the references given there, and the references in *those* references. This has been a hot topic recently! See also [32].)

Books on the subject have always had a highly personal flavor. Some, like those by the mathematical physicists Jeffreys and Heading [28] and [22], emphasize methods immediately applicable to the physical sciences, like the method of stationary phase, so important in wave mechanics. Copson's book [13], because of his interest in complex variables, stresses the method of steepest descents. The earlier books tended to treat specialized subjects. Ford [20] was concerned with the algebraic properties of asymptotic series, and the asymptotic behavior induced on an entire function when the form of its Taylor coefficients was prescribed.

The number of books seems to be growing exponentially. Obviously, publishers have found them a very marketable item. They have become increasingly specialized. Several recent books emphasize differential equations, either from the point of view of systems theory or turning point problems [19, 43, 47, 48, 52]. Some deal with abstruse and specialized topics, like applications of nonstandard analysis to fields of formal series, or to fiberings over manifolds [4, 27, 31]. Others treat the asymptotics of eigenvalues of Toeplitz matrices [9, 24], or of difference equations [26]. Asymptotics are becoming increasingly important in queuing theory, number theory, and statistics [1, 10, 14, 16, 25, 33]. One of the most exciting recent developments is the application of statistical methods for asymptotic estimation to problems in analysis. A good discussion of these techniques can be found in Van Assche's book [51], where he shows how a remarkably ingenious application of the local limit theorem for lattice distributions (for example, see Petrov [40]) serves to generate the asymptotics for many of the classical orthogonal polynomials. My experience with these methods has been less than completely satisfactory; I have found it difficult to aim the methods in the right direction, in other words, to define the probability space and the associated distribution functions so that the theory generates the desired asymptotics. In general, the method gives you what it chooses to give you, not necessarily what you want.

Matched asymptotic expansions has become a popular topic recently, with an inevitable brood of books [17, 29]. The literature swells when one considers that perturbation theory is nothing more than a disguised form of asymptotics (for $\varepsilon$ read $1/n$ or $1/z$), special in its application to differential equations. The books [36, 38] are good examples of some of the recent publications in the field. For numerical applications see [23].

Some recent books so completely reflect an engineer's viewpoint, or are so mired in a specialized set of physical problems, that nonspecialists may well find them meaningless [8, 21, 35, 44, 46, 50, 54]. There is at least one book on the asymptotic analysis of ocean currents [45]! Almost every physical discipline—fluid mechanics, wave propagation, etc.—seems to have spawned its own set of ad hoc techniques for asymptotic analysis, some of them resting on fragile mathematical foundations. I have the feeling that the territories of these disciplines are never breached, and these scientists will go from one rediscovery to another, carrying—like Sisyphus—the same stone again and again up the hill.

Consequently, the need is greater now than ever before for balanced, comprehensive accounts of the progress we have made in asymptotic methods. For the reader interested in a general treatment, there are some superb books to choose among: [11, 15, 18, 22, 28, 30, 34, 39, 41, 49]. The reader will note that the books are getting larger, simply because the field has developed so rapidly. Jeffrey's 1962 book is a mere 140 pages, Heading's only 160; but Olver's, published in 1974, is 560 pages. Each of the books in the above list has its own virtues. Of these I believe Olver's is the most useful because it is current, it covers such an abundance of material and it is well organized and beautifully written.

Now the above are joined by another comprehensive reference, namely, the present work. The title suggests that it is just another treatise on a specialized topic. But that is far from the truth. Many problems in asymptotics having little apparent connection with integrals can be formulated in terms of integrals by the exercise of a little ingenuity, for instance, the asymptotics of the partition function $p(n)$ can be obtained by an application of the circle method to a contour integral representation for $p(n)$. The computations are made possible by the use of a $q$-function identity. Often an integral transform may be obtained for the quantity in question, and this integral may then be examined by a variety of techniques.

Books on asymptotics differ widely in their approach to rigor. The book by de Bruijn [11] has intermittent lapses of rigor, is occasionally reduced to hand waving and foot shuffling. The book by Erdélyi [18] is fastidiously rigorous. Yet both are compulsively readable. In this subject there seems to be little relationship between rigor and readability. The problems de Bruijn chooses to attack are uncommonly interesting, and his book has been very influential. In few other fields of mathematics, in fact, are the standards of exposition so imposing; and the book by Wong joins the best of these. Because he comes down soundly on the side of rigor, his arguments are more difficult than they would otherwise be, but the added complexity is mostly offset by the great clarity of the writing and organization. Wong's book is not the first to concentrate on the asymptotic estimation of integrals—see [7, 12, 42]—but it is, I think, by far the best.

Some of the material in Wong's book is covered in other books. But there are a number of ways that the book makes a unique and substantial contribution to the literature. The book starts with a chapter on the fundamentals of asymptotics. This chapter deserves high praise. It can be understood by a good calculus student; yet the examples are provocative. The reader will get an idea of the tenor of the writing from the following passage:

"In analysis and applied mathematics, one frequently comes across problems concerning the determination of the behavior of a function as one of its parameters tends to a specific value, or of a sequence as its index tends to infinity. The branch of mathematics that is devoted to the investigation of these types of problems is called asymptotics. Thus, for instance, results such as

$$\log n! \sim (n+1/2)\log n - n + 1/2\log 2\pi,$$

$$H_n \equiv 1 + \frac{1}{2} + \frac{1}{3} + \cdots + \frac{1}{n} \sim \log n,$$

and

$$L_n \equiv \frac{1}{\pi}\int_0^\pi \frac{|\sin(n+\frac{1}{2})t|}{\sin\frac{1}{2}t}\,dt \sim \frac{4}{\pi^2}\log n,$$

are all part of this subject."

Notice how the author has stated formulas before he has defined what "$\sim$" means. A reader unfamiliar with the field will infer from context what asymptotic equality means, and the introduction provides what is really needed in the beginning of any viable treatment of the subject: examples to motivate and intrigue. A complete array of definitions soon follows. Then the author discusses generalized asymptotic expansions and some basic techniques for deriving them, including integration by parts, the Euler-Maclaurin summation formula, and Watson's lemma.

The second chapter discusses the classical procedures for obtaining asymptotic representations from integrals, including Laplace's method, the principle of stationary phase, and Perron's method. There follows a welcome discussion of Darboux's method and the formula of Hayman, methods for deducing the asymptotics of the Taylor coefficients of a function, and not directly related to problems involving integrals.

In many instances the function whose asymptotics are desired has a representation as an integral transform. Much of what has been accomplished on this problem is due to the author, and the discussion in Chapters 3 and 4 constitutes the most extensive resource in the literature for dealing with Mellin, Fourier and Hankel transforms.

Chapter 5 treats the theory of distributions. Since many applied mathematicians are still unfamiliar with this material, the author provides a self-contained treatment—one of the most readable treatments I have seen. (It could easily be used as a text in a discussion of the subject in a real variable course.) The results obtainable with the distributional approach are many and varied, and every applied mathematician should consider such methods part of his set of tools. In Chapter 6 the author applies the previous theory to Stieltjes transforms, Hilbert transforms, Laplace and Fourier transforms, and fractional integrals. Although Wong makes no special claims for himself in having initiated the use of distributional tools and attributes much of the work in this chapter to writers such as Lighthill, Jones, Durbin and Zayed, he has, in fact, been a very productive researcher in the area, particularly in association with J. P. McClure.

Chapter 7 treats the difficult problem of obtaining uniform asymptotic expansions, for example, in the case where the integrand has coalescing saddle points. The most general kind of an integral which occurs is

$$I(\lambda) = \int_C g(t; \alpha_1, \alpha_2, \ldots, \alpha_m) e^{-\lambda f(t; \alpha_1, \alpha_2, \ldots, \alpha_m)}\, dt.$$

It is an extremely difficult task to obtain asymptotic expansions in $\lambda$ which are uniform in the $\alpha_i$'s. The author discusses several simple (but important) situations, namely those in which two or three of the critical points of the integrand (zeros of $f'$) are allowed to approach each other. A number of illuminating examples are given, including applications to Bessel functions and Laguerre polynomials. Despite the usefulness of results due to Chester, Ursell, Friedman, Olver, Erdélyi, and others, it must be admitted that the derivation of uniform expansions is still as much of an art as a science; most situations have to be treated on an ad hoc basis, and the analysis of even simple examples usually involves a major computational effort. The discussion in the present book, along with the relevant portions of Olver's book (where the problem is formulated in terms of differential equations), constitutes a major addition to the literature.

Double and higher-dimensional integrals are treated in Chapters 8 and 9. In Chapter 8 the behavior of double integrals of the form

$$I(\lambda) = \iint_D g(x, y) e^{i\lambda f(x, y)} \, dx \, dy$$

is investigated. One wishes such integrals and all their associated complexities—degenerate cases, boundary critical points, curves consisting entirely of stationary points—would just go away. But unfortunately such integrals are crucial in many disciplines, including, nowadays, combinatorics. Higher-dimensional integrals are discussed in Chapter 9; one of the most frequently occurring cases is that of multiple Fourier transforms. This is the first time the material has been treated fully in a reference work, and it is very welcome. For a number of years I have been hounded by engineer colleagues who wanted asymptotic analyses of multidimensional integrals. And now I have a source of nearly one hundred pages on the subject to refer them to.

The book has many virtues, as I have mentioned. One of the most striking is one of the least expected: the exercises. No other book, past or present, can approach Wong's book in its profusion of carefully selected exercises. The exercises vary from the quick and easy to the research-problem variety. Where the going gets tough (regularization of divergent integrals, tempered distributions) the number of exercises actually increases, rather than decreases. Only a writer profoundly familiar with his field could lavish such pedagogical goodies on his readers.

The book is beautifully bound and printed, in the best standards of Academic Press, and replete with a sixteen-page bibliography, a list of symbols (highly useful), and a thorough index. Every applied mathematician (indeed most physical scientists) will want to own this book. There is simply no other book that does as much as it does as well as it does.

JET WIMP

Department of Mathematics and Computer Science
Drexel University
Philadelphia, Pennsylvania 19104

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**12[65-04].**—William H. Press, Brian P. Flannery, Saul A. Teukolsky & William T. Vetterling, *Numerical Recipes in Pascal—The Art of Scientific Computing*, Cambridge Univ. Press, Cambridge, 1989, xxii + 759 pp., 24 cm. Price $47.50.

This is a Pascal version of the original *Numerical Recipes*, published in 1986 and reviewed in [1]. To quote from the authors' Preface: "Pascal was not, of course, entirely neglected in the original version of *Numerical Recipes.* An Appendix, at the back of that book, contained Pascal translations of all the FORTRAN subroutines and functions. These translations were workable, but not very stylish, and they were printed in a condensed, unreadable format, without comments. Many Pascal users let us know, in no uncertain terms, what they thought of that!

"In this edition, therefore, all of the procedures have been completely rewritten, in a consistent Pascal style, one which makes use of pointers, dynamic memory allocation, and other features not found in the original FORTRAN subroutines."

W. G.

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**13[65-01].**—Francis Scheid, 2000 *Solved Problems in Numerical Analysis*, Schaum's Solved Problems Series, McGraw-Hill, New York, 1990, v + 698 pp., $27\frac{1}{2}$ cm. Price: Softcover $19.95.

This is an extensive collection of problems in numerical analysis, at the upper undergraduate or beginning graduate level, with complete solutions provided for each problem. The coverage is rather complete, although differential equations (ordinary and partial) are underrepresented relative to their importance. The extent of coverage can be gathered from the following summary: Machine arithmetic and errors (Chs. 1–2): 28 pages; Summation of series (Ch. 9): 45 pages; Interpolation (by polynomials and spline functions, Chs. 3–6): 128 pages; Numerical differentation and integration (Chs. 7–8): 101 pages; Approximation (least squares, minimax and $L_1$, by polynomials, rational, and trigonometric functions, Chs. 12–15 and 19): 124 pages; Numerical linear algebra (linear systems, matrix inversion and eigenvalues, Ch. 17): 57 pages; Nonlinear equations (Ch. 16): 35 pages; Optimization (Ch. 18): 29 pages; Difference equations (Ch. 10): 36 pages; Ordinary differential equations (initial and boundary value problems, Ch. 11 and Ch. 20.1–4): 85 pages; Partial differential equations (boundary value problems, Ch. 20.5–8): 17 pages; Monte Carlo methods (Ch. 21): 8 pages. There is a 4-page subject index, but no bibliography.

The majority of problems are of the pencil-and-paper and desk computer variety. The inclusion of substantial computer assignments, and more emphasis on questions of numerical stability that such assignments could help illustrate, would have enhanced the value of this collection. Even so, it represents a rich source of problems. Instructors and students alike will benefit from it, the former in selecting homework problems, the latter for reviewing a particular subject area and acquiring the necessary problem-solving skills in it.

W. G.

**14[65U05, 40A15, 65B10, 41A21].**—K. O. BOWMAN & L. R. SHENTON, *Continued Fractions in Statistical Applications*, Statistics: Textbooks and Monographs, Vol. 103, Dekker, New York, 1989, x + 330 pp., $23\frac{1}{2}$ cm. Price \$89.75.

This book is a unique addition to the relatively small collection of books on continued fractions. It is also probably the only book devoted to the use of continued fractions in statistical applications. It presents a very readable and enticing account of the authors' extensive research involving application of continued fractions and Padé sequences to convergent and divergent series representations that occur in statistics. The emphasis throughout the book is on computational accuracy with modest computational resources. A valuable and unique feature of the book is that it points out the potential usefulness of computer algebra systems such as MACSYMA or REDUCE for obtaining expressions. Students will find the book very useful because many insightful comments are given as results are developed. Results are not just presented, but motivated and explained in a very personal style. The student reading through the many comments and examples will gain numerical analytic experience and knowledge that are usually only obtained through many hours or years of computational experiment.

The book is self-contained and incorporates the necessary results from many interesting references. It places a great amount of emphasis on the work of Stieltjes. It starts out with an introduction, using statistical examples, to continued fractions, summability theory, and the moment problem. These basic concepts are then successively extended to increasingly complex applications. Complete and computationally detailed examples are used extensively in the book and can serve as references for many applications in statistics. The book does not consider multivariate problems, as the authors found that there are many outstanding problems in the univariate situation. This book should be required reading for all statisticians, and will be informative reading for numerical analysts and engineers. For the mathematically inclined the book reads like a novel, a very good novel.

MARIETTA J. TRETTER

Business Analysis and Research
Texas A & M University
College Station, Texas 77843

**15[62-04].**—PETE DIGBY, NICK GALWEY & PETER LANE, *Genstat* 5: *A Second Course*, Clarendon Press, Oxford, 1989, xi + 233 pp., $24\frac{1}{2}$ cm. Price \$65.00 hardcover, \$29.95 paperback.

This book is a follow-up to *Genstat 5–An Introduction* [1] . It introduces the use of GENSTAT for advanced statistical techniques that the program can carry out directly, and describes how to write procedures in the GENSTAT command language, enabling the user to define new commands in order to perform operations or analyses for which no canned software is available.

The advanced statistical techniques discussed include regression analysis of grouped data, nonlinear regression, analysis of experimental designs, analysis of contingency tables, principal component analysis, principal coordinate analysis, cluster analysis, and time series analysis.

Unfortunately, the book makes a common mistake of most introductions to the use of statistical software: it gives a pseudo-introduction to the statistical technique, instead of providing appropriate references to textbooks and explaining how GENSTAT can be used to do the numerical calculations. Everyone who has seen students "learn" statistics from an SPSS handbook knows what I mean. This is particularly evident in Chapter 5, entitled "The analysis of variation in several variables", where principal component analysis is introduced and illustrated in a way that will be incomprehensible to the novice, and redundant to those who know the basic notions on which the method is based. In contrast, the more technical chapters on how to write programs and procedures in the GENSTAT language appear to be readable and useful.

BERNHARD FLURY

Department of Mathematics
Indiana University
Bloomington, Indiana 47405

1. Peter Lane, Nick Galwey & Norman Alvey, *Genstat 5–An Introduction*, Clarendon Press, Oxford, 1987. {Review **4**, Math. Comp. **52** (1989), 252.}

**16[94-01, 94Bxx, 05B05].**—VERA PLESS, *Introduction to the Theory of Error-Correcting Codes*, 2nd ed., Wiley, New York, 1989, xiii + 201 pp., $23\frac{1}{2}$ cm. Price \$34.95.

This is the second edition of a book which has already secured a prominent place in the textbook literature on algebraic coding theory. The attractive feature of the first edition, namely that of providing a leisurely introduction to the field which is at the same time mathematically rigorous, is again present here. New sections on BCH codes, on Reed-Muller codes, and on the binary Golay code have been added in the second edition, and the bibliography has been updated. There is a certain stress on cyclic codes, and there is ample material on self-dual codes, a subject that is apparently close to the heart of the author. An appendix contains the necessary background in linear algebra. The author manages to convey the beauty and the elegance of the subject, and she writes in an engaging style. The book can be strongly recommended as a course textbook and for self-study. My only criticism concerns the layout which is not optimal: for instance, on pp. 126–127 the statement of Theorem 80 is interrupted by Table 8.1, and on pp. 160–161 the statement of Theorem 97 is interrupted by Table 9.3.

H. N.

**17[62-01, 62J10, 62Kxx, 65Fxx].**—Richard M. Heiberger, *Computation for the Analysis of Designed Experiments*, Wiley Series in Probability and Mathematical Statistics—Applied Probability and Statistics, Wiley, New York, 1989, xv + 683 pp., 24 cm. Price $59.95.

ANOVA (analysis of variance) programs form an important part of statistical software packages. This book discusses in great detail how ANOVA programs are constructed and how their components work. Broader issues in the design of software systems for statistical applications are also treated quite extensively. The book is divided into five parts which cover statistically designed experiments, programming systems, least squares and ANOVA, the interpretation of design specifications, and the analysis of statistically designed experiments.

The treatment of these topics is very much oriented towards application and computation, with little emphasis on the development of the underlying theory. Most concepts are introduced by examples and few words are lost on basic ideas like Latin squares or block designs. An introductory chapter on the theoretical underpinnings would have done no harm. The author gives a lot of useful advice on programming style and on the handling of program systems on the user level. The book contains a generous supply of programs in FORTRAN, BASIC, APL, and C and many worked-out examples illustrating computational procedures. Compilable source codes for all programs are included in a floppy disk, which is packaged with the book and formatted for the IBM PC or compatible computers. For the numerical analyst, the most interesting part of the book is Chapter 11, which describes how techniques of numerical linear algebra such as QR factorizations, Householder reflections, Cholesky factorizations, and LU factorizations can be applied to least squares problems.

The book is eminently suitable as a guide for the practitioner because of its careful expository style and its stress on "hands-on" computations. The mathematical prerequisites are elementary linear algebra and a first course in statistics. Fluency in at least one programming language is assumed.

H. N.

**18[11-01, 11A51, 11Y05, 11Y11].**—David M. Bressoud, *Factorization and Primality Testing*, Undergraduate Texts in Mathematics, Springer, New York, 1989, xiii + 237 pp., 24 cm. Price $45.00.

Is it possible to teach an undergraduate, beginning number theory course by focusing almost entirely on factoring and primality testing? The thought is that these topics use so much number theory that little in a standard course would be left out. This is Bressoud's premise and his book is a text for such a course. Of course, some factorization and primality testing algorithms would be difficult to present at this level, but enough remains to make a nicely rounded book. Bressoud even gets to the elliptic curve method for factoring, minus most proofs though.

There is a definite "hands-on" flavor to the book. The algorithms presented are meant to be tried out by the students. (It is assumed that one has access to high-level software that can deal with long integers.) Actual programs are given for many algorithms, written in a kind of shorthand Pascal, that should be easily translatable into code by someone who knows programming. Some of the more advanced topics reached include the $p \pm 1$ factoring methods, the rho method, the quadratic sieve and continued fraction factoring algorithms, pseudoprimes, the $p \pm 1$ primality tests, and, as mentioned above, elliptic curve factoring.

One unfortunate omission is random compositeness testing. It would have been a simple matter for Bressoud to have developed the Solovay-Strassen (random Euler) test or the Miller-Rabin (random strong probable prime) test. The latter is just barely missed—see the comments on p. 78 and exercise 6.21. Sometimes poor advice is offered. For example, on p. 70, Bressoud seems to say that the $p-1$ and rho methods should be tried with several random seeds, rather than pushed further with one seed. This is apt advice for elliptic curve factoring, but not for $p-1$ or rho.

There are several typographical conventions that were glaring to my eye. One is the consistent use of $\times$ as a times sign—we consistently see expressions like $2 \times k$ for $2k$ and $a \times b$ for $ab$. Another is the use of $\partial$ as a group operation and # for group exponentiation—I suppose this is to favor neither multiplicatively nor additively presented groups. Nevertheless, equations such as $x\#3 = x \partial x \partial x$ are jarring.

If you want a book delving deeply into the theory and practice of factoring and primality testing, this is not a good choice (nor does it purport to be). If you want to teach a beginning number theory course to computer-literate students and get to many interesting and powerful methods, this book is your text. Overall, the style is very friendly and inviting, and I think students who like to program will enjoy it.

C. P.

**19[11-01, 11A41, 11D09, 11E25, 11R37, 11G15].**—David A. Cox, *Primes of the Form* $x^2 + ny^2$ *: Fermat, Class Field Theory, and Complex Multiplication*, Wiley, New York, 1989, xi + 351 pp., 24 cm. Price \$42.95.

Number theory, perhaps more than any other branch of mathematics, is organized around great problems. It is characterized not by the techniques used, which may come from algebra, analysis or geometry, but rather by the questions which are asked. For this reason, instead of biting off some general theory to write about, the author of a number theory textbook is often tempted to choose a tantalizing conjecture, or old riddle, as the book's unifying theme.

The classical problem around which David Cox organizes his book is: Given a positive integer $n$, find a way of telling which primes $p$ can be written in the form $x^2 + ny^2$ for some integers $x$, $y$. The simplest and best known case is $n = 1$, where the answer, due to Fermat, is: if and only if $p = 2$ or $p \equiv 1 \bmod 4$.

The goal is the following theorem, which in some sense solves the problem: *Given $n$, there exists a polynomial $f_n(x)$ with integer coefficients such that a prime $p$ can be written in the form $p = x^2 + ny^2$ if and only if $-n$ is a quadratic residue modulo $p$ and $f_n(x) \equiv 0 \bmod p$ has a solution.*

The book starts by developing the elementary methods that suffice for certain values of $n$: quadratic, cubic and biquadratic reciprocity; reduction of positive definite quadratic forms; and genus theory. The next part is devoted to class field theory. Using a classical approach to the subject, the author shows how the basic theorem about the existence of the polynomials $f_n(x)$ follows from the Artin Reciprocity Theorem. The final third of the book is concerned with the problem of constructing the $f_n(x)$ in the theorem for a given $n$; this leads to elliptic functions, complex multiplication, and properties of the $j$-function.

The book concludes with a discussion of the Goldwasser-Kilian-Atkin primality test using elliptic curves over finite fields and the theory of complex multiplication. There is an ample supply of exercises throughout the text.

The book is a welcome addition to the expository literature. The writing is informal, enthusiastic, and enriched with extensive historical information.

The reader should be warned, however, that few sections of the book are self-contained. For most of the central theorems either the entire proof or the hardest part of the proof is not given (but references are always supplied). For example, the basic theorems of class field theory, the results of Deuring, Gross, and Zagier on the class equation, and even cubic and biquadratic reciprocity, are all stated without proof.

The author's primary purpose is not to give proofs, but rather to motivate, to trace the threads of a fascinating story, and along the way to explain some difficult ideas of modern algebraic number theory in a down-to-earth way. In this he succeeds admirably.

The author claims that the first third of the book is suitable as a supplementary text in a beginning undergraduate number theory course. I disagree. The pace of exposition and the difficulty of exercises are likely to leave undergraduates more confused than enlightened. In his impatience to get to advanced topics, the author shortchanges the more routine matters.

For example, the first proof in the book is of the theorem that an odd prime $p$ can be written as $x^2 + y^2$ if and only if $p \equiv 1 \bmod 4$. The proof starts out by stating the two basic assertions to be proved:

*Descent Step*: If $p | x^2 + y^2$, $\gcd(x, y) = 1$, then $p$ can be written as $x^2 + y^2$.

*Reciprocity Step*: If $p \equiv 1 \bmod 4$, then $p | x^2 + y^2$, $\gcd(x, y) = 1$.

Both statements are likely to confuse a student—the first because the $x$ and $y$

at the end are not the same as the $x$ and $y$ at the beginning, and the second because of missing quantifiers. Moreover, the proof that follows contains a gap: one needs to know that a nontrivial divisibility $p|x^2 + y^2$ implies that $p \equiv 1 \bmod 4$, since otherwise (middle of p. 11) one cannot rule out the possibility that $N$ is the product of $p$ and a prime $q \equiv 3 \bmod 4$.

This type of imprecision, while not likely to discourage a sophisticated reader, does diminish the book's value for undergraduates.

To summarize, David Cox's book is an excellent textbook and reference for people at the graduate level and above.

NEAL KOBLITZ

Department of Mathematics
University of Washington
Seattle, Washington 98195

**20[11-00, 65-00].**—JONATHAN BORWEIN & PETER BORWEIN, *A Dictionary of Real Numbers*, Wadsworth & Brooks/Cole Advanced Books & Software, Pacific Grove, California, 1990, viii + 424 pp., $28\frac{1}{2}$ cm. Price $69.95.

This book is quite accurately described by its title. The authors have catalogued approximately 100,000 "interesting" real numbers, indexing them lexicographically according to the first eight digits following the decimal point. Both pure mathematical constants and some constants of physics are included.

Such a listing could be quite valuable as a reference book for a mathematician, computer scientist or physicist who in the course of a calculation, either theoretical or empirical, produces some number whose identity is not known. If such a number is found in this listing, and computation to higher precision confirms its equality to the constant in the list, then one can pursue a rigorous proof with a high level of confidence. In other words, a listing of this sort could enable mathematicians to employ "experimental" techniques in their research.

To their credit, the authors give a definition in the Introduction of exactly what numbers are included in the list. It is worth briefly summarizing this definition. Their listing is based on a "standard domain" of 4,258 numbers. These numbers consist of

1. Certain simple rational numbers.
2. Rational multiplies of certain irrational and transcendental constants.
3. Square roots and cube roots of small integers and simple rational numbers.
4. Elementary functions evaluated at certain simple values.
5. Sums and differences of square roots of certain small integers.
6. Rational combinations of certain constants.
7. Euler's constant, Catalan's constant, and some constants from physics.

All other numbers in the listing are either combinations of values of elementary functions, real roots of cubic polynomials with small coefficients, or special functions evaluated at numbers in the "standard domain" defined above.

As an exercise in evaluating how useful this listing might be in actual practice, this reviewer made a listing of 16 real constants (before studying the authors' definition of the book's contents). Of these, all but four were actually indexed in the book. It is worth mentioning, and briefly discussing, the four exceptions:

1. $4.6692016091 =$ Feigenbaum's $\delta$ constant (from the theory of chaos). This constant was not included in the standard domain.
2. $14.1347251417 =$ the imaginary part of the first zero of Riemann's $\zeta$ function. Note the "Real" in the title of the book.
3. $0.4619397663 = \frac{1}{2}\sin(3\pi/8) = \frac{1}{4}\sqrt{2+\sqrt{2}}$. Twice this constant is included.
4. $0.9772498680 = P(2)$, where $P(\cdot)$ denotes the cumulative Gaussian probability function. Twice this constant is included, since $P(x) = \frac{1}{2}[1 + \mathrm{erf}(x/\sqrt{2})]$, and the values of erf are indexed.

This exercise underscores both the strengths and the weaknesses of the book. On one hand, it appears that most numbers ordinarily encountered in mathematics are included. However, there are some holes in the list. Most notable are those cases where the constant one is looking for differs from some special function result by a simple rational factor, such as one-half.

Twenty years ago, tables of mathematical functions were widely used by both pure and applied mathematicians. In the intervening years such compilations have been rendered obsolete by the widespread availability of scientific calculators and subroutine libraries that can evaluate even fairly esoteric functions. One wonders if eventually the same fate will come to a book such as this. Already part of this table, namely the tabulation of roots of simple polynomials, has been rendered obsolete by the availability of relation-finding algorithms in packages such as Mathematica. These routines detect polynomial roots by searching for small integer relations in the vector $(1, x, x^2, x^3, \ldots, x^{n-1})$. Perhaps eventually such routines can be expanded in power to search for many other possible relationships. But in the meantime the Borwein book is all we have.

David H. Bailey

Numerical Aerodynamic Simulation Systems Division
NASA Ames Research Center
Moffett Field, California 94035

**21[11-01, 11B39].**—S. Vajda, *Fibonacci & Lucas Numbers, and the Golden Section: Theory and Applications*, Ellis Horwood Series in Mathematics and its Applications, Wiley, New York, 1989, 190 pp., $24\frac{1}{2}$ cm. Price $64.95.

Although there is a journal devoted to Fibonacci numbers and their generalizations, few monographs deal with this subject alone. There is a short book about Fibonacci and Lucas numbers by V. E. Hoggatt, Jr. [1] and a little book by N. N. Vorob′ev [2]. Thus, Vajda's book, which is longer than both of these books combined, is a welcome addition to the literature of the subject.

The introduction lists some problems in which Fibonacci numbers arise, from biology to computer science and from poetry to probability. In the main body of the book, scores of identities involving Fibonacci and Lucas numbers are derived. The important ones are numbered and repeated at the end of the book for easy reference. Some of the topics considered are Pell's equation, paradoxical dissection of rectangles, the golden section, finite sums involving Fibonacci and Lucas numbers and binomial coefficients, divisibility properties, distribution of Fibonacci numbers modulo $m$, search for extrema of real functions, and analysis of games. One chapter was written by B. W. Conolly; it deals with *Meta-Fibonacci sequences* such as $H(1) = H(2) = 1$, $H(n) = H(n - H(n-1)) + H(n - H(n-2))$ for $n > 2$. An appendix gives results from number theory which are used in the main text.

The reader should beware of many typographical errors and even a few factual errors. For example, formula (77) on p. 60 states that

$$\sum_{i=1}^{\infty} 1/F_i = 3 + \sigma = 4 - \tau = \frac{7-\sqrt{5}}{2}.$$

In fact, it is a famous unsolved problem to evaluate this sum in closed form or to decide whether it is transcendental. The proof which follows (77) actually demonstrates the true formula

$$\sum_{i=0}^{\infty} 1/F_{2^i} = \frac{7-\sqrt{5}}{2}.$$

S. S. Wagstaff, Jr.

Department of Computer Sciences
Purdue University
West Lafayette, Indiana 47907

1. V. E. Hoggatt, Jr., *Fibonacci and Lucas numbers*, Houghton Mifflin, New York, 1969.
2. N. N. Vorob′ev, *The Fibonacci numbers*, Translated from Russian by Halina Moss, Blaisdell, New York, 1961.

**22[65-06, 65L05, 65P05].**—R. E. Bank, R. Bulirsch & K. Merten (Editors), *Mathematical Modelling and Simulation of Electrical Circuits and Semiconductor Devices*, International Series of Numerical Mathematics, Vol. 93, Birkhäuser, Basel, 1990, xv + 297 pp., 24 cm. Price $59.00.

These are the proceedings of a conference held at the Mathematics Research Institute in Oberwolfach, October 30–November 5, 1988. There are eight contributions on circuit simulation, most of them dealing with the numerical treatment of differential-algebraic equations, and 13 contributions on device simulation, discussing the numerical solution of the semiconductor equations by finite element, discretization, linearization, and other techniques.

W. G.

**23[65-06, 65F10].**—David R. Kincaid & Linda J. Hayes (Editors), *Iterative Methods for Large Linear Systems*, Academic Press, Boston, 1990, xxx + 319 pp., $23\frac{1}{2}$ cm. Price \$49.95.

This book contains 17 lectures presented at a conferen^e held October 19-21, 1988, at the University of Texas at Ausin, honoring David M. Young, Jr. on the occasion of his 65th birthday. Surveying a wide spectrum of topics in iterative methods—from the analysis of preconditioners to issues of implementation on vector and parallel computers—this volume is an impressive and worthy tribute to one of the pioneers of the field.

W. G.