REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS

The numbers in brackets are assigned according to the American Mathematical Society classification scheme. The 1980 Mathematics Subject Classification (1985 Revision) can be found in the December index volumes of Mathematical Reviews.

38[65-01].—HEINZ RUTISHAUSER, *Lectures on Numerical Mathematics* (translated from the German by Walter Gautschi), Birkhäuser, Boston, 1990, xv+ 546 pp., $23\frac{1}{2}$ cm. Price \$49.50.

H. Rutishauser was a successful pioneer in numerical analysis at the beginning of the computer age. The German edition of his lecture notes was prepared posthumously for publication in two volumes by M. Gutknecht at the suggestion of, and with advice from, P. Henrici, P. Läuchli, and H.-R. Schwarz. The engineers and other scientists who attended Rutishauser's classes learned how the calculus and the elements of linear algebra could be skillfully used for a wide variety of applications. Algorithms and short sections of ALGOL codes were developed and analyzed for problems in optimization, linear algebra, approximation, numerical integration, ordinary differential equations, partial differential equations, etc. To confirm Rutishauser's fine taste in choice of topics, it suffices to observe that he gives a treatment of Floquet's theory for a linear ordinary differential equation with periodic coefficients. Throughout he is concerned with developing robust, stable algorithms and computer programs. The text has many illustrative examples that expose the underlying principles of numerical analysis and computer science.

This translation into fluent English with its editorial corrections makes the material more widely available. However, the inclusion of new supplementary notes and bibliographical references at the end of each of the thirteen chapters brings every section up to date! Hence, the book is now a most useful reference for the student, teacher, engineer, and scientist. W. Gautschi achieved this remarkable resurrection with the acknowledged help of C. de Boor, H. Brunner, B. N. Parlett, F. A. Potra, J. K. Reid, H. J. Stetter, and L. B. Wahlbin. A number of people had suggested that Gautschi update the work and he also gives credit for this to G. W. Stewart's review of the German edition in *Bull. Amer. Math. Soc.*, vol. 84, no. 4, July, 1978, pp. 660–663.

The book concludes with a self-contained 65-page appendix "An axiomatic theory of numerical computation with an application to the quotient-difference algorithm." This material is divided into five chapters and is a not quite completed monograph that Rutishauser had been preparing until his untimely death.

E. I.

39[65–01].—LARS ELDÉN & LINDE WITTMEYER-KOCH, *Numerical Analysis: An Introduction*, Academic Press, Boston, 1990, x+347 pp., $23\frac{1}{2}$ cm. Price $39.95.

This book is intended for an introductory course in numerical analysis at the advanced undergraduate level. The student or reader is thus supposed to have prior knowledge in calculus and preferably in linear algebra. The book consists of ten chapters with the headings:

Chapter 1: Introduction.
Chapter 2: Error analysis and computer arithmetic.
Chapter 3: Function evaluation.
Chapter 4: Nonlinear equations.
Chapter 5: Interpolation.
Chapter 6: Differentiation and Richardson extrapolation.
Chapter 7: Numerical integration.
Chapter 8: Systems of linear equations.
Chapter 9: Approximation.
Chapter 10: Differential equations.

The short Chapter 1 is devoted to a discussion of the relationships between mathematical models and corresponding numerical problems.

In Chapter 2 we find a discussion of various sources of errors and derivation of the well-known formulas for error propagation. This topic is dealt with in most textbooks in numerical analysis. But the authors also give an interesting discussion of the IEEE standard for floating-point arithmetic and about pipelined floating-point operations.

Next, Chapter 3 deals with fairly modern topics, such as the CORDIC algorithm for evaluating trigonometric functions. We find here also some useful classical results such as the estimation of the remainder of a truncated alternating series by means of Leibniz's theorem and the integral estimate for truncated positive series.

Chapters 4 and 5 give a useful treatment of classical topics which have many applications.

Chapter 6 presents the important Richardson extrapolation, which should be known by almost everyone working in the area of numerical calculations. Its application to numerical differentiation is treated here, but in subsequent chapters this extrapolation is applied to numerical integration and the treatment of differential equations.

In Chapter 7 we find the classical Romberg scheme for numerical integration. Perhaps the authors could have mentioned in this context that there are situations (e.g., in the integration of periodic functions, or when the interval of integration is infinite) when the trapezoidal rule gives very accurate results which cannot be improved by means of Richardson extrapolation. On the other hand, the authors give an interesting treatment of adaptive quadrature.

Chapter 8 gives a very good introduction to the numerical treatment of linear systems of equations and Gaussian elimination. The cases of general, i.e., nonstructured matrices, positive definite and band matrices are dealt with in detail. Iterative improvement is also discussed. It would perhaps have been worthwhile for the authors to include a short description of the Gauss-Seidel and Jacobi iteration schemes, which now are of interest in the context of the popular multigrid methods. We also find in Chapter 8 a discussion of perturbation, for which the authors introduce vector and matrix norms. The discussion of high-performance computers is also valuable.

In Chapter 9 normed function spaces are introduced in an elementary way, and some classical results on orthogonal polynomials are presented. The reader is also informed that interpolation at the Chebyshev points in general gives a good polynomial approximation.

Chapter 10 is devoted to ordinary differential equations and describes a few important methods for initial value and boundary value problems.

The book is well written. The discussion is clear and easy to follow. The authors present central topics in numerical analysis and the book should be useful for anyone who is interested in numerical calculations.

F.S.

40[65N30, 76D05].—MAX D. GUNZBURGER, *Finite Element Methods for Viscous Incompressible Flows: A Guide to Theory, Practice, and Algorithms*, Computer Science and Scientific Computing, Academic Press, Boston, 1989, xvii +269 pp., $23\frac{1}{2}$ cm. Price \$44.50.

The book by Gunzburger will appeal to a broad range of people interested in understanding algorithms for solving the model equations of fluid flow. This could be someone who wants to write new software, someone interested only in using existing software, but wanting to use it more intelligently, or someone looking for research problems in the field. Gunzburger has attempted to present the body of mathematical results currently available on the subject to such people.

The casual reader might comment that the book lacks proofs. This is not an accurate statement, even though the preface says that "no detailed proofs are given." For example, a fairly complete outline of a key issue, "divergence stability," is given (Chapter 2); details are in the papers cited. Even research mathematicians may appreciate this approach—it tells them what is important and where to find out more. However, the approach may make the book inappropriate as a text for a graduate course in mathematics.

There are several books with subject matter related (and complementary) to the book under review. The monographs of Girault and Raviart [2, 3] present more mathematical details, but without the scope available in Gunzburger's book. The forthcoming book by Brezzi and Fortin [1] will also offer a more mathematical perspective. Glowinski's monograph [4] is closer in spirit, in that it addresses many algorithmic details essential in a complete implementation of a computer code. However, [4] is more advanced technically, whereas Gunzburger has attempted something more expository.

Gunzburger addresses many of the algorithmic issues related to model complexities that arise in practical applications, such as mixed boundary conditions, non-Newtonian models, nonsimply connected domains, to name just three. Often ignored in theoretical treatments, such complexities lead to nontrivial perturbations in an algorithm and may cause significant problems for code modifications if not anticipated in the planning stage. The inclusion of such topics makes the book a valuable reference.

Not all topics critical to successful simulations are covered as fully as one might like. Indeed, the last chapter is devoted to a discussion of a number of "omitted" topics, and it provides some excellent guidance for future research topics. One of the topics discussed there that may surprise theoretically oriented computational mathematicians, yet is of crucial practical importance, concerns test problems. Software for solving partial differential equations in complex geometries is difficult to write and debug, so the testing of it is very important. One test problem for two-dimensional Navier-Stokes codes has recently been proposed in [6].

One cannot assume that all available mathematical theory regarding the modelling of "viscous incompressible flows" has been represented in the book. As an example, one of the earlier chapters is devoted to time discretizations. The material presented is the essential introductory theory for the most basic time-stepping techniques, yet much more could have been said on the subject. For example, there is no reference to the important series of papers by Heywood and Rannacher [5] which discuss some severe limitations to the accuracy of standard methods unique to incompressible flows, nor is there a discussion of the techniques introduced by Glowinski and coworkers [4].

Whether for getting an introduction to the subject, finding a detailed algorithm for a complicated model, or just seeking the philosophical viewpoint of one of the experts in the field, this book will be very valuable. It has appeared at a time when the subject is maturing and some definitive guidance can be given regarding code development and use. It also proposes numerous directions for further research in this continually developing field.

R. Scott

1. Franco Brezzi and Michel Fortin, *Mixed and hybrid finite element methods* (to appear).

2. Vivette Girault and Pierre-Arnaud Raviart, *Finite element approximation of the Navier-Stokes equations*, Lecture Notes in Math., vol. 749, Springer-Verlag, Berlin, 1979.

3. ——, *Finite element methods for Navier-Stokes equations, theory and algorithms*, Springer-Verlag, Berlin, 1986.

4. Roland Glowinski, *Numerical methods for nonlinear variational problems*, Springer-Verlag, Berlin, 1984.

5. John Heywood and Rolf Rannacher, *Finite element approximation of the nonstationary Navier-Stokes problem. Part* I: *regularity of solutions and second-order error estimates for spatial discretization*, SIAM J. Numer. Anal. **19** (1982), 275–311; *Part* II: *stability of solutions and error estimates uniform in time*, SIAM J. Numer. Anal. **23** (1986), 750–777; *Part* III: *smoothing property and higher order error estimates for spatial discretization*, SIAM J. Numer. Anal. **25** (1988), 489–512; *Part* IV: *error analysis for second-order time discretization*, SIAM J. Numer. Anal. **27** (1990), 353–384.

6. A. S. Lodge, W. G. Pritchard, and L. R. Scott, *The hole-pressure problem*, IMA J. Appl. Math. **46** (1991), 39–66.

41[65-01, 35A15, 65N25, 65N30].—M. Křížek & P. Neittaanmäki, *Finite Element Approximation of Variational Problems and Applications*, Pitman Monographs and Surveys in Pure and Applied Mathematics, vol. 50, Wiley, New York, 1990, viii+239 pp., 24 cm. Price $95.00.

According to the authors' introduction the purpose of this book is two-fold: to present a condensed and elementary form of the theory of the finite element method, and to extend standard treatises to cover a number of recent special developments.

The first part of the book (Chapters 2–7) is an introduction to the mathematics of the finite element method. The starting point is Chapter 2, on the variational formulation of second-order elliptic problems, including a concise section on Sobolev spaces. Chapter 3 introduces the finite element method and describes several standard finite element spaces, and Chapter 4 discusses their approximation properties and associated convergence results in energy and L_2-norms. Chapters 5 and 6 address quadrature aspects and the generation of the stiffness matrix, and the relatively lengthy Chapter 7 is devoted to the solution of the resulting systems of linear equations.

In this part of the book some basic results are proved, but for the more sophisticated ones the reader is referred to other treatises. For instance, the Bramble-Hilbert lemma is stated and used extensively, but for a proof the authors quote the book of Ciarlet, which also is frequently referenced in the sections on quadrature and elsewhere. In the chapter on solution of the linear algebraic systems, different subsections treat the Jacobi method, the Gauss-Seidel method, SOR, steepest descent, and conjugate gradient methods. Preconditioning is mentioned without being pursued, and more modern notions, such as methods based on FFT, domain decomposition, and multigrid methods, are not mentioned.

The remaining part of the book, corresponding to the second purpose stated, has twelve chapters on topics such as methods for increasing the accuracy in standard finite element approximation (Chapter 8), fourth-order elliptic problems (Chapter 9), parabolic and hyperbolic problems (Chapters 10–11), methods and problems associated with divergences and rotations, including solution of Stokes, Maxwell's, and Helmholtz's equations (Chapters 12–14), nonlinear problems (Chapters 15 and 18), and eigenvalue and bifurcation problems (Chapters 16–17).

The selection of topics appears motivated mainly by the authors' own research interests, and no attempt at a wide coverage of recent developments appears to have been made. The chapters are often quite short and sometimes characterized by a certain lack of substance and precision, with simple computed examples carrying the burden of persuasion. In view of the expressed purpose of being accessible to readers with limited background, the presentation is often on the terse side.

Although the list of references is extensive, many household names and relevant works from the field are missing, conveying the impression that the authors are somewhat less than well informed about some of the areas they describe, whereas no less than 19 papers by the authors themselves are quoted.

To give some examples of the above points, in a short chapter on eigenvalue problems, the authors reproduce an incorrect error estimate for eigenvalues from the early literature. The subsequent related four-page chapter on a bifurcation problem is too short to accomplish its purpose of explaining the rather difficult problem, describing a method for its solution, and presenting numerical evidence. In the chapter containing a discussion of superconvergence, on which the authors have written an exhaustive survey article, one is surprised to find the book by Axelsson and Barker from 1984 to be the basic reference on nodal superconvergence, the only other reference being the survey just mentioned.

The short chapter on hyperbolic problems considers only two specialized topics, perhaps not the obvious first choices, namely the solution of the wave equation with nonhomogeneous Dirichlet boundary conditions for $t > 0$, and a stationary, convection-dominated diffusion problem. In the former a penalty approximation is presented, where the nonhomogeneous boundary condition is replaced by a Newton-Robin type condition with a small coefficient in front of the normal derivative. The resulting equations are then discretized by standard finite elements in space and a symmetric difference scheme in time. No analysis of the method is presented, nor any claim about its properties, but some computations are described with tables and pictures. In the latter part of the chapter a Petrov-Galerkin method is applied to a linear one-dimensional singularly perturbed problem. Again, no analysis or convergence statement is given, only a computation hinting at the superiority of the Petrov-Galerkin method with the appropriate trial space over the standard Galerkin method.

In summary, the reviewer feels that the two purposes of being both an elementary introduction and a survey of the authors' research interests have turned out not to be quite compatible. In the first part, for simplicity and brevity, the analysis has been cut down to such a minimum that the second part would be very difficult to penetrate for someone with only this as a background. However, the appearance of a new book in such a central and active field of numerical analysis as finite elements always arouses interest and curiosity. The present one will certainly serve as a useful survey of work of the authors and their collaborators and compatriots, often not easily accessible otherwise.

V.T.

42[65–02, 35L65, 76–08].—RANDALL J. LEVEQUE, *Numerical Methods for Conservation Laws*, Lectures in Mathematics, ETH Zürich, Birkhäuser, Basel, 1990, ix+214 pp., 24cm. Price: Softcover $24.50.

The title seems too general. The book does not cover all the aspects of numerical methods for conservation laws. For example, shock-fitting methods, or finite elements and spectral methods, are not discussed. However, the text does provide an up-to-date coverage on recent developments of shock-capturing finite difference methods, which is one of the most active research areas in numerical solutions for conservation laws.

The first part of this book summarizes the mathematical theory of shock waves. It also covers topics related to gas dynamics equations. Although the material is available in other books listed in the references, a somewhat more elementary approach with the help of graphs is provided here. It should prove helpful to students with a limited background in partial differential equations, but who want to get some feeling about the theory in order to read the second part regarding numerical methods.

Part two of this book is about recent developments of shock-capturing finite difference methods. In the past fifteen years there has been a lot of activity in designing and analyzing stable and accurate shock-capturing finite difference methods for conservation laws whose solutions contain shocks. The developments have been following quite a different path than the traditional linear stability analysis based on smooth solution assumptions. Tools for nonlinear stability such as TVD (total-variation-diminishing) methods have been developed, and high-order nonlinearly stable methods have been designed to resolve shocks and other complicated flow structures. Unfortunately, most of the results have been available only in isolated, sometimes hard-to-read journal articles. Books or Lecture Notes in this area are notably rare. This book is therefore rather unique and should prove to be a valuable reference and textbook in this area.

The text is carefully prepared. I have used a report version of it for a graduate reading course at Brown University, and students feel that it is well written and on the whole easy to understand. Misprints are rare, although the first sentence in the Preface misses an "are" right after the first two words. Another not-so-obvious mistake is in Exercise 17.1 on page 199: the minmod function in (16.51) would have to be changed to a minimum-in-absolute-value function to establish the claimed agreement.

In view of its limited scope, this book is not suitable for a general numerical analysis course for partial differential equations, or for computational fluid dynamics. However, it would serve as an excellent textbook for a graduate seminar course for mathematics or engineering students who are interested in shock calculations, and as a general reference book for researchers. The fact that it contains exercises and is available in relatively inexpensive soft cover is another welcome feature.

One comment on the organization of the material: the first part on theory of conservation laws and gas dynamics equations seems too lengthy, since most of the material can already be found in many good books. It would seem worthwhile to condense the first part and to expand the second part on numerical methods. One gets the impression, when reading through the book, that it is gradually running out of steam: towards the end, the description becomes more and more sketchy.

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43[65M60, 65N30, 65N35].—CLAUDIO CANUTO, M. YOUSSUFF HUSSAINI, ALFIO QUARTERONI & THOMAS ZANG, *Spectral Methods in Fluid Dynamics*, Springer Series in Comput. Phys., Springer, New York, 1988, xv+557 pp., 24 cm. Price $90.50.

The authors present here a comprehensive, up-to-date treatment of spectral methods: "when to use them, how to implement them, and what can be learned from their rigorous theory."

The distinguishing feature of this book is its synthesis between the description of spectral *algorithms* that are successfully implemented in Computational Fluid Dynamics (CFD) problems, and whatever rigorous theory is currently available to support their numerical results.

After the introductory material in Chapter 1, the content of the book can be grouped into three related parts.

The first part outlines the basic ingredients which are involved in spectral solution of PDE's. In includes a bird's eye view on the fundamental concepts of spectral expansions in Chapter 2. Chapter 3 is concerned with accurate spatial differentiation of these spectral expansions in the context of linear and nonlinear PDE's. Temporal discretizations of spectral approximations to time-dependent problems are treated in Chapter 4, and the solution of implicit spectral equations (which arise owing to full spectral differentiation matrices) is studied in Chapter 5.

The second part of the book is devoted to applications of spectral methods for CFD problems. It includes a detailed description of spectral algorithms for unsteady incompressible Navier-Stokes equations in Chapter 7, and for compressible Euler equations in Chapter 8.

The third part of the book deals with the rigorous analysis of spectral methods. Error estimates for spectral expansions are presented in Chapter 9, which is followed by the linear stability and convergence analysis of spectral methods in

Chapter 10. Chapters 11 and 12 demonstrate applications of the general theory to steady Navier-Stokes and time-dependent problems, respectively. The final Chapter 13 addresses the issue of spectral computations in general geometries via domain decomposition methods.

The main text concludes with an impressive list of references.

The class of spectral methods is an umbrella for a family of certain *projection* methods. I now proceed with a brief description of these methods, touching upon various aspects that are highlighted in the book.

The underlying problem consists of one or more PDE's

$$Lu = F \tag{1}$$

augmented with side conditions—initial conditions, boundary conditions, etc., which make the problem *well posed*. The N-dimensional spectral approximation of (1) reads

$$P_N L u_N = F_N, \tag{2}$$

together with appropriate N-dimensional side conditions. Different spectral methods are identified with different N-dimensional projections P_N. The distinctive character of spectral methods is their use of *global, highly accurate* projections. Typically, these projections are represented by N-term orthogonal expansions, e.g., Fourier, Chebyshev, Legendre, etc. Chapters 2–5 describe how to proceed with efficient solution of (2). In particular, the FFT can be used to carry out algebraic and differential operations with the spectral expansions. Preconditioning and multigrid techniques are used for the solution of the full, often ill-conditioned linear systems associated with the linearized equation (2).

The behavior of the approximate spectral solution, u_N, depends on the familiar notions of consistency, stability, and convergence. In the linear case, one obtains from (1) and (2) that $e_N = u_N - P_N u$ satisfies the error equation

$$P_N L P_N e_N = P_N L(I - P_N)u + F_N - P_N F. \tag{3}$$

Consistency requires that the approximation error on the right-hand side of (3) becomes small as N increases. An attractive feature of the spectral projections, analyzed in Chapter 9, is their so-called spectral (or "infinite-order") accuracy, which means that the approximation error decays as fast as the global smoothness of the exact solution u permits. The spectral method (2) is *stable* if $P_N L P_N$ is boundedly invertible. In this case, the error equation (3) implies that u_N is spectrally accurate with $P_N u$ and hence, by consistency, u_N *converges* spectrally to u itself, provided the latter is sufficiently smooth. The consistency, stability, and convergence of spectral methods hinge on several key issues which make the spectral algorithm work successfully. Particular attention should be paid to the aliasing phenomenon; the use of filtering techniques in the presence of nonsmooth data; the appropriate choice of a spectral projection for a given problem, and the choice of discrete collocation points to realize this projection; the correct treatment of boundary conditions; and the behavior of the eigenproblems associated with the linearized equations (2). The fundamental role of these and other issues in the context of spectral methods is explained in Chapters 3–5. Practical implementations are demonstrated in Chapters 7–8. The importance of these issues is reemphasized in the stability analysis presented in Chapters 9–13.

The book is *not* easy to read. This is in part due to its large scope, and in part due to the subject itself: the details involved in spectral algorithms are inherently less 'clean' than those of, say, finite difference methods. The reader is therefore required to be familiar with spectral methods. As the authors indicate in the preface, they intend to reach audiences of both users and theorists of spectral methods. In this respect, the expert will find *Spectral Methods in Fluid Dynamics* to be a valuable source of material. The classical 1977 monograph of Gottlieb and Orszag [1] is, naturally, out of date in view of the enormous amount of activity that took place during the last decade in the area of spectral methods, and the present book will most likely replace it as the standard reference on the subject.

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1. D. Gottlieb and S. Orszag, *Numerical analysis of spectral methods: Theory and applications*, SIAM, Philadelphia, PA, 1977.

44[65-02, 65N05, 65N10, 65N15].—BERND HEINRICH, *Finite Difference Methods on Irregular Networks: A Generalized Approach to Second Order Elliptic Problems*, International Series of Numerical Mathematics, vol. 82, Birkhäuser, Basel, 1987, 206 pp., 24 cm. Price $49.00.

The subtitle of this work is: A generalized approach to second order elliptic problems. Actually, the book is basically concerned with one particular method of approximation, viz., the "finite volume method," also known as the "box integration method," the "box method," the "finite control volume method," or, as the author prefers to call it, the "balance method." Mainly selfadjoint problems in the plane are treated.

In brief, the balance method investigated in this book is as follows: Given a primary mesh, write down the weak formulation of the elliptic partial differential equation against a constant function in a control volume, locally associated with the primary mesh. This procedure has a "physical" interpretation; hence the name balance method. Then approximate the resulting relation using values of the unknown function at nodal points of the primary mesh; equations for these nodal values ensue. Variations of the basic scheme differ according to how the control volumes are constructed from the primary mesh and also according to whether a triangular or rectangular primary mesh is used.

Three types of primary meshes are considered: A. Uniform (regular), B. almost uniform, i.e., uniform except for a small layer around curved boundaries, say, and C. quasi-uniform but globally irregular meshes. Given the title of the book, the last case is clearly the most important.

The book is to a large extent a summary of the work of the author and others on how this balance method behaves.

There is at present a discussion among numerical analysts about the role of truncation error analysis for irregular meshes in "ordinary" finite difference and finite element methods. Some of the issues are very nicely illustrated in this monograph, as I hope the following outline of some main ideas will show.

First, the ensuing equations for nodal values alluded to above are treated in a classical manner, well known to anyone who has studied the five-point operator on a uniform mesh. A maximum principle is derived and thereafter the exact solution is put through its paces through the equation for nodal values, resulting in a "truncation error." Using the maximum principle, any bound on the maximum norm of the "truncation error" yields a maximum norm bound on the actual error.

This analysis works all right for meshes of type A, and, with some exertions inspired by finite difference work in the late sixties, for meshes of type B, provided the solution is sufficiently smooth.

However, in the case C of totally irregular meshes, the "truncation error" is merely bounded in the maximum norm; thus the above analysis via the maximum principle does not even show convergence. This not unusual observation for irregular meshes is made on page 107 in this book.

Noting that the pointwise analysis via the maximum principle does not work for irregular meshes, to rescue the situation, the author switches to an analysis in L_2 dual norms, basically, with minor technical deviations now and then, using that the actual H_1 error is bounded by the H_{-1} error in the "truncation error." The program is carried through with aplomb for the three mesh situations A, B, and C. In particular, in the case A of globally regular meshes, so-called superconvergent results, $O(h^2)$ for first-order difference quotients, are obtained, as also, to a lower order, in the case of meshes of type B.

As for Chapter 6, nonsymmetric problems, I have little to say. Although the author uses upwinding techniques for the first-order convection terms, the results there say little about a truly singularly perturbed, convection-dominated case, as far as I could ascertain.

The style of this book is readable, although I suspect that some numerical analysts will find the collections of notation for meshes and geometry of domains in the beginning somewhat overpowering. Many technical points are relegated to the literature. I note with pleasure that the author very carefully takes into account the treatment of curved boundaries. The typography is pleasant, with hand-written symbols carefully executed.

In conclusion, this is a careful monograph on error analysis of the balance method.

L. B. W.

45[20-06, 20-04, 05-04, 08-04, 16-04].—MARTIN C. TANGORA (Editor), *Computers in Algebra*, Lecture Notes in Pure and Appl. Math., vol. 111, Dekker, New York and Basel, 1988, viii+162 pp., $25\frac{1}{2}$ cm. Price \$89.75.

This volume contains papers presented at a conference on Computers in Algebra held in December 1985 at the University of Illinois at Chicago. The sixteen papers illustrate various aspects of the use of computers in algebra, either to prove theorems or to do calculations which lead to simple proofs by hand. Most contributions come from the area of computational group theory. Overview papers discuss algorithms for p-groups and permutation groups, polynomial-time algorithms for permutation groups and the computer-aided determination of Galois groups. The use of the computer to obtain or prove presentations for finite simple groups is discussed in several papers, including work on the conjectured Y_{555}-presentation connected to the monster simple group. Three papers are concerned with the computer-aided determination of cohomology groups and rings, used for example to help the enumeration of perfect groups of small order. Other topics discussed include coding theory, proof of solvability of word problems in universal algebra, and Hopf algebras.

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