REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS

The numbers in brackets are assigned according to the American Mathematical Society classification scheme. The 1991 Mathematics Subject Classification can be found in the annual subject index of *Mathematical Reviews* starting with the December 1990 issue.

6[65-01, 65Dxx, 65Fxx, 65Gxx, 65Hxx, 65Kxx].—GÜNTHER HÄMMERLIN & KARL-HEINZ HOFFMANN, *Numerical Mathematics* (Translated by Larry Schumaker), Undergraduate Texts in Mathematics, Springer, New York, 1991, xi + 422 pp., $23\frac{1}{2}$ cm. Price: Softcover \$39.95.

This is a straight translation of the first German edition (reviewed in [1]) incorporating, however, a few very minor corrections.

W. G.

1. W. Gautschi, Review **10**, Math. Comp. **55** (1990), 391–392.

7[35R30, 65M30, 93B30].—H. T. BANKS & K. KUNISCH, *Estimation Techniques for Distributed Parameter Systems*, Systems & Control: Foundations & Applications, Vol. 1, Birkhäuser, Boston, 1989, xiii + 315 pp., $23\frac{1}{2}$ cm. Price \$42.00.

A typical problem addressed in this book is the following: Given certain "observations" z of a "state" $u = u(x, t; q)$ which satisfies

$$(1) \qquad \begin{cases} u_t = (qu_x)_x \equiv A(q)u, & 0 < x < 1, \ 0 < t, \\ u(x, 0) = \phi(x), & 0 < x < 1, \\ \text{and boundary conditions}, \end{cases}$$

can one recover the unknown coefficient $q = q(x, t)$?

A typical general scheme for this problem is as follows: First, select a criterion for, hopefully, nailing down q; say, the "output least squares error criterion,"

$$(2) \qquad \underset{q}{\text{Min!}}\ |u(\cdot, \cdot; q) - z|^2.$$

Here, $|\cdot|$ denotes a suitable seminorm, e.g., the deviations at some discrete points (x_i, t_j). Then fix the "admissible parameter set" $\widetilde{Q}$, typically involving constraints on q motivated from the problem, e.g., $q(x, t) \geq \gamma > 0$, and perhaps even $q =$ constant. Some additional conditions, e.g., norm bounds, are also typically involved for making $\widetilde{Q}$ a compact subset of a suitable metric space. Then select finite-dimensional approximations Q_M to $\widetilde{Q}$, and also

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approximations $A^N(q)$ to $A(q)$, in some finite-dimensional state spaces; the latter lead to (semi) discrete solution schemes for (1).

Finally, solve the finite-dimensional version of (1), (2), thus, hopefully, getting $\overline{q}_M^N$.

A typical result of this book gives conditions on the setup to guarantee so-called "function space parameter convergence." Among other things, this means that a) the $\overline{q}_M^N$ exist; b) every convergent subsequence of $\overline{q}_M^N$, as $M, N \to \infty$, converges to a solution q^* of (2); and c) there exists at least one such convergent subsequence.

In practice, some basic method has to be used for the discrete optimization problem (2), and some time-discretization for (1), which of course intervenes in (2). Practical aspects of this are discussed in the book. In the brief description above I have left out an "observation operator" $\mathscr{T}$ that the authors include throughout; this means that z is compared to $\mathscr{T}u$ in (2), rather than to u itself.

The contents of the book are as follows. Chapter I gives many examples of problems such as (1) occurring in applied science. Chapter II is devoted to preliminary material from semigroup theory. The aim here is to "provide a useful summary for quick reference." It is noted that "those for whom this is new material may wish to supplement it with readings in the references." Chapter III contains a thorough general setup and analysis along the lines I have indicated above for (1), (2), with examples of concrete approximations (with more to come in Chapter V). A long Chapter IV discusses identifiability and stability, including regularization approaches. Very loosely, these are questions about existence and uniqueness of solutions to (2), and their stability under changes in z. Section 2 of this chapter contains ten carefully chosen examples which greatly adds to the understanding of the concepts and issues. In Chapter V more details, mainly on practice and implementation, are given for the linear parabolic case. Eleven numerical examples with numerically generated observations z from a "true" q^* are given, which nicely illustrates the theory. Finally, in this chapter, examples with purely experimental data are considered. A discussion of the use of the methods is given for answering, in a statistical sense, questions such as: Is convection important in the model? Is diffusion spatially and/or temporally varying?

In Chapter VI the authors consider parameter identification in linear elliptic problems, discussing many methods, not only based on (2), and again generously sprinkling the text with interesting examples.

As the authors remark, the material covered is not complete or completely up-to-date. An extra bibliography, Chapter VII, is intended to atone for this. Five short appendices on splines conclude the book.

One aspect of not being completely up-to-date is that standard finite element methodology based on weak formulations for solving (1) is not treated (although Example V.3 comes close). Most approximations to (1) use $A^N = P^N A$ with P^N a projection to a spline or eigenfunction space which is in the domain of A (in L_2). The authors give references for the standard finite element approximation, and, in the elliptic case, they are being used.

The writing is lucid and, as should be clear from the above, concepts are thoroughly motivated and discussed. Pitfalls and imperfections in theory and practice are clearly pointed out. I found only a few trivial misprints. The book is eminently understandable to someone who knows basic spline theory and a bit more than basic functional analysis. (Facts about compactness and metrizability in the weak * topology of L_∞ are used, with careful referencing.)

A particular aspect of this work is the constant interplay between a carefully erected abstract framework, in which convergence is proven, and very concrete examples which sometimes stretch the theory to its limits and beyond. In this, the book is not only of interest to people in the area of parameter estimation, but serves up a nice slice of life of present-day applied and numerical mathematics which may be enjoyed by a wider audience.

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8[65-06, 65P05, 65H20, 35B32, 58F14].—Dirk Roose, Bart de Dier & Alastair Spence (Editors), *Continuation and Bifurcations: Numerical Techniques and Applications*, NATO ASI Series, Series C: Mathematical and Physical Sciences, Vol. 313, Kluwer, Dordrecht, 1990, xiii + 426 pp., 24 $\frac{1}{2}$ cm. Price \$132.00/Dfl.220.00.

The numerical solution of parameter-dependent problems has become an extremely important branch of scientific computing and numerical analysis. The theoretical understanding of bifurcation phenomena permits the development of powerful methods automatically detecting qualitative changes in solution behavior.

This book contains 26 articles and 10 abstracts of talks given at a workshop held at the Katholieke Universiteit Leuven, Belgium, in September 1989. The authors cover a wide range from theoretical investigations to numerical algorithms and applications to real-world problems.

Several articles are concerned with low-dimensional representations of the behavior of systems described by partial differential equations. The approaches discussed by the authors include the construction of approximate inertial manifolds as well as spectral methods. The resulting finite-dimensional systems are used for the computation of bifurcation diagrams for problems such as the Kuramoto-Sivashinski equation or systems of reaction-diffusion equations.

A group of articles is concerned with bifurcation in the presence of symmetries. Apart from theoretical investigations, numerical methods using information about symmetries are presented.

Methods for the computation of heteroclinic and homoclinic orbits and their use for the detection of global bifurcations are discussed. Further numerical aspects include the effect of time-discretization on the global attractor as well as the computation of Hopf bifurcations. Continuation and bifurcation software is presented, and the desirable features of such software are discussed in an article which tries to initiate a discussion about standards for continuation codes.

Several interesting applications are presented. The author of this review learned that cubature formulae can be constructed via continuation methods. Other articles describe a classification of flow structures for the Navier-Stokes equations, the dynamics of the Maxwell-Bloch equations for passive optical systems, Marangoni convection in crystal growth problems, image processing via the dynamics of reaction-diffusion systems and the stability of a robot.

This book provides a very nice picture of the variety of problems which can be attacked by continuation and bifurcation methods as well as the corresponding theoretical and numerical approaches. The articles are well written and provide a useful source of information for anybody doing research in the field or just having an interest in the subject.

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9[65-02, 65F05, 65F10, 65N22].—JACK J. DONGARRA, IAIN S. DUFF, DANNY C. SORENSEN & HENK A. VAN DER VORST, *Solving Linear Systems on Vector and Shared Memory Computers*, SIAM, Philadelphia, 1991, x + 256 pp., 23 cm. Price: Softcover $19.75.

The last twenty years have seen a revolution in scientific computing as a result of the emergence of parallel and vector computers. Particularly within the last decade, as these machines have become widely available, there has been increasing interest in algorithms capable of achieving the full potential of such computers.

One of the most frequent and important problems in scientific computing is the solution of linear systems of equations, the topic of this book. Although the title limits the scope to shared memory machines, there is, in fact, much in the book of explicit or implicit relevance to distributed memory machines also.

A fairly long Chapter 1 sets the stage with a discussion of many hardware features such as pipelining, chaining, RISC and VLIW machines, cache and other memory organization issues, connection topologies for parallel machines, ending with a few programming techniques such as loop unrolling. This is followed by two short chapters, one surveying various past and current machines, and the second discussing such issues as data dependency and control flow graphs, load balancing, synchronization, and indirect addressing. Chapter 4 gives additional general background including Amdahl's law, speed-up and scaled speed-up, and Hockney's $n_{1/2}$ and r_∞ parameters with examples of these for various machines.

Chapter 5 begins the main subject matter with a treatment of various forms of LU decomposition for dense matrices, as well as related topics such as LDL^T decomposition for indefinite matrices and QR decomposition. The main theme is the necessity of proper data management on machines with hierarchical memories (registers, cache, etc.). This leads in a natural way, and through the use of performance examples, to the desirability of blocked forms of the decompositions in which operations are performed on submatrices as much as possible. Three block organizations of LU decomposition are considered and their characteristics compared. Throughout, there is discussion of the role of the BLAS (vector operations), level-2 BLAS (matrix-vector operations), and level-3 BLAS (matrix-matrix operations). Overall, this chapter gives a good background and motivation for the development of LAPACK, the project led by the first author to replace LINPACK and EISPACK by a collection of subprograms suitable for parallel and vector machines.

Chapter 6 considers direct methods for sparse systems, a topic associated with the second author. After a discussion of sparse data structures and the problems of fill and pivoting, various examples, using the code MA28, are given to show that hardware gather-scatter operations are not a cure for the indirect-addressing problem. Attention is then focused on the symmetric problem and the use of graph theory, especially cliques. Next, there is discussion of the frontal method with examples on CRAY machines, followed by the multi-frontal method and elimination trees with examples of parallelism. The chapter ends with a short survey of other approaches to parallelism.

Chapter 7 deals with iterative solution of sparse linear systems, an area associated with the last author. There is a review of primarily conjugate gradient type methods, especially for nonsymmetric systems (least squares formulations, biconjugate gradient, conjugate gradient squared, GMRES, etc.). A key part of any conjugate gradient method is matrix-vector multiplication, and there is an interesting discussion of efficient ways to do this, depending on the sparsity structure and the machine. Another key aspect of CG methods is preconditioning, and various possibilities (ILU, polynomial, approximate inverse, etc.) are reviewed along with their pros and cons on various architectures. The emphasis in this chapter is more on vector than parallel machines, but the last section deals with several parallel issues and examples.

The book ends with a glossary, instructions for obtaining software through NETLIB or from libraries such as NAG, and three other appendices on further hardware information, the BLAS, and operation counts for the BLAS and various decompositions. There is a bibliography of 173 titles.

Although each of the authors is well known for a particular research area, the book is well integrated, and relatively easy to read. Given the rate at which high-performance computer architecture is changing, much of the information in the book may have a relatively short lifetime, but hopefully at least some of the main principles of algorithm development will endure. All in all, the book is an excellent introduction to its subject matter and a welcome contribution to this field.

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10[33-01, 33C50, 41-01, 11-01].—THEODORE J. RIVLIN, *Chebyshev Polynomials: From Approximation Theory to Algebra and Number Theory*, 2nd ed., Wiley, New York, 1990, xiii + 249 pp., $24\frac{1}{2}$ cm. Price $49.95.

The Chebyshev polynomials are defined by $T_n(x) = \cos n(\arccos x)$. They were introduced by Chebyshev in the mid-nineteenth century, who sought the solution to the following problem: Find the polynomial of degree $n-1$ which best approximates x^n on $[-1, 1]$ in the uniform norm, or, equivalently, minimize the uniform norm of a monic polynomial of degree n on $[-1, 1]$. As is well known, the Chebyshev polynomials have an incredible number of remarkable properties in approximation theory and classical analysis. They are a prototype of orthogonal polynomials on $[-1, 1]$ (the Jacobi polynomials are a generalization of those of Chebyshev), and they provide the solution to an unusual number of extremal problems. They also play an important role in interpolation theory and numerical integration. Less well known are properties of these polynomials which have a basis in modern analysis, especially ergodic theory, as well as in algebra and number theory.

The book under review not only supplies the important results concerning Chebyshev polynomials in these areas, but also contains a basic introduction to approximation and interpolation theory. Written in a lucid style, which illuminates the beautiful topics covered, the author has enhanced his work by the inclusion of over 300 interesting exercises. As a result, the book can easily be used as a text, especially in a seminar, but it also should be read by anyone wishing to learn about this fascinating subject.

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11[41A99].—PAUL NEVAI & ALLAN PINKUS (Editors), *Progress in Approximation Theory*, Academic Press, Boston, 1991, xi + 916 pp., $23\frac{1}{2}$ cm. Price $189.00.

This is a collection of 62 research papers that have been submitted to, and accepted by, the *Journal of Approximation Theory* and, with the authors' permission, have been assembled in this volume in order to alleviate the current backlog of the journal. Accordingly, a great variety of topics, both in pure and applied approximation theory, are being addressed, and the ordering of the papers alphabetically with respect to authors only accentuates this diversity. In character, the papers range from a short 3-page note to a substantial 74-page memoir. The printing conforms exactly to that of the journal, except that no received dates are given.

W. G.

12[68Q40, 65Y15, 65Y25, 11-04, 12-04, 13-04, 14-04, 30-04, 33-04].—STEPHEN WOLFRAM, *Mathematica—A System for Doing Mathematics by Computer*, 2nd ed., Addison-Wesley, Redwood City, California, 1991, xxiv + 961 pp., 23 cm. Price $48.50 hardcover, $33.50 paperback.

Mathematica is an interactive computer software system and language intended for solving problems in mathematics. Prominent features are numerical and symbolic mathematical manipulation, elaborate plotting software using the PostScript display technology, and a versatile programming language. While few of the features are entirely novel or "state of the art," the combination of all of these facilities in an accessible package has made it popular.

Any serious user of Mathematica (version 2.0, corresponding to the system described in this publication) will probably wish to have this book close at hand. A purchaser of the software will presumably already own one copy of this manual. It is the principal reference for the commands, and the on-line help is not an adequate substitute. Furthermore, a user will most likely continue to need occasional assistance in the use of this complex program.

For those who have no access to the program, this volume is effective as a convincing sales tool for the program: it includes dramatic color plots, numerous examples of interesting and clever mathematical manipulations, and few intimations of any limitations on its capabilities.

In nearly 1000 pages, this text includes a brief "Tour of Mathematica," a "Graphics Gallery" (26 pages of color), and four major sections: a practical introduction, a more detailed description of the features of the system and language, "advanced mathematics," and a reference section. This last portion is a 150-page alphabetical list of nearly all the mathematical functions, language components, and system commands. There is an excellent 50-page index.

The text has been produced through computer typesetting, and benefits from the integration of Mathematica with the PostScript picture-description language. The apparently effortless inclusion of plots, computer input and output, and mathematical text is a model for how such material should be presented.

Although it is not possible in a brief review to summarize the material in this text, some general critical comments seem to be in order:

The disparity in the level of treatment of issues may be distressing to the mathematician. For example, over 100 pages are devoted to graphics, and there are over 50 options to the "Plot3D" command. Yet the "definition" of the natural logarithm is given on one line, and neglects to mention the treatment or even existence of a branch cut in the complex plane. Does this mean the treatment is so obvious and correct that it needs no explanation? Not really, since (for example) one cannot deduce from the documentation the meaning Mathematica would assign to the function $f(z) := \log(z^3) - 3\log(z)$. Nor can one deduce Mathematica's answer if one were to ask it to solve the equation $f(z) = 0$ for z. The current version claims there is no solution, although $-\pi/3 < \arg z \leq \pi/3$ would be more correct.

Other aspects of the program that are hardly documented but can substantially compromise the correctness of computational results include significance arithmetic in the numerical model, strange treatment of asymptotic order, limits and infinities, interval arithmetic, and inequalities. In fact, a subtle implication of the design seems to be that it is acceptable for exact algebraic computation to sometimes produce wrong answers, much as though "experimental error" played a role in arithmetic or algebra.

Purchasers of this book may have various expectations about its contents. In order to avoid disappointment, I am indicating some of the areas **not** covered:

(a) There is no survey, brief or otherwise, of the fields of computer algebra, numerical analysis, graphics, or programming. The author provides bibliographic citations only for books using Mathematica, not source material for understanding the technology. This seems to reinforce the author's viewpoint that Mathematica is a new and unique "black box" for "doing mathematics."

Readers of this review may be aware of a substantial literature on numerical computation as well as graphics, although references on symbolic computation (for example [2], [4]) may be less well known.

(b) There are no descriptions of, or references for, the algorithms that are used. There is virtually no description of how the system performs any of its mathematical tasks. Knowledge of the details of these algorithms, from arbitrary-precision numerical evaluation of special functions, to symbolic definite integration, is of interest for many reasons. Even within the goal of providing information to users of the program, such information would make it possible to avoid some of the flaws in the methods. Admittedly, the kind of extensive details given in Buchberger et al. [2] would be out of place in a mere users' manual, but this same volume is also the system's only reference document. By comparison, Maple's manual [3] indicates the algorithms used and has much more detailed information and references. Maple goes further by providing much of its source code.

(c) There is insufficient material to provide a detailed understanding of the program. The welter of detail on system "global flags" and special commands such as Together, Expand, ComplexExpand, PowerExpand, etc. should not obscure the fact that about the only way to tell if an utterance in the language is syntactically or semantically meaningful is to type it into the system and (in some cases) check the results by some independent means.

(d) There is insufficient information to provide an explanation of what parts of mathematics are "known" to the system. There are numerous "built-in" mathematical functions from the natural logarithm to generalized hypergeometric functions. In some cases the system can do no more than evaluate the function at a numeric point. Some functions can be differentiated. Some can appear in integrands. Some can be rewritten in terms of other functions or simplified in various ways. Usually, such implicit transformations are undocumented, and implicit or explicit transformations may make unwarranted range assumptions so as to be, in fact, incorrect. Although there are many instances in which Mathematica gives wrong answers, Wolfram admits such a possibility on page 89, only with respect to definite integrals: "... Mathematica may give results, which, while formally correct, will lead to incorrect answers when you substitute particular values for variables." One need not be devious to find problems. In a problem recently given to calculus students here, Mathematica failed to find the arc length of the parametric curve $c(t) := (t - \sin t, 1 - \cos t)$ from $t = 0$ to 2π.

(e) There is an insufficiently precise description of the programming language. It is nearly self-evident that it is important to have a clear description of an algorithmic language. This is vital in Mathematica because many users inevitably find it necessary to build up collections of procedure definitions to augment the built-in interactive commands. Since the language draws from many previous languages, several equivalent programming "paradigms" coexist. The choice of the wrong technique can make a program quite slow. In fact the favored technique of using patterns and rules is appealing in simple cases, but it can cause subtle errors, inconsistencies and gross inefficiencies.

A computer scientist trying to understand the language may be hindered by the omission of a formal definition of the syntax in Backus-Naur Form (BNF). Wolfram's treatment of modules, blocks, contexts, and packages is confusing, but perhaps no more so than is required by the apparently confused design of scopes, evaluation, and variables. For example, the system does not understand that the scope of the x in $\int f(x)\,dx$ is local to the integral.

Other books, especially those by Maeder [6] and Blachman [1] provide alternative perspectives on the language and point out some traps and pitfalls for the unwary. A simple such example is that no distinction is made between a column-vector and a row-vector.

Although we have tried to restrict our comments here to the book under review, readers may wish to refer to reviews of the Mathematica *program* as well. Simon [7] compares Mathematica to several other systems in terms of computational speed and convenience on a set of problems. Another review [5] discusses at length how well the program fulfills the goals set out by the designers.

Readers who have become aware, through Mathematica, of the general capabilities of symbolic manipulation programs may find it beneficial to examine literature on alternative systems such as Derive, Macsyma, Maple, Reduce, and Theorist. Determining the "best" system is necessarily dependent on individual circumstances. At the very least, it appears that some of these other systems get correct answers when Mathematica does not. Quoting further specific flaws or discrepancies between the documentation and the system may be pointless because the behavior of the system may be changed freely.

What we have observed is consistent with the disclaimer on the inside title page: "The author, Wolfram Research and Addison-Wesley shall not be responsible under any circumstances for providing information on or corrections to errors and omissions discovered at any time in this book or the software it describes, whether or not they are aware of the errors or omissions. The author, Wolfram Research and Addison-Wesley do not recommend the use of the software described in this book for applications in which errors or omissions could threaten life, injury or significant loss."

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6. Roman Maeder, *Programming in Mathematica*, Addison-Wesley, 1989.
7. Barry Simon, *Four computer mathematical environments*, Notices Amer. Math. Soc. **37** (1990), 861–868.

13[68–06].—DAVID V. CHUDNOVSKY & RICHARD D. JENKS (Editors), *Computers in Mathematics*, Lecture Notes in Pure and Appl. Math., Vol. 125, Marcel Dekker, New York and Basel, 1990, vii + 410 pp., 25 cm. Price $89.75.

This volume contains 14 contributions to the International Conference on Computers and Mathematics, which took place July 29–August 1, 1986, at Stanford University. The papers deal with the role of computers in subjects as diverse as number theory, analysis, special functions, algebraic geometry, topology and mathematical physics. Topics include: factorization, polynomial root finding, algorithms for solving differential equations, computer animation, automated theorem proving, symbolic computation, computer algebra, and several others.

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