REVIEWS AND DESCRIPTIONS OF TABLES AND BOOKS

The numbers in brackets are assigned according to the American Mathematical Society classification scheme. The 1991 Mathematics Subject Classification can be found in the annual subject index of *Mathematical Reviews* starting with the December 1990 issue.

1[65–02, 65Gxx, 65Nxx, 65Rxx, 65F05, 65F10].—SOLOMON G. MIKHLIN, *Error Analysis in Numerical Processes* (Translated from the German edition by Reinhard Lehmann), Pure and Applied Mathematics, A Wiley-Interscience Series of Texts, Monographs, and Tracts, Wiley, Chichester, 1991, 283 pp., $23\frac{1}{2}$ cm. Price \$87.95.

This book is an extended and somewhat reorganized translation of the author's book "Fehler in numerischen Prozessen", published by the Akademie-Verlag Berlin in 1985. In it, a classification of errors in numerical methods into four different types, named the approximation error, the perturbation error, the algorithm error, and the rounding error, is used and applied in several of the major problem areas in numerical analysis. In the case of the standard finite element method for an elliptic boundary value problem, the approximation error is the error between the exactly computed solution of the discretized problem and the continuous solution, the perturbation error results from not solving the discretized problem exactly, for instance by using quadrature rules for the inner products occurring in its formulation, an algorithm error could be the error resulting from using a finite number of iterations in an iterative method for solving the system of linear equations, and the rounding error, of course, derives from the finite precision in the computer used. The discussion of the perturbation error is based on notions of stability of numerical processes with respect to associated sequences of pairs of Banach spaces.

The different types of error are analyzed for various numerical approaches to ordinary and partial differential equations and to integral equations, including singular ones, and for direct and iterative methods in linear algebra. In the differential equations area, variational methods are emphasized but finite difference and collocation methods are also touched upon.

The latter part of the book is devoted to numerical methods for nonlinear problems such as unilateral variational problems. One of the chapters in this part contains an appendix written by B. V. Tyukhtin, surveying estimates for the approximation error in finite element methods applied to obstacle problems, which have been developed in the Western literature.

Most of the material of the book is quoted from earlier books and papers by the author and members of his school. Sometimes one discerns a certain lack of regard for relevant work from outside this circle. In reading the book,

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however, one has to be impressed by the width of the scope of the study and the very considerable mathematical culture of academician Mikhlin. This makes for quite pleasant reading.

V. T.

2[65-01, 65N30].—BARNA SZABÓ & IVO BABUŠKA, *Finite Element Analysis*, Wiley, New York, 1991, xv +368 pp., 24 cm. Price $59.95.

Babuška and Szabó have pioneered the use of high-order polynomials as an alternative to mesh refinement in the finite element method. That this is a significant part of the subject of the book is therefore no surprise. However, given the different primary disciplines represented by the two authors, one could not predict a priori whether such a book would take a mathematical or an engineering approach, or perhaps some novel combination of the two. The order of authorship hints that an engineering approach is to be emphasized, and this is reflected in the "matrix method" notation that is used, but the use of mathematical ideas is also an essential part of the text.

The goals of the book are best stated by the authors themselves in the Preface.

> Our purpose in writing this book is to introduce the finite element method to engineers and engineering students in the context of the engineering decision-making process. Basic engineering and mathematical concepts are the starting points. Key theoretical results are summarized and illustrated by examples. Focus is on the developments in finite element analysis technology during the 1980s and their impact on reliability, quality assurance procedures in finite element computations, and performance. The principles that guide the construction of mathematical models are described and illustrated by examples.

Numerous books on the finite element method with a variety of objectives have appeared recently, so many that it would be quite lengthy to compare even a representative number of them. However, we will venture one comparison with a mathematical audience in mind. The present book has extensive detail with regard to examples, and its coverage of topics in linear elasticity is exhaustive. Both of these are essential for the book to be successful with an engineering audience. The book by Claes Johnson [2], on the other hand, offers a more conventionally mathematical approach to the subject. That book covers a more diverse set of topics than do Babuška and Szabó, but it lacks details of computer implementation and omits many practical issues covered by them.

In addition to providing an up-to-date survey of high-order-polynomial methods, the book includes other material not previously available in textbook form. The book presents one chapter on efficient computation of stresses using post-processing techniques to enhance the quality of the solution, and another on error estimation and control.

There is a chapter on "Miscellaneous Topics" that gives some hints about areas for further study. Included are a discussion of mixed methods and a sample nonlinear problem. However, there are some standard topics not covered in the book that may be of interest to students, such as that of viscous fluid flow

[1] (although there is one chapter on potential flow) and nonlinear models in solid mechanics, such as plasticity.

L. R. S.

1. Max D. Gunzburger, *Finite element methods for viscous incompressible flows*, Academic Press, Boston, 1989. [Review **40**, Math. Comp. **57** (1991), 871–873.]

2. Claes Johnson, *Numerical solutions of partial differential equations by the finite element method*, Cambridge Univ. Press, 1987. [Review **1**, Math. Comp. **52** (1989), 247–249.]

3[65Cxx, 65Mxx, 65Nxx].—W. E. SCHIESSER, *The Numerical Method of Lines: Integration of Partial Differential Equations*, Academic Press, San Diego, 1991, xiii + 326 pp., $23\frac{1}{2}$ cm. Price $69.95.

The numerical method of lines for time-dependent PDEs consists of forming a spatially discrete system of ordinary differential (or possibly differential-algebraic) equations in time and then calling a suitable ODE or DAE integrator. The method is quite powerful and versatile, and is widely used in conjunction with the wide array of high-quality ODE and DAE solvers now available.

This book presents, mainly by example, one approach, or paradigm, for the numerical method of lines (NUMOL). It fills in the details of the method by establishing a complete Fortran program template in which the discrete representation of spatial operators is done by finite differences in a certain set of Fortran software (the DSS routines), and the coding for problem specification and I/O is to be inserted into a few specific spots, with heavy use of certain COMMON structures. Relegating the discretization to a set of black boxes and imposing a structure on the problem-specific coding makes it relatively straightforward to set up and solve a problem. But the price for this is a certain loss of flexibility in the areas of spatial discretization, boundary condition representation, and treatment of PDE systems.

Aside from certain limitations, and some minor errors and omissions, the book provides a very good introduction to the numerical method of lines. It assumes very little in the way of technical background of the reader. For example, the concepts of a PDE as a physical model, of Taylor series approximation, and of eigenvalues of a matrix, are all introduced from scratch as needed. Any scientist or engineer with a little Fortran background can read this book and quickly learn to solve some interesting problems. As an introduction to the subject, the book does not treat some of the more advanced aspects, such as mixed derivatives, irregular regions, or outflow boundary conditions, and it only briefly touches on the issues of nonuniform grid selection, differential-algebraic system problems, and the efficient treatment of large stiff systems. Fans of finite-element-type versions of the method of lines will not be accommodated by this book.

Chapter 1 does a good job of introducing the basic ideas of NUMOL, with the heat equation as the (much repeated) example. The various steps are explained in great detail, although some features of the procedure could use even more discussion.

A disturbing practice that appears in Chapter 1, and is continued later, is that of forming the discrete second derivative by two successive applications of a (central) first-derivative approximation ("stagewise"), rather than by direct differencing. This goes against the nearly unanimous opinion of the numerical PDE community. In the standard example, it makes the problem bandwidth five, instead of three, as in the standard central differencing, and has the potential for introducing spurious spatial oscillations (which can be demonstrated for the heat equation). Direct differencing for second derivatives is introduced in §3.4, but it is given less than equal emphasis. The relative simplicity and small size of the problems solved seems to allow stagewise differentiation to perform well here.

The first chapter ends by setting up the Fortran template for all subsequent examples. It is based on a largely fixed main program and interface routine FCN, problem-dependent routines INITAL, DERV, and PRINT; and a set of COMMON blocks with a fixed overall structure but problem-dependent details. Many readers will quibble with the programming style, but the template does do the job, and provides a start for the novice. One of the largest quibbles will be the nondynamic nature of the COMMON structure. Despite the appearance of problem size NEQN in the data file in all examples, this size cannot be varied at run time (except for a 1-D scalar PDE), because mesh sizes are built into the COMMON blocks. More serious is the fact that for a *system* of PDEs, the ordering of the dependent variables is by PDE variable first and then by mesh point, making the problem bandwidth far from minimal. In the stiff case (assuming fewer PDEs than mesh points), the solution with banded treatment of the Jacobian is far less costly if that ordering is transposed.

The issue of estimating and controlling the errors in a NUMOL solution is discussed, but somewhat too briefly. Early in Chapter 1 there is a good discussion of the important issue of setting error tolerances for the integrator. But it comes after an example where pure relative error control is specified on a problem that includes a vanishing solution component. Somehow this was not fatal, but it was partly to blame for high costs that are instead attributed to stiffness. Anyway, this mistake does not occur again after §1.6, as all subsequent examples use mixed tolerances. But the use of a scalar absolute tolerance throughout sets a dangerous example for one faced with a system having a wide range of magnitudes in the solution components. In Chapter 2, following the first nonlinear example, there is a short discussion on checking the validity of the NUMOL solution. It mentions use of physical intuition, but fails to mention an obvious numerical approach: refining the spatial and/or tightening the time integration tolerances. This usually gives a good idea of the error level, but of course is not guaranteed to.

The author's procedure for enforcing Dirichlet boundary conditions is strange at first sight, but as eventually adopted it is quite valid, though not thoroughly explained. Early on, this is done by resetting values of the dependent variable in the FCN routine, in violation of the usage instructions for most ODE integrators. However, with the introduction of the COMMON structure in §1.8, what is actually being done is to load boundary values into the appropriate components of the temporary arrays in COMMON/Y/, where they are used in the evaluation of all the other derivatives. The derivatives of those boundary variables are returned to the integrator as zero. Thus the ODE solver is integrating a dummy equation (with a constant solution) for each such boundary condition, while the true boundary value (which may or may not be constant) is absorbed into the remaining ODEs as needed. (Cases of nonconstant boundary values appear in §3.5 for ramp and pulse functions, and in Chapter 6 for the Burgers equation.) The correct boundary values get printed by the PRINT routine by virtue of the call to DERV just before the call to PRINT. In fact, this practice is not dangerous or erroneous, provided that it follows the given program structure. The alternative of having the ODE solver integrate the derivative of the boundary values can be impractical in the nonconstant case.

Chapter 2 gives several substantial examples that make the case for the ease of setup and application of NUMOL. These range from 1-D linear scalar PDEs to 2-D nonlinear systems, with hyperbolic and elliptic examples as well as parabolic. The wave equation example and the example with two coupled PDEs illustrate how two different forms (generic and problem-specific) of the COMMON blocks work together.

Chapter 3 is a primer on finite difference representations of 1-D derivatives, coupled with some of the DSS routines that implement them. Most of it assumes a uniform grid, though this is not clearly stated at the outset. For an introductory book, there is unexpected emphasis on higher-order differences. For example, the simplest case of direct second derivatives (three-point second-order, corresponding to DSS042) is not given, only the more involved five-point fourth-order case. Three-point differencing appears only in passing in §5.2. The final section of the chapter gives a good description of the obstacles involved with advection equations, and introduces noncentral difference schemes.

An interesting feature of all of the differentiation routines used is that they sacrifice bandwidth at the endpoints for the sake of preserving the order of accuracy. For example, for a parabolic problem with Neumann boundary conditions, DSS042 gives the standard 3-point second derivative approximation at each interior point, but also a 3-point difference expression at each boundary, making the system Jacobian pentadiagonal instead of tridiagonal. This tradeoff is probably a bad one in the stiff case, as the loss of efficiency is likely to offset the gain in accuracy at the boundary. In fact, one can easily have the best of both by keeping the tighter bandwidth but refining the mesh at the boundary if necessary, but doing this with the existing DSS routines would be awkward.

Chapter 4 does a good job of introducing the basic notions of explicit and implicit ODE methods, stability of ODE systems, and stability of numerical ODE methods. Moreover, it does this in a way that encourages the use of available ODE software. The important issue of stiffness in the NUMOL context is discussed, motivating the introduction of BDF methods. Following that are sections describing a number of available sophisticated ODE solvers. There are a few minor flaws in the presentation. In the sections on stability of the Euler methods, the equations connecting the problem eigenvalues λ to the characteristic growth factors β could have been obtained for *general* coefficients, by noting that the characteristic equation for β with the change of variable $\beta = 1 + \lambda\Delta t$ (explicit case) or $\beta = 1/(1 - \lambda\Delta t)$ (implicit case) is exactly the characteristic equation for λ. The section on BDF methods misstates the usual procedure for the initial guess; it is actually to extrapolate from existing data at order q, rather than use the base point. The section on the LSODE integrator incorrectly states that for a problem with banded coupling, programming a dense Jacobian is harder than a banded one; in fact both involve the same programming—of nonzero elements only.

The first two sections of Chapter 5 derive the von Neumann stability conditions for the advection equation (with centered and upwind differencing) and the heat equation with centered differencing. The results are correct, but the attempt to link them to actual NUMOL integrations is flawed. First, there is no note of the fact that, because the von Neumann analysis ignores boundary conditions, the actual eigenvalues of the ODE systems solved are somewhat different. There are two notes that attempt to make the connection in the case of the advection equation. But the first note incorrectly cites the real stability interval, whereas the better stability of RKF45 on the imaginary axis (vs explicit Euler) probably does account for its good performance. The second note gives undue credit to the variable stepsize algorithm for rescuing an otherwise unstable integration method. In all cases, the correct CFL condition is given, leading to conclusions on increasing stiffness with mesh refinement, but there is no discussion of convergence or Lax equivalence.

The bulk of Chapter 5 is devoted to the most complicated example problem in the book, a 1-D humidification column with three PDE variables plus a control variable. The example is noteworthy for its complexity and the fairly complete solution given. However, there is one improper feature, and a few omissions. The DERV routine includes lines that reset negative values of dependent variables. This is dangerous because it can make the integration unstable; the small negative values are harmless and if desired can be replaced by zero in the printing of output. The data lines displayed have four new entries ("1000 1 1 1 REL") that are not discussed. The computed spectrum displays damping modes with $\mathrm{Re}(\lambda) \sim -5000$, while the final time is $t = .5$. This raises questions of stiffness and efficiency that are not discussed. The solution was probably done with a nonstiff solver at a high price. The details of solving the problem with a stiff solver, including the use of the sparse structure of the Jacobian, would have made the example much more interesting and useful.

Chapter 6 begins with a heuristic classification of PDEs in terms of the appearance of dependencies, rather than rigorously in terms of discriminants of coefficient matrices. But for the novice this is probably more appropriate. The rest of the chapter gives various treatments of the Burgers equation that illustrate (again) handling different boundary conditions and setting up 2-D and 3-D problems. Finally, the adaptive grid solution of a Burgers equation provides an interesting and useful departure from all of the previous fixed-uniform-grid examples. But the program structure appears unable to permit the extension of the adaptive grid scheme to a system of PDEs.

In the appendices are (A) equations for the Laplacian in three coordinate systems, (B) a list of the DSS spatial differencing routines available, and (C) a list of over 200 applications from various disciplines, for which NUMOL solution programs are available.

Numerous typographical errors have crept into the book. I have a one-page list of corrections from the author, and have generated another (slightly longer) list in my own reading. Most of the errors are fairly innocuous, but a few are not. For example, in an exercise at the end of Chapter 1, the coding given for a modified Euler method has two errors (one in the time variable in the FCN call, and one in the dependent variable in the final loop). An error of a different sort was also made in my copy of the book: the binding was attached upside-down!

A. C. HINDMARSH

Mathematics Division
Lawrence Livermore Laboratory
Livermore, CA 94551

4[34–00, 35–00].—DANIEL ZWILLINGER, *Handbook of Differential Equations*, 2nd ed., Academic Press, Boston, 1992, xx + 787 pp., $23\frac{1}{2}$ cm. Price $54.95.

The first edition of this book was published in 1989 and has been reviewed in [1]. That a second edition is appearing just three years after the first attests to the success of, and continued demand for, the book.

The principal changes made by the author are as follows. In Part I, dealing with basic concepts and transformations, new paragraphs have been added on chaos in dynamical systems, existence and uniqueness theorems in ODEs and PDEs, inverse problems, normal form of ODEs, and stability theorems for ODEs. Prüfer and modified Prüfer transformations, which originally appeared in Parts II and III, respectively, have been moved to Part I. Part II on exact methods has a new paragraph on exact first-order PDEs. The most extensive changes occur in Part IV, dealing with numerical methods, where one finds a reworked paragraph on available software, a long new paragraph on software classification, including excerpts from the GAMS manual, and new sections on finite difference methodology, grid generation, stability concepts in numerical ODEs, multigrid methods, parallel computer methods, and lattice gas dynamics (particle methods). In addition, many minor improvements have been made throughout the book: new examples, additional notes, and updated bibliographies. All in all, the text has expanded from the original 635 pages to 760 pages.

It should be clear from this brief review that the new edition of this reference work continues to be a useful aid to scientists and engineers and will be indispensable to anybody who needs to solve differential equations.

W. G.

1. W. F. Ames, Review **1**, Math. Comp. **54** (1990), 479–480.

5[65–06, 65Y05, 68–06].—JACK DONGARRA, PAUL MESSINA, DANNY C. SORENSEN & ROBERT G. VOIGT (Editors), *Parallel Processing for Scientific Computing*, SIAM, Philadelphia, PA, 1990, 454 pp., $25\frac{1}{2}$ cm. Price: Softcover $49.50.

This collection of 83 papers and short abstracts from the 1989 SIAM Conference on Parallel and Scientific Computing covers five areas: matrix computations, numerical methods, differential equations, massive parallelism, and performance and tools. Papers range from theoretical studies to performance evaluation to descriptions of software systems. Many of the major researchers in these fields are represented, and these papers give a good overview of research in this fast-changing area as of 1989. Many of the topics are still current, and thus many of the papers remain valuable. In this short review we will simply list the topics covered, since the number of papers is too large to mention each one individually.

In the section on matrix computations there are papers on block algorithms for dense matrix problems, sparse ordering and factorization algorithms, symmetric and nonsymmetric eigenvalue problems using QR, Lanczos, and bisection algorithms, and condition estimation.

The numerical methods papers cover bifurcation computations, asynchronous PDE solvers, linear and nonlinear optimization, homotopy methods, weather modeling, Navier-Stokes solvers, power-flow computations, ODE solvers, and parallel FFTs.

The differential equations papers cover multicolor elliptic solvers, fast Poisson solvers, elliptic solvers using domain decomposition, implicit and explicit parabolic solvers, stiffness matrix generation, nonlinear hyperbolic solvers, and aerodynamic applications.

The section on massive parallelism includes finite element computations, computational fluid dynamics, computing sparse approximate inverses, transportation optimization and molecular dynamics on the CM-2, load balancing, interprocessor connection networks, scheduling recurrence solvers, solving systems of conservation laws, the DINO language, and systolic arrays.

The section on performance and tools covers the PARTI runtime support system, automatic blocking of linear algebra codes, the CONLAB parallel simulator, the Linda coordination language, the Seymour data parallel language, performance modeling, and workload metrics.

J. W. D.

6[65D07, 65Dxx, 41A15].—Willard M. Snyder & Richard H. McCuen, *Numerical Analysis With Sliding Polynomials*, Lighthouse Publications, Mission Viejo, California, 1991, x + 561, pp., $25\frac{1}{2}$ cm. Price. $58.00.

"Sliding polynomials" are piecewise polynomial functions of a special form. The authors emphasize two types, the "four-point" and the "six-point". The former can be described as follows. An increasing set of knots x_i is given, accompanied by corresponding ordinates, y_i. On a typical interval $[x_i, x_{i+1}]$, the four-point sliding polynomial will be a cubic polynomial p determined by the four conditions $p(x_i) = y_i$, $p(x_{i+1}) = y_{i+1}$, $p'(x_i) = a$, $p'(x_{i+1}) = b$. Here a is the slope at x_i of the quadratic interpolant of the ordinates at x_{i-1}, x_i, and x_{i+1}. The value b is the slope at x_{i+1} of the quadratic that interpolates the given ordinates at x_i, x_{i+1}, and x_{i+2}. The definition leads to a composite function that is of class C^1. The six-point sliding polynomial is similar; it is a piecewise quintic polynomial and is of class C^2.

The book emphasizes the case of equally-spaced knots, and provides formulas and codes for doing various tasks with these sliding polynomials in the equally-spaced case. There are ten chapters, dealing with such topics as numerical differentiation, numerical integration, smoothing, contouring, differential equations, integral equations, and finite elements. Multidimensional sliding polynomials are needed for the latter. Each chapter contains many examples employing realistic data. Appendix A (17 pages) gives numerical constants useful in working with these functions. Appendix B (140 pages) gives program listings in the language BASIC.

E. W. C.

7[65–06, 65D07, 65D17, 68U05].—GERALD FARIN (Editor), *NURBS for Curve and Surface Design*, SIAM, Philadelphia, PA, 1991, ix + 161 pp., $25\frac{1}{2}$ cm. Price: Softcover $33.50.

This book contains a collection of twelve papers on the theory and application of NURBS. Most of the papers are based on lectures given at a SIAM conference on geometric design held in Tempe, Arizona, in 1990.

The word NURBS is an acronym for *nonuniform rational B-splines*. The use of this name perpetuates a problem of nomenclature which has arisen regarding the term B-spline. Strictly speaking, B-splines are nonnegative locally supported, smooth piecewise polynomial functions with some very special properties which make them ideal *basis* functions for certain linear spaces S of polynomial splines. They go back to a paper of Schoenberg in 1946. The idea is that each spline s in S can be written uniquely as a linear combination of the given B-splines. Unfortunately, for many in the CAGD community, a B-spline is the linear combination itself. To help avoid confusion, such an object is sometimes referred to as a *B-spline curve*. They are of considerable interest as tools for CAGD. For many applications, however, it turns out to be useful to consider the more general class of curves which arise when each coefficient c_i in the B-spline expansion is multiplied by a weight w_i, and the overall expansion is divided by the weighted sum of the basis functions. Curves of this type are called NURBS. They are piecewise rational functions, where the term *nonuniform* refers to the fact that the basis splines may be constructed on a nonuniform knot sequence. Surfaces can also be modelled using NURBS in a standard tensor-product framework.

NURBS have several advantages for CAGD applications, such as the fact that conics can be exactly represented, and there are many who would argue that they are becoming the standard working tool in industry. The aim of this collection of papers is to contribute to the mathematical development of the theory of NURBS. Topics treated include Bézier patches on quadrics, G^1 surface interpolation over irregular meshes, curves and surfaces on projective domains, reparameterization and degree elevation, constrained interpolation, parametric triangular patches based on generalized conics, generalized NURBS surfaces, the rational Overhauser curve, B-spline and Bézier representations, linear fractional transformations, curvature-continuous NURBS, and approximation of NURBS by polynomial curves. The authors of the individual papers are W. Boehm & D. Hansford, H. Chiyokura, T. Takamura, K. Konno & T. Harada, T. DeRose, G. Farin & A. Worsey, T. Goodman, B. Ong & K. Unsworth, B. Hamann, G. Farin & G. Nielson, D. Hoitsma & M. Lee, J. Jortner, D. Lasser & A. Purucker, M. Lucian, H. Pottmann, and T. Sederberg & M. Kakimoto.

LARRY L. SCHUMAKER

Department of Mathematics
Vanderbilt University
Nashville, TN 37235

8[14Qxx, 65–06, 65D17, 68U05].—ROBERT E. BARNHILL (Editor), *Geometry Processing for Design and Manufacturing*, Geometric Design Publications, SIAM, Philadelphia, PA, 1992, x + 211 pp., $25\frac{1}{2}$ cm. Price: Softcover $36.50.

This book contains a collection of nine papers relating to geometry processing, along with a bibliography on surface-surface intersection methods. Most of the papers are based on lectures given at a SIAM conference on geometric design held in Tempe, Arizona, in 1989. The editor defines geometry processing to be the calculation of geometric properties of already constructed curves, surfaces, and solids. His stated aim for the book is to spur a unifying development of the subject.

We paraphrase here the editor's descriptions of the papers: 1) R. Farouki develops special offset curves which can be parameterized by rational functions, 2) R. Barnhill, T. Frost, and S. Kersey use a combination of geometric and numerical techniques to find self-intersections of networks of general triangular or rectangular parametric patches, 3) J. Hoschek and F.-J. Schneider develop suitable approximations for conversions between B-spline surfaces, 4) G. Farin compares knot removal and degree reduction as tools for fairing B-spline curves and surfaces, 5) E. Brechner presents general envelope methods for determining offsets, 6) R. Barnhill, B. Bloomquist, and A. Worsey develop adaptive contouring algorithms for the contouring of surfaces which are networks of triangular polynomial patches, 7) L. Piegl discusses algorithms for dealing with surfaces having special mathematical forms (such as natural quadrics and extruded surfaces), 8) N. Patrikalakis develops surface-surface intersection algorithms for the implicit-parametric and parametric-parametric cases, and 9) K. Wang discusses intersection problems for rational parametric surfaces. The SSI bibliography of G. Farin contains about 50 articles.

LARRY L. SCHUMAKER

Department of Mathematics
Vanderbilt University
Nashville, TN 37235

9[49M45, 49M40, 49M37, 03D15].—STEPHEN A. VAVASIS, *Nonlinear Optimization: Complexity Issues*, The International Series of Monographs on Computer Science, Vol. 8, Oxford Univ. Press, New York, 1991, x + 165 pp., 24 cm. Price $39.95.

This is a well-written, concise, and accurate book covering a range of subjects in nonlinear optimization and complexity. The writing style is lively and holds the reader's attention.

Chapter 1 contains introductory material concerning convexity and optimality conditions. Chapter 2 starts with a brief introduction to complexity theory, Turing machines and models of computation, the P and NP classes of problems, and NP-completeness. In addition, several problems are proved to be NP-complete, including the general quadratic programming problem.

Chapter 3 is devoted to convex quadratic programming. The author starts with a nice example, presenting a strongly polynomial-time algorithm for separable quadratic knapsack problems. Next, an interior-point algorithm is given for the general convex quadratic minimization problem. The interior-point algorithm presented (first proposed by Renegar and Shub) is a primal dual method with logarithmic barriers. It follows a central path and takes small steps. The chapter ends with a discussion on strongly polynomial-time algorithms.

In the case of nonconvex quadratic programming, there is no known, efficient, general-purpose algorithm for computing the global minimum, since this problem is NP-complete. Chapter 4 begins with special cases of nonconvex quadratic problems that remain NP-complete, such as problems defined on the unit simplex and problems with box constraints. The problem of minimizing a quadratic function with an ellipsoid constraint is then considered. Although robust algorithms have been proposed (e.g., in trust region methods) for this problem, the complexity of the problem was not analyzed until Ye provided a polynomial-time algorithm in 1988. A similar algorithm is also credited to Karmarkar in 1989. The chapter ends with a discussion on a simple enumerative algorithm for the general indefinite quadratic problem.

Local optimization and complexity is the focus of Chapter 5. In the first section of the chapter, it is shown that the general problem of local optimality for nonconvex quadratic problems is NP-hard. This is a surprising result since, in many classical optimization algorithms, it has been assumed that local optima are easy to find. In fact, the reviewer has shown that even the problem of checking the existence of a Kuhn-Tucker point for a nonconvex quadratic problem is also NP-hard. In the other two sections of the chapter some theorems are proved that characterize local minima, and a strongly polynomial algorithm is presented for computing a local minimum of separable quadratic knapsack problems.

The last part of the book (Chapter 6) is dedicated to the black box model. In many practical optimization problems the objective function $f(x)$ is not provided analytically, but is available as a black box; that is, given x, the black box returns the value $f(x)$. In this model, it is possible to prove lower-bound complexity results based on how much information an algorithm can obtain about the objective function (information-based optimization). In the first section, it is shown that the worst-case complexity for global minimization, in the information model, is exponential in the number of variables and the number of digits of accuracy. It is then shown that, in the case of local minimization, the dependence on the number of digits remains exponential, but the running time is polynomial in the number of variables. Regarding convex minimization, however, it is shown that there is an efficient information-based algorithm for this problem.

Each chapter of the book ends with a list of carefully selected problems that complete and extend the material presented. As we can see, the book presents interesting material regarding complexity issues in nonlinear optimization. I highly recommend this book. It will be a valuable source for researchers and students in nonlinear optimization. The detailed index and the rich bibliography enhance the usefulness of this elegant publication.

PANOS PARDALOS

Department of Industrial Engineering
University of Florida
Gainesville, FL 32611

10[65–02, 65C05, 65C10].—ISTVÁN DEÁK, *Random Number Generators and Simulation*, Mathematical Methods of Operations Research, Vol. 4, Akadémiai Kiadó, Budapest, 1990, 342 pp., $24\frac{1}{2}$ cm. Price $46.00.

Several books on random number generation and simulation methods have appeared in recent years, so the potential reader of this one will want to know, first of all, what distinguishes it from the rest of the crowd. A prominent feature of this monograph is certainly the strong emphasis on computational and algorithmic aspects. All algorithms for random number and random variate generation and other computational procedures are carefully described in terms of step-by-step prescriptions and are often followed by examples with illustrative numerical data. Furthermore, the multivariate case receives a more thorough treatment in this book than is customary. In view of the current trend towards parallelization in simulation methods, this is definitely a very useful and timely feature. The reader will also find some well-chosen practical examples of simulation methods, such as the optimal water level regulation of a lake which is modeled by a stochastic programming problem.

The book starts out with providing background on probability theory and statistics in Chapter 1. The next chapter is mainly devoted to uniform pseudorandom numbers, but there is also a brief discussion of quasi-Monte Carlo methods and quasirandom points. Chapter 3 turns to general methods for random variate generation, both for continuous and for discrete distributions. A detailed treatment of random variate generation for special classes of distributions, such as normal distributions, exponential distributions, and beta distributions, is given in Chapter 4. Methods for random vector generation, including methods for generating uniformly distributed points in special domains such as simplices, balls, and spheres, are discussed in Chapter 5. Chapter 6 contains the basics of the Monte Carlo method as well as special sampling techniques and methods for variance reduction, while Chapter 7 describes several Monte Carlo techniques developed by the author for computing the distribution function of multivariate normal distributions. The last chapter offers a cross section of various types of applications of simulation methods, ranging from large systems of linear equations to simulated annealing.

With its ample supply of examples and its many useful hints for actually carrying out simulations, the book is geared towards the practitioner. The author takes great care to lead the applications-oriented reader to a stage where he/she can implement simulation methods concretely. The underlying theory is treated only to the extent that is necessary to understand the algorithms.

H. N.

11[68–01, 68Q40].—F. BRACKX & D. CONSTALES, *Computer Algebra with LISP and REDUCE: An Introduction to Computer-Aided Pure Mathematics*, Mathematics and Its Applications, Vol. 72, Kluwer, Dordrecht, 1991, xii + 264 pp., $24\frac{1}{2}$ cm. Price $54.00/Dfl.160.00.

The REDUCE Computer Algebra system has a long history of wide distribution on a variety of computers. Its international community continues to use and improve the program, under the coordination of its original author, A. C. Hearn at the RAND Corp.

This text, which is based on a series of lectures on the LISP programming language and on REDUCE, appears to target an audience of persons who have acquired a REDUCE system but are (a) completely unfamiliar with algorithms for symbolic computation, and are (b) interested in seeing some applications.

Chapter 1 is a very brief (17-page) introduction to computer algebra systems' general capabilities. See Buchberger et al. [1] for a collection of more accurate reports and bibliographic information.

Chapter 2 (88 pages) describes "Standard LISP". Since REDUCE is written in this language (actually, Portable Standard LISP), it is necessary to know LISP to gain an in-depth understanding of the internal operation of REDUCE. Yet persons learning LISP for the first time should certainly seek an alternative to this treatment. The authors dwell on those features that should be avoided in writing programs in LISP, yet ignore important concepts such as data abstraction.

Since Brackx and Constales go to the effort of presenting LISP, one might reasonably expect to learn how the system REDUCE is written, preferably described in layers of abstraction covering up the gritty implementation details. Such a description could demonstrate to the novice how easy it is to (say) differentiate expressions. But this is entirely missing.

Chapter 3 is a summary of the REDUCE user manual.

Chapter 4 is a collection of brief programming examples, and Chapter 5 is a 50-page description of a set of programs dealing with Euclidean geometry (for example, given a certain description of a figure, rotate it). The 9-entry bibliography and the 6-page index do not strengthen the book.

I found the title of this book misleading since there is virtually no description of how REDUCE performs any of its computer algebra, nor is there any indication of what is easy or difficult to automate in mathematics generally, or why. The promise of an "introduction to computer-aided pure mathematics" is unfortunately not fulfilled.

RICHARD J. FATEMAN

Computer Science Division, EECS Dept.
University of California
Berkeley, CA 94720

1. B. Buchberger, G. E. Collins, and R. Loos (eds.), *Computer algebra: Symbolic and algebraic computation*, 2nd ed., Springer-Verlag, Wien and New York, 1983.

12[13Pxx, 14Qxx, 20B40, 20C40, 68Q40].—G. M. PIACENTINI CATTANEO & E. STRICKLAND (Editors), *Topics in Computational Algebra*, Dordrecht: Kluwer Academic Publishers, 1990, iii + 261 pp., $24\frac{1}{2}$ cm. Price \$99.00/Dfl.160.00.

Computational algebra is a quickly expanding area, in which good books are still rare. Hopeful expectation put me in a forgiving mood when I started to review this book.

Therefore, the graphic design of *Topics in Computational Algebra* will not be ascribed to the lack of taste of anybody in particular; the amateuristic artwork on the cover (showing a personal computer that must have looked old-fashioned at least a decade ago) starkly contrasts with the choice of expensive acid-free paper and a plasticized hard cover by the publisher. Likewise, I am willing to assume that the use of at least a dozen different fonts and a bottom margin that allows you to prove just about every theorem in it are consequences of the fact that this book is a reprint of the numbers 1 & 2 of Vol. 21 of *Acta Applicandae Mathematicae.*

We should be grateful to the editors Piacentini Cattaneo and Strickland for putting so much energy in first organizing a Semester on Computational Algebra in Rome, and next squeezing enough papers out of ten invited speakers to fill this book with. Surely, after coming up with an appropriate title, they could impossibly have found the time to write a proper Introduction to the book, for instance to substantiate their view (expressed in the half-page Foreword) that "the Semester largely succeeded in reaching its intended objectives", which were "to give an update on interesting techniques based on algorithms in different branches of algebra, with the idea of emphasizing the computational aspects of each branch". Mind you, they do not dare to state that the same objective applies to this book. (In fact the task of finding a common denominator to the papers is a hard one and reminded me of the impossible high school exams for which one had to discover a 'theme' in some experimental novel.) Hence, one can hardly blame contributors for not submitting the material that best suited these objectives. At least in one case such material did exist, even though it is not presented here, but luckily we are reassured by Neubüser that one can recover most of it by tracing back the literature in the bibliography to his joint paper with Celler and Wright, *Some remarks on the computation of complements and normalizers in soluble groups.* This paper, in which new methods and implementations in the Aachen GAP system for the calculation of classes of complements of a normal subgroup and of normalizers of a subgroup in a soluble group are described, and compared with existing implementations in the CAYLEY language, could certainly make a nice article in a journal on algorithms and computations in algebra. But in this book I would have preferred to see the survey that Neubüser apparently delivered in three lectures under the promising title *Computing in Soluble Groups* in another form than a list of references.

The next paper (*Methods for computing in algebraic geometry and commutative algebra* by Stillman) does not even give us a complete list of references ("Find Reference" is listed as item [12] instead)—however, we will not blame the author for this, nor for photocopying three pages of computer listings with his remarks scribbled in the margin, making a fresh copy of the book look secondhand already. But at least this contribution does not need a bibliography to make it an 'interesting and suitable update on interesting techniques', in this case all based on Gröbner basis calculations, 'with the idea of emphasizing the computational aspect'. On his way, the author explains what the main goals are in computational algebraic geometry and commutative algebra, and how these are achieved in the Macaulay system (of Stillman and Eisenbud). As an example the algorithm for calculating the radical of an ideal in a polynomial ring is described. In *Computing with characters of finite groups* by H. Pahlings, the application of another important technique, the 'LLL' method of finding short vectors in lattices, to the calculation of ordinary characters of finite groups is very clearly outlined. N. White goes out of his way to explain the importance of Cayley algebras and Cayley factorization of bracket polynomials in the algebraic interpretation of geometric statements, in his *Cayley factorization and a straightening algorithm.*

To offset these three relevant papers, there are two or three whose inclusion in the book I found hard to justify, no matter how lenient the mood. Formanek's *The Nagata-Higman theorem* describes the history of the theorem which states that if in an algebra over a field of characteristic zero every nth power vanishes, then for some $N = d(n)$ every product of N elements vanishes. This theorem is 35 years old and with respect to the only aspect that with my good will I could call computational (that of determining $d(n)$) no progress is reported in 15 years: but "the reader may feel that, given the inequality $n(n+1)/2 \le d(n) \le n^2$ the exact value of $d(n)$ no longer is of sufficient interest", as Formanek puts it. Perhaps afraid that the same qualification would apply to his contribution, he concludes with a vaguely related (but at least recent) result concerning Engel conditions. Although obtaining multiplicities of representations in a decomposition of an irreducible representation of a group with respect to a subgroup could have a computational flavor, Kac and Wakimoto succeed very well in avoiding calculations and mention of the word algorithm in the description of the particular case they are interested in (*Branching functions for winding subalgebras and tensor products*). Especially since this is the very first paper of the volume it seems a trifle too specialistic: in the smallest of all fonts used in the book, the Introduction and the Preliminaries take up almost half of the 37 pages of the paper. On the other hand, *Supersymmetric bracket algebra and invariant theory* could have done with an introduction, for instance to highlight the computational and the new aspects of the subject to the reader, but the authors (Huang, Rota and Stein) may have considered that quite rightly to be a task for the editors.

Buchsbaum, in *Aspects on characteristic-free representation theory of* GL_n*, and some application to intertwining numbers*, explicitly "tried to choose topics that indicate a strong bridge between representation theory and computational algebra": he illustrates the use of Schur complexes in the calculation of certain intertwining numbers. In *The expansion of various products of Schur functions*, Remmel outlines recent developments on efficient combinatorial algorithms for computing various products of symmetric functions, and the application to the representation theory of the symmetric group. Finally, Drensky purports to give a quantitative description of the polynomial identities of algebras of 2×2 matrices over a field of characteristic zero, without using invariant theory.

Perhaps you think this is not a proper book review; well, I am of the opinion that this is not a proper book: some collections of papers, however well-intended, put (and bound) together, still don't make up more than a journal. Hopefully you agree with me, in a final flattering moment of forgiving, that this is just as good a review as any journal deserves.

WIEB BOSMA

School of Mathematics and Statistics
University of Sidney
Sydney, NSW 2006
Australia